DEFINITE INTEGRALS – CLASS XII

SUJITHKUMAR KP

1. $\int_a^b f(x)dx = \int_a^b f(t)dt$	2. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx,$ $a < b < c$	4. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
5. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$	6. $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
7. $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, f(2a-x) = f(x)$	8. $\int_0^{2a} f(x)dx = 0,\ f(2a-x) = -f(x)$
9. $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx, if\ f(-x) = f(x)$	10. $\int_{-a}^a f(x)dx = 0, if\ f(-x) = -f(x)$

1 The value of $\int_0^{\frac{\pi^2}{4}} \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx =$ ______

(A) -2 (B) 1 (C) 2 (D) π

ANS: (C) 2

2 Evaluate : $\int_1^3 \frac{cos(logx)}{x} dx.$

(A) $cos(log3)$ (B) $sin(log3)$ (C) $\cos 3$ (D) $-sin(log3)$

ANS: (B) $sin(log3)$

3 Evaluate : $\int_0^{\frac{\pi}{2}} \frac{dx}{1+sinx} =$ ____

(A) -2 (B) 1 (C) 2 (D) 0

ANS: (B) 1

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1+sinx} = \int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)} = \int_0^{\frac{\pi}{2}} \frac{1}{2}\, sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right) dx = \frac{1}{2}.\left[\frac{tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{-\frac{1}{2}}\right]_0^{\frac{\pi}{2}} = -tan(0) + tan\left(\frac{\pi}{4}-0\right)$$

$= 1$

4 Let $l_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}\ and\ l_2 = \int_1^2 \frac{dx}{x},\ then$

(A) $l_1 = l_2$ (B) $l_1 > l_2$ (C) $l_1 < l_2$ (D) $l_1 > 2l_2$

ANS: (C) $l_1 < l_2$

$$1 + x^2 > x^2 \ \Rightarrow\ \sqrt{1+x^2} > x \quad x \in [1,2]$$

$$\Rightarrow \frac{1}{\sqrt{1+x^2}} < \frac{1}{x} \ \Rightarrow\ \int_1^2 \frac{dx}{\sqrt{1+x^2}} < \int_1^2 \frac{dx}{x}, \quad \Rightarrow \quad l_1 < l_2$$

5 Evaluate : $\int_8^{13} \frac{\sqrt{21-x}}{\sqrt{x}+\sqrt{21-x}} dx =$ ___

(A) -5 (B) 5 (C) $\frac{5}{2}$ (D) 10

ANS: (C) $\frac{5}{2}$ $I = \int_8^{13} \frac{\sqrt{21-x}}{\sqrt{x}+\sqrt{21-x}} dx = \int_8^{13} \frac{\sqrt{21-(21-x)}}{\sqrt{(21-x)}+\sqrt{21-(21-x)}} dx$

$$I = \int_8^{13} \frac{\sqrt{x}}{\sqrt{21-x}+\sqrt{x}}$$

Add $2I = \int_8^{13} dx = [x]_8^{13} = 5 \quad \therefore \quad I = \frac{5}{2}$

6 The value of $\int_0^2 x[x]dx =$ ___

(A) 3 (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{3}{4}$

ANS: (B) $\frac{3}{2}$

$\int_0^2 x[x]dx = \int_0^1 x[x]\, dx + \int_1^2 x[x] = 0 + \int_1^2 x\, dx = \left[\frac{x^2}{2}\right]_1^2 = \frac{1}{2}\,(4-1) = \frac{3}{2}$

7 Evaluate the following integral $\int_1^e logx =$ ___

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{5}{2}$ (D) -1

ANS: (A) 1

$\int_1^e logx\, dx = \int_1^e logx.1\, dx$

Apply Product rule,

$I = logx.x - \int_1^e x.\frac{1}{x}\, dx = [xlogx - x\,]_1^e = 1$

8 Evaluate : $\int_0^1 x(1-x)^{89}\, dx$

(A) $\frac{1}{8190}$ (B) $\frac{1}{8290}$ (C) $\frac{1}{8100}$ (D) 8190

ANS: (A) $\frac{1}{8190}$

9 Evaluate: $\int_1^2 e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)\, dx$

ANS: $\frac{e^2}{2} - e$

10 Assertions (A) : if $\int_0^a f(x)dx = 5$ then $\int_0^a f(a-x)dx = 5$

Reason (R) : $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

ANS: (A)

11 Evaluate: $\int_1^e \frac{1}{x\sqrt{1-(logx)^2}}\, dx$

A) 0 B) π C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$

ANS: C) $\frac{\pi}{2}$ $\int_1^e \frac{1}{x\sqrt{1-(logx)^2}}\, dx = \int_0^1 \frac{1}{\sqrt{1-t^2}}\, dt$

$= \sin^{-1} 1 - \sin^{-1} 0 = \frac{\pi}{2}$

Let $logx = t$

$\frac{1}{x}dx = dt$

$x = 1, t = 0$

$x = e\,, t = 1$

12 Evaluate : $\int_0^{\frac{\pi}{4}} (tan^2x + tan^4x)\, dx$

ANS: $\int_0^{\frac{\pi}{4}} (tan^2x + tan^4x)\, dx$

$= \int_0^{\frac{\pi}{4}} tan^2x(1 + tan^2x)\, dx = \int_0^{\frac{\pi}{4}} tan^2x\, sec^2x\ \ dx$

$tanx = t\,,\ sec^2x\ \ dx\, dt$

$I = \int_0^1 t^2\ \ dt\ = \frac{1}{3}$

13 Evaluate : $\int_0^{\frac{\pi}{4}} \frac{tan^3x}{1 + cos2x}\, dx$

ANS: $I = \int_0^{\frac{\pi}{4}} \frac{tan^3x}{1 + cos2x}\, dx = \int_0^{\frac{\pi}{4}} \frac{tan^3x}{2cos^2x}\, dx \ = \frac{1}{2}\int_0^{\frac{\pi}{4}} tan^3x\,.sec^2xdx$

$tanx\ = t, sec^2xdx = dt, \quad x = \frac{\pi}{4}, \quad t = 1$ and $x = 0\ then\ t = 0$

$I = \frac{1}{2}\int_0^1 t^3dt = \frac{1}{8}$

14 Show that $\int_0^{\frac{\pi}{4}} \log(1+tan\theta)\,d\theta = \frac{\pi}{8}\,log2$

$I = \int_0^{\frac{\pi}{4}} \log(1+tan\theta)\,d\theta = \int_0^{\frac{\pi}{4}} \log\left(1+tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

$\int_0^{\frac{\pi}{4}} \log\left(1+\frac{1-tan\theta}{1+tan\theta}\right)d\theta = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+tan\theta}\right)d\theta$

$= \int_0^{\frac{\pi}{4}} \log(2) - \log(1+tan\theta)\,d\theta$

$I = log2 \int_0^{\frac{\pi}{4}} 1\,d\theta - I$

$2I = log2 \int_0^{\frac{\pi}{4}} 1\,d\theta = \frac{\pi}{8}\,log2$

15 Using properties of integrals, evaluate : $\int_0^{\pi} \frac{4\,x\,sinx}{1+cos^2x}\,dx$

$I = \int_0^{\pi} \frac{4x\,sinx}{1+cos^2x}\,dx = \int_0^{\pi} \frac{4\,(\pi-x)\,\sin(\pi-x)}{1+cos^2(\pi-x)}\,dx = \int_0^{\pi} \frac{4\,(\pi-x)\,sinx}{1+cos^2x}\,dx$

Adding 2 I = $4\int_0^{\pi} \frac{\pi\,sinx}{1+cos^2x}\,dx$

I = $2\int_0^{\pi} \frac{\pi\,sinx}{1+cos^2x}\,dx$ $\quad sinx = t \;\; cosx\,dx = dt$

I = $2\pi\int_1^{-1} \frac{-dt}{1+t^2}$ simplify

$2\pi(\tan^{-1}1 - \tan^{-1}(-1)) = 2\pi\left(\frac{\pi}{4}+\frac{\pi}{4}\right)$

$= \pi^2$

16 Evaluate: $\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{3+sin2x}\,dx$

ANS: $\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{3+sin2x}\,dx = \int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{4-(1-sin2x)}\,dx$

$\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{4-(sinx-cosx)^2}\,dx = \int_{-1}^{0} \frac{1}{4-t^2}$

$\frac{1}{4}log\left[\frac{2+t}{2-t}\right]$ Simplify with limit -1 to 0

$= \frac{1}{4}log3$

$sinx - cosx = t$

$(sinx+cosx)dx = dt$

$x = 0, t = -1$

$x = \frac{\pi}{4}, t = 0$

17 Evaluate: $\int_0^{\frac{\pi}{2}} \frac{cosx}{(1+sinx)\,(2+sinx)}\,dx$

$\int_0^{\frac{\pi}{2}} \frac{cosx}{(1+sinx)\,(2+sinx)}\,dx = \int_0^1 \frac{1}{(1+t)\,(2+t)}\,dt$ $\quad sinx = t \quad cosx\,dx = dt$

$= \int_0^1 \frac{1}{t^2+3t+2}\,dt = \int_0^1 \frac{1}{\left(t+\frac{3}{2}\right)^2-\frac{9}{4}+2}\,dt$

$\int_0^1 \frac{1}{\left(t+\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}\,dt$

$= log\left(\frac{t+1}{t+2}\right)$ Simplify with limit 0 to 1

$= \log\frac{4}{3}$

18 Using properties of integrals, evaluate : $\int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1}+\sqrt[3]{11-x}}\,dx$

ANS: Apply $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

I= $\int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1}+\sqrt[3]{11-x}}\,dx = \int_2^8 \frac{\sqrt[3]{10-x+1}}{\sqrt[3]{10-x+1}+\sqrt[3]{11-10+x}}\,dx$

I = $\int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1}+\sqrt[3]{11-x}}\,dx$

$2I = \int_2^8 1 \; dx$

I = 3

19 Evaluate : $\int_2^5 \{ |x-2| + |x-3| + |x-5| \} dx$

ANS: $\int_2^5 \{|x-2| + |x-3| + |x-5| \} \, dx$

$$= \int_2^5 (x-2)\, dx + \int_2^3 (3-x)\, dx + \int_3^5 (x-3)\, dx + \int_2^5 (5-x)\, dx$$

$$\left[\frac{x^2}{2} - 2x\right]_2^5 + \left[3x - \frac{x^2}{2}\right]_2^3 + \left[\frac{x^2}{2} - 3x\right]_3^5 + \left[5x - \frac{x^2}{2}\right]_2^5$$

Simplify

= $\left(\frac{9}{2} - 0\right) - \left(0 - \frac{1}{2}\right) + (2-0) - \left(0 - \frac{9}{2}\right) = \frac{23}{2}$

20 (i) Evaluate : $\int_0^4 \{ |x| + |x-2| + |x-4| \} dx$ ANS: 20

(ii) Evaluate : $\int_1^3 \{ |x-1| + |x-2| + |x-3| \} dx$ ANS: 5

21 Evaluate: $\int_1^5 \frac{\sqrt{x}}{\sqrt{6-x} + \sqrt{x}} \, dx$

ANS: $I = \int_1^5 \frac{\sqrt{x}}{\sqrt{6-x} + \sqrt{x}} \, dx = \int_1^5 \frac{\sqrt{6-x}}{\sqrt{6-(6-x)} + \sqrt{6-x}} \, dx$

$I = = \int_1^5 \frac{\sqrt{6-x}}{\sqrt{x} + \sqrt{6-x}} \, dx$

Adding

$2I = \int_1^5 1 dx = [x]_1^5$ = 4

$I = \frac{4}{2} = 2$

22 Evaluate : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{tanx}} \, dx$

23 Evaluate: $\int_0^e \frac{dx}{x\sqrt{1-(\log x)^2}}$.

24 Evaluate: $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-x^2}} dx$

ANS: $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-x^2}} dx$ = $[\sin^{-1} x]_0^{\frac{1}{\sqrt{2}}} = \sin^{-1} \frac{1}{\sqrt{2}} - 0 = \frac{\pi}{4}$

25. Evaluate : $\int_0^{\pi} |\cos x|$

ANS: $\int_0^{\pi} |\cos x| = \int_0^{\frac{\pi}{2}} cosx dx - \int_{\frac{\pi}{2}}^{\pi} cosx \, dx$

$= [sinx]_0^{\frac{\pi}{2}} - [sinx]_{\frac{\pi}{2}}^{\pi}$ = 2

Type 1: Evaluate the following definite integrals.

1. $\int_0^{\frac{\pi}{4}} \sqrt{1+sin2x}\,dx$

2. $\int_0^{\frac{\pi}{2}} cos3xcos2x\,dx$

3. $\int_0^{\frac{\pi}{2}} cos^2x\,dx$

4. $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

5. $\int_2^3 \frac{1}{x^2+1}\,dx$

6. $\int_0^1 \frac{2x+3}{5x^2+1}\,dx$

7. $\int_0^{\frac{\pi}{4}} \sec x\sqrt{\frac{1-sinx}{1+sinx}}\,dx$

8. $\int_1^2 \frac{5x^2}{x^2+4x+3}\,dx$

9. $\int_0^2 \frac{5x+1}{x^2+4}\,dx$

10. $\int_{\frac{\pi}{2}}^{\pi} \frac{1-sinx}{1-cosx}\,dx$

11. $\int_0^1 xe^x\,dx$

12. $\int_1^3 \frac{1}{x^2(x+1)}\,dx$

13. $\int_0^{\frac{\pi}{4}} 2tan^3x\,dx$

14. $\int_0^1 \sin^{-1}x\,dx$

15. $\int_0^1 \frac{1}{2x-3}\,dx$

16. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos\theta\, cosec^2\theta\,d\theta$

17. $\int_0^{\frac{\pi}{2}} \frac{sin^2\theta}{(1+\cos\theta)^2}d\theta$

18. $\int_0^{\frac{\pi}{2}} cos^2\frac{\theta}{2}d\theta$

19. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{1-sin2x}\,dx$

20. $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-x^2}}dx$

21. $\int_0^1 \frac{1}{\sqrt{2x+3}}dx$

22. $\int_0^1 \frac{2x}{1+x^2}dx$

23. $\int_0^1 \frac{e^x}{1+e^{2x}}dx$

ANSWERS

1. 1
2. $\frac{3}{5}$
3. . $\frac{\pi}{4}$
4. $\frac{\pi}{2}$
5. $tan^{-1}3 - tan^{-1}2$
6. $\frac{1}{5}log\,6 + \frac{3}{\sqrt{5}}tan^{-1}\sqrt{5}$
7. $2-\sqrt{2}$
8. $5 + 10\,log\frac{8}{15} + \frac{25}{2}log\frac{6}{5}$
9. $\frac{5}{2}log\,2 + \frac{\pi}{8}$
10. 1 - log2
11. 1
12. $\frac{2}{3} + log\frac{2}{3}$
13. 1- log2
14. $\frac{\pi}{2} - 1$
15. $-\frac{1}{2}\,log3$
16. $\sqrt{2}-1$
17. $-\frac{\pi}{2}+2$
18. $\frac{\pi+2}{4}$.
19. $\sqrt{2}-1$
20. $\frac{\pi}{4}$
21. $\sqrt{5}-\sqrt{3}$
22. log2
23. $tan^{-1}e - \frac{\pi}{4}$

HOME WORK

1. $\int_{\frac{\pi}{2}}^{\pi} \frac{1+cosx}{1-cosx}\,dx = -\frac{\pi}{2}+2$

2. $\int_0^{\frac{\pi}{2}} sinx\,sin2x\,dx = \frac{2}{3}$

3. $\int_0^{\frac{\pi}{4}} secx\,dx = \log(\sqrt{2}+1)$

4. $\int_3^5 \frac{x^2}{x^2-4}\,dx = 2 + log\frac{15}{7}$

5. $\int_1^2 \frac{1}{(x+1)(x+2)}\,dx = log\frac{9}{8}$

6. $\int_1^3 \frac{1}{7-2x}dx = \frac{1}{2}log5$

7. $\int_0^{\frac{\pi}{4}} \frac{1}{1+cos2x}\,dx = \frac{1}{2}$

8. $\int_0^a 3x^2\,dx = 8$, find a

9. $\int_0^{\frac{\pi}{2}} \sqrt{1-cos2x}\,dx = \sqrt{2}$

10*. $\int_0^a \frac{dx}{4+x^2} = \frac{\pi}{8}$ find a

11*. $\int_0^{\frac{\pi}{2}} e^x(sinx - cosx)dx$

12. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cotx\,dx = \frac{1}{2}log2$

13. $\int_3^4 \frac{1}{x^2-4}\,dx = \frac{1}{4}log\frac{5}{3}$

14. $\int_0^{\pi} \frac{1}{1+\sin x}\,dx = 2$

15. $\int_0^{\frac{\pi}{2}} cos^2dx = \frac{\pi}{4}$

Type 2: By substitution

19. $\int_1^3 \frac{\cos(\log x)}{x}\,dx$ 20. $\int_0^{\frac{\pi}{4}} \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx$ 21. $\int_1^e \frac{\sin(\pi \log x)}{x}\,dx$ 22. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}\,dx$

23. $\int_0^1 \frac{e^{2x}}{1+e^{4x}}\,dx$ 24. $\int_0^1 \frac{x}{1+\sqrt{x}}\,dx$ 25. $\int_0^1 \frac{x^5}{1+x^6}\,dx$ 26. $\int_0^1 \sin^{-1}\frac{2x}{1+x^2}\,dx$

27. $\int_0^1 (\cos^{-1} x)^2\,dx$ 28. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log \sin x dx$ 29. $\int_0^{\frac{\pi}{4}} \frac{1}{4+5\cos^2 x}\,dx$ 30. $\int_0^{\frac{\pi}{2}} \frac{1}{4\sin^2 x+5\cos^2 x}\,dx$

31. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1+\cos x}} dx$ 32. $\int_0^{\frac{\pi}{2}} 2\tan^3 x dx$

ANSWERS

19. sin log3 *20. 2* *21.* $\frac{2}{\pi}$ *22.* $\frac{\pi}{4}$

23. $\frac{1}{2}\left(\tan^{-1} e^2 - \frac{\pi}{4}\right)$ *24.* $2\left(\frac{5}{6} - \log 2\right)$ *25.* $\frac{1}{6}\log 2$ *26.* $\frac{\pi}{2}$ - $\log 2$

27. $\pi - 2$ *28.* $\frac{1}{4}\log 2 - \frac{\pi}{8} + \frac{1}{4}$ *29.* $\frac{1}{6}\tan^{-1}\frac{2}{3}$ *30.* $\frac{\pi}{4\sqrt{5}}$.

31. $2(\sqrt{2}-1)$ *32.* $1 - \log 2$

HOME WORK

1. $\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta}\ \sin^3\theta\, d\theta = \frac{8}{21}$ 2. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}} dx = \frac{3}{2}$ 3. $\int_0^{\frac{\pi}{2}} \frac{1}{4+9\cos^2 x}\,dx = \frac{\pi}{4\sqrt{13}}$.

4. $\int_0^{\frac{\pi}{2}} \frac{1}{5+4\sin x}\,dx = \frac{2}{3}\tan^{-1}\frac{1}{3}$. 5. $\int_0^{\frac{\pi}{4}} 2\tan^3 x\,dx = 1 - \log 2$ 6. $\int_1^2 \frac{e^x(1+x\log x)}{x}\,dx = e^2 \log 2$

7. $\int_0^1 \frac{\tan^{-1} x}{1+x^2}\,dx = \frac{\pi^2}{32}$. 8. $\int_1^2 \frac{\cos(\log x)}{x}\,dx = \sin \log 2$ 9. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{4-\sin^2 x}\,dx = \frac{\log 3}{4}$ 10*. $\int_e^{e^2} \frac{dx}{x\log x}$

11*. $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1+3\sin^2 x}\,dx$ Hint : $\int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{(1+\tan^2 x)(1+4\tan^2 x)}\,dx$ 12* $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{3+\sin 2x}\,dx$ Hint: $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{(\sin x-\cos x)^2-4}\,dx$

13. $\int_0^{\frac{\pi}{4}} \frac{1}{\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x}\,dx = \frac{\pi}{2}$

Type-3 Using Properties, evaluate:

31. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx = \frac{\pi}{4}$ 32. $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\cos^5 x+\sin^5 x}\,dx = \frac{\pi}{4}$ 33. $\int_0^{\frac{\pi}{2}} \log\tan x\,dx = 0$

34. $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx = \frac{\pi}{8}\log 2$ 35. $\int_0^1 x(1-x)^{99}\,dx = \frac{1}{10100}$ 36. $\int_0^2 x\sqrt{2-x}\,dx = \frac{16\sqrt{2}}{15}$

37. $\int_0^{\pi} \frac{x}{1+\sin x}\,dx = \pi$ 38. $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x}\,dx = \frac{\pi}{2\sqrt{2}}\log(\sqrt{2}+1)$ 39. $\int_0^{\pi} x\sin^3 x\,dx = \frac{2\pi}{3}$

40. $\int_0^{\pi} \frac{x\tan x}{\sec x+\cos x}\,dx = \frac{\pi^2}{4}$ 41. $\int_0^{\pi} \frac{x\tan x}{\sec x\,\mathrm{cosec}\,x}\,dx = \frac{\pi^2}{4}$ 42. $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\,dx = \pi\left(\frac{\pi}{2}-1\right)$

43. $\int_0^{\frac{\pi}{2}} \frac{sinx - cosx}{1+sinx\,cosx}\,dx = 0$

44. $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt[4]{tanx}}\,dx = \frac{\pi}{4}$

45. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cosx\,dx = 2$

46. . $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{tanx}}\,dx = \frac{\pi}{4}$

47. $\int_0^{\pi} \frac{x\,sinx}{1+cos^2x}\,dx = \frac{\pi^2}{4}$

48. $\int_0^3 x^2(3-x)^{\frac{1}{2}}\,dx = \frac{144\sqrt{3}}{35}$

49. $\int_2^5 |x-5|\,dx = \frac{9}{2}$

50. $\int_0^3 |3-2x|\,dx = \frac{9}{2}$

51. $\int_0^{\pi} \frac{e^{cosx}}{e^{cosx}+e^{-cosx}})\,dx = \frac{\pi}{2}$

52. $\int_2^3 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx = \frac{1}{2}$

53. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin|x|\,dx = 2$

54. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x|\,dx = 2$

55. $\int_0^1 \log\left(\frac{1}{x}-1\right) dx = 0$

56. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^5x\,dx = 0$

57. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{cotx}}\,dx = \frac{\pi}{12}$

58. $\int_1^4 \{|x-1| + |x-2| + |x-4|dx = \frac{23}{2}$

59*. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{cotx}}\,dx = = \frac{\pi}{12}$

60. $\int_0^{\frac{\pi}{2}} sin2x \log tanx\,dx = 0$

61. $\int_0^1 \cot^{-1}(1-x+x^2)\,dx = \frac{\pi}{2} - log2$

HOME WORK: 3

1. $\int_0^{\frac{\pi}{2}} \frac{1}{1+tanx}\,dx = \frac{\pi}{4}$

2. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{tanx}}{1+\sqrt{tanx}}\,dx = \frac{\pi}{4}$

3. $\int_0^1 x(1-x)^n\,dx = \frac{1}{(n+1)(n+2)}$

4. $\int_0^{\frac{\pi}{2}} (2\log sinx - \log sin2x)\,dx = -\frac{\pi}{2}\,log2.$

5. $\int_0^{\frac{\pi}{2}} \frac{sin^2x}{sinx+cosx}\,dx = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$

6. $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx = \frac{a}{2}$

7. $\int_{-2}^2 |x+1|\,dx = 5$

8. $\int_{-6}^6 |x+2|\,dx = 40$

9. $\int_0^{2\pi} |cosx|\,dx = 4$

10 $\int_0^{\pi} \frac{x}{a^2cos^2x + b^2sin^2x}\,dx = \frac{\pi^2}{2ab}$

11. $\int_0^4 x(4-x)^{3/2} = \frac{512}{35}$

12. $\int_0^{\pi} \frac{x\,sinx}{1+cos^2x}\,dx = \frac{\pi^2}{4}$

13. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{cotx}}\,dx = \frac{\pi}{12}$

14. $\int_0^{\pi} \frac{x\,sinx}{1+sinx}\,dx$

15. $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$, $hint: x = tan\theta.$

16. $\int_0^{\frac{\pi}{4}} \frac{sinx\,cosx}{cos^4x + sin^4x}\,dx = \frac{\pi}{8}.$

HOTS

1. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{sinx+cosx}{\sqrt{sin2x}}\,dx$

2. $\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{9+16sin2x}\,dx$

3. $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\,dx = a\pi$

4. $\int_0^{\infty} \frac{x^3}{(1+x^2)^{9/2}}\,dx$

5. $\int_0^a \frac{x^4}{\sqrt{a^2-x^2}}\,dx$

6. $\int_0^{\frac{1}{2}} \frac{1}{(1-2x^2)\sqrt{1-x^2}}\,dx$

7. $\int_0^{\frac{\pi}{2}} \frac{x\,sinx\,cosx}{sin^4x + cos^4x}\,dx$

8. $\int_0^{2\pi} \frac{1}{1+e^{sinx}}\,dx$

9. $\int_0^{\frac{\pi}{4}} \sqrt{tanx} + \sqrt{cotx}\,dx$

SUJITHKUMAR K