

LIMITS AND DIFFERENTIATION

CLASS XI

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Evaluate the following:

1. $\lim_{x \rightarrow -1} \frac{x^3+1}{2x^2+5x+3}$
2. $\lim_{x \rightarrow 3} \frac{x^4-81}{x^2-9}$
3. $\lim_{x \rightarrow \sqrt{2}} \frac{x^2-2}{x^2+\sqrt{2}x-4}$
4. $\lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{4}{x^3-2x^2} \right)$
5. $\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{2}{x^2-4x+3} \right)$
6. $\lim_{x \rightarrow 3} \frac{x^2-x-6}{x^3-3x^2+x-3}$
7. $\lim_{x \rightarrow 2} \frac{x^{5/2}-2^{5/2}}{x^{3/2}-2^{3/2}}$
8. $\lim_{x \rightarrow 0} \frac{\sqrt{1-x}-1}{x}$
9. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$
10. $\lim_{x \rightarrow -2} \frac{3-\sqrt{7-x}}{2+x}$
11. $\lim_{x \rightarrow 4} \frac{x-4}{3-\sqrt{13-x}}$
12. $\lim_{x \rightarrow 0} \frac{\sqrt{2-x^2}-\sqrt{2+x^2}}{5x^2}$
13. $\lim_{x \rightarrow 1} \frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}$
14. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{2+3x}-\sqrt{2-3x}}$
15. $\lim_{x \rightarrow 7} \frac{4-\sqrt{9+x}}{1-\sqrt{8-x}}$
16. $\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{(n+5)^2}$
17. $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\frac{\pi}{2}-\theta}{\sin 2\theta}$
18. $\lim_{x \rightarrow 0} \frac{10^x-5^x-2^x+1}{x^2}$
19. $\lim_{x \rightarrow 0} \frac{\sin 3x \cdot \tan 4x}{x^2}$
20. $\lim_{\theta \rightarrow 0} \frac{1-\cos \theta}{\theta}$
21. $\lim_{\theta \rightarrow 0} \frac{1-\cos 6\theta}{1-\cos 4\theta}$
22. $\lim_{x \rightarrow 0} \frac{1-\cos 4x}{x^2}$
23. $\lim_{x \rightarrow 0} \frac{x \tan x}{1-\cos x}$
24. $\lim_{x \rightarrow 0} \frac{\tan 2x-x}{2x-\sin x}$
25. $\lim_{x \rightarrow 0} \frac{x \sin x}{1-\cos x}$
26. $\lim_{\theta \rightarrow 0} \frac{1-\cos m\theta}{1-\cos n\theta}$
27. $\lim_{\theta \rightarrow 0} \frac{1-\cos \theta}{\theta \cdot \sin \theta}$
28. $\lim_{x \rightarrow 0} \frac{1-\cos x}{\sin^2 x}$
29. $\lim_{x \rightarrow 0} \frac{\sin 2x (1-\cos 2x)}{x^3}$
30. $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$
31. $\lim_{x \rightarrow 0} \frac{\tan 3x - \sin x}{3x - \sin^2 x}$
32. $\lim_{\theta \rightarrow 0} \frac{\cos a\theta - \cos b\theta}{\theta^2}$
33. $\lim_{x \rightarrow 0} \frac{\cos 6x - \cos 2x}{\cos 8x - \cos 4x}$
34. $\lim_{x \rightarrow 0} \frac{\sin 2x - \sin 4x}{\sin 4x - \sin 8x}$
35. $\lim_{x \rightarrow 0} \frac{1 - \cos 2x + \tan^2 x}{x \sin x}$
36. $\lim_{x \rightarrow 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x}$
37. $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$
38. $\lim_{x \rightarrow 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x}$
39. $\lim_{x \rightarrow 0} \frac{x^3 \cot x}{1 - \cos x}$
40. $\lim_{x \rightarrow 0} \frac{\sqrt{7} - \sqrt{6 + \cos x}}{x \sin x}$
41. $\lim_{x \rightarrow 0} \frac{x \tan 2x}{1 - \cos 2x}$
42. $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\frac{\pi}{2} - \theta}{\sin 2\theta}$
43. $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2\theta}{(\pi - 2\theta)^2}$
44. $\lim_{\theta \rightarrow \frac{\pi}{4}} \frac{\sin \theta - \cos \theta}{\theta - \frac{\pi}{4}}$
45. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\left(\frac{\pi}{2} - x\right)^2}$
46. $\lim_{x \rightarrow 1} \frac{1 + \cos \pi x}{(1-x)^2}$
47. $\lim_{\theta \rightarrow \frac{\pi}{2}} (\sec \theta - \tan \theta)$
48. $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$
49. $\lim_{x \rightarrow \theta} \frac{\sin^2 x - \sin^2 \theta}{x^2 - \theta^2}$
50. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos^2 x}$
51. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$
52. $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{3} - \tan x}{\pi - 3x}$

$$53. \lim_{x \rightarrow 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$$

$$54. \lim_{x \rightarrow 1} \frac{x^2-1}{\sqrt{x+8}-3}$$

$$55. \lim_{x \rightarrow \sqrt{2}} \frac{\sqrt{3+2x} - (\sqrt{2}+1)}{x^2-2}$$

$$56. \lim_{x \rightarrow 3} \frac{x^5-243}{x^3-27}$$

$$57. \lim_{n \rightarrow \infty} \frac{n^3}{(1^2+2^2+\dots+n^2)}$$

$$58. \lim_{x \rightarrow 0} \frac{\sin(2+x) - \sin(2-x)}{x}$$

$$59. \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$$

$$60. \lim_{x \rightarrow 0} \frac{x^3 \cot x}{1 - \cos x}$$

$$61. \lim_{y \rightarrow 0} \frac{(x+y) \sec(x+y) - x \sec x}{y}$$

$$62. \lim_{x \rightarrow 1} \frac{1-x^2}{\sin \pi x}$$

$$63. \lim_{x \rightarrow 1} (1-x) \tan \frac{\pi x}{2}$$

$$64. \lim_{x \rightarrow 0} \frac{e^{2x}-1}{x}$$

$$65. \lim_{x \rightarrow 0} \frac{e^{\sin x}-1}{x}$$

$$66. \lim_{x \rightarrow 0} \frac{a^x-b^x}{x}$$

$$67. \lim_{x \rightarrow 0} \frac{e^x-x-1}{x}$$

$$68. \lim_{x \rightarrow 0} \frac{\sin 3x}{3^x-1}$$

$$69. \lim_{x \rightarrow 0} \frac{e^{ax}-e^{bx}}{x}$$

$$70. \lim_{x \rightarrow \frac{\pi}{2}} \frac{e^{\cos x}-1}{\cos x}$$

$$71. \lim_{x \rightarrow 0} \frac{3^x-2^x}{\tan x}$$

$$72. \lim_{x \rightarrow 0} \frac{e^x+e^{-x}-2}{x^2}$$

$$73. \lim_{x \rightarrow 0} \frac{6^x-2^x-3^x+1}{\sin^2 x}$$

$$74. \lim_{x \rightarrow 0} \frac{\log(2+x)-\log(2-x)}{x}$$

$$75. \lim_{x \rightarrow 0} \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$$

$$76. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1}$$

$$77. \lim_{x \rightarrow 0} \frac{x^2 + \sin 3x}{2x - \sin x}$$

$$78. \lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$$

$$79. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$$

$$80. \lim_{x \rightarrow 0} x \sin \frac{1}{x}$$

81. Differentiate the following by **first principle**.

i) $\tan 2x$ ii) $\sqrt{2x+3}$ iii) $x \sin x$ iv) $\sin(2x+1)$ v) $\sqrt{\sin x}$ vi) $\sqrt{\tan x}$

vii) $\sin \sqrt{x}$ viii) $\cos \sqrt{x}$ ix) $\sin^3 x$ x) e^x xi) x^3 xii) e^{2x} xiii) $\frac{x+1}{x-1}$

82. Find the derivative of:

i) $x^3 \tan x$ ii) $(1+x^2) \cdot \cos x$ iii) $e^x \log x$ iv) $\sec x \cdot \tan x$ v) $\frac{2x^2-4}{3x^2+7}$ vi) $\frac{x + \sin x}{x + \cos x}$

vii) $\frac{\sqrt{x}}{x+1}$ viii) $\frac{1}{\sin x - \cos x}$ ix) $\frac{\log x}{1+x \log x}$ x) $x \cdot \sin x \cdot \log x$ xi) $e^x \cot x$.

83. Differentiate with respect x

i. $\log \sin(x^2)$ ii. $e^{\sin 2x}$ iii. $e^{x \sin x}$ iv. $\log \tan \left(\frac{\pi}{4} + \frac{x}{2}\right)$ v. $\cos(\log \sin x)$ vi. $(\log \sin x)^2$

vii. $3^{x \log x}$ viii. $\cos \sqrt{x}$ ix. $\frac{e^x}{\sin 2x}$ x. $\log(\cos e^x)$ xi. $\cos(\log x + e^x)$ xii. $\log(x + \sqrt{x^2 + a^2})$.

xiii) $(6-7x)^{5/6}$ xiv) $\frac{1}{\sqrt{7x-9}}$ xv) $\sqrt{1+\tan x}$ xvi) $\sin x^2 \cdot \tan x^3$ xvii) $\cos(\sin x^2)$ at $x = \sqrt{\frac{\pi}{2}}$

xviii) $e^{\sqrt{3x}}$ xix) e^{8-4x} xx) $x e^{\sin x}$