

SAMPLE PAPER
PERIODIC TEST – 2

Class: X

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Subject: Mathematics (041)

Date : 01- 09 - 2025

M.M : 80

Time : 3 Hours

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory.
2. Section A has 18 MCQ's and 02 Assertion – Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) – type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) – type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) – type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.
7. All Questions are compulsory. However, an internal choice in 2 questions of 5 marks, 2 questions of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION A

1. The next term of the AP $\sqrt{18}, \sqrt{50}, \sqrt{98}, \dots$ (1)
(A) $\sqrt{152}$ (B) $\sqrt{160}$ (C) $\sqrt{121}$ (D) $\sqrt{162}$
ANS: (D) $\sqrt{162}$
2. If $\tan\theta = 1$ and $\sin\alpha = \frac{1}{\sqrt{2}}$, find the value of $\cos(\theta + \alpha)$, θ and α are both acute angles. (1)
(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$
ANS: (A) 0
3. The sum of exponents of prime factors in the prime factorisation of 250 is ____ (1)
(A) 3 (B) 4 (C) 5 (D) 6
ANS: (B) 4
4. If $\tan\left(\frac{5\theta}{2}\right) = \sqrt{3}$ and θ is acute, then find the value of 2θ . (1)
(A) 40° (B) 24° (C) 48° (D) 120°
ANS: (C) 48°
5. In triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when ____ (1)
(A) $\angle B = \angle E$ (B) $\angle A = \angle D$ (C) $\angle B = \angle D$ (D) $\angle A = \angle F$
ANS: (C) $\angle B = \angle D$
6. If the roots of quadratic equation $ax^2 + bx + c = 0$ are equal in magnitude but opposite in sign (1)
then find the value of b .
(A) -2 (B) 2 (C) 0 (D) -1
ANS: (C) 0
Let one root be α other roots be $-\alpha$
Now, sum of roots $= -\frac{b}{a} \Rightarrow \alpha + (-\alpha) = -\frac{b}{a} \Rightarrow b = 0$
7. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first (1)
triangle is 9 cm., what is the corresponding side of the other triangle ?
(A) 5.4cm (B) 4 cm (C) $\frac{5}{3}$ cm (D) 3cm
ANS: (A) 5.4cm
If $\triangle ABC \sim \triangle PQR$ then $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AB+BC+AC}{PQ+QR+PR} = \frac{25}{15}$

$$\frac{AB}{PQ} = \frac{25}{15} = \frac{5}{3} \Rightarrow \frac{9}{PQ} = \frac{5}{3} \Rightarrow PQ = \frac{27}{5} = 5.4 \text{ cm}$$

8. If $\tan \theta = \frac{a}{b}$, then find the value of $\frac{a \sin \theta + b \cos \theta}{a \sin \theta - b \cos \theta}$ is _____ (1)
- (A) $\frac{a^2 - b^2}{a^2 + b^2}$ (B) $\frac{a^2 + b^2}{a^2 - b^2}$ (C) $\frac{a}{a^2 + b^2}$ (D) $\frac{b^2}{a^2 + b^2}$

ANS: B) $\frac{a^2 + b^2}{a^2 - b^2}$

$$\frac{a \sin \theta + b \cos \theta}{a \sin \theta - b \cos \theta} = \frac{a \tan \theta + b}{a \tan \theta - b} \Rightarrow \frac{a \times \frac{a}{b} + b}{a \times \frac{a}{b} - b} = \frac{a^2 + b^2}{a^2 - b^2}$$

9. Quadratic polynomial whose zeros are $3 - 2\sqrt{3}$ and $3 + 2\sqrt{3}$ is _____ (1)
- (A) $x^2 - 6x - 3$ (B) $x^2 - 6x + 3$
(C) $x^2 + 6x - 3$ (D) $x^2 + 3x - 6$

ANS: (A) $x^2 - 6x - 3$

$$\alpha + \beta = 3 - 2\sqrt{3} + 3 + 2\sqrt{3} = 6,$$

$$\alpha\beta = (3 - 2\sqrt{3})(3 + 2\sqrt{3}) = 9 - 12 = -3$$

Required polynomial = $x^2 - sx + p$ ie $x^2 - 6x - 3$

10. Line joining $(-1, 1)$ and $(5, 7)$ is divided by a line $x + y = 4$ in the ratio of _____ (1)
- (A) 1:2 (B) 1:3 (C) 3:4 (D) 1:4

ANS: (A) 1:2

Let line $x + y = 4$ divides the line joining the points $(-1, 1)$ and $(5, 7)$ at C in the ratio

$k : 1$. Coordinates of C are $\left(\frac{5k-1}{k+1}, \frac{7k+1}{k+1}\right)$ \because C lies on the line

$$x + y = 4 \Rightarrow \frac{5k-1}{k+1} + \frac{7k+1}{k+1} = 4 \Rightarrow k = \frac{1}{2}$$

Hence ratio 1 : 2.

11. Half the perimeter of a rectangular garden, whose length is 12 m more than its width is 60 m. Find the length of the garden. (1)

(A) 36 (B) 20 (C) 18 (D) 72

ANS: (A) 36

Let length of the garden = x m and breadth of the garden = y m

$$x = y + 12 \Rightarrow x - y = 12$$

and $\frac{1}{2} \times \text{Perimeter} = 60$

$$\Rightarrow \frac{1}{2} \times 2(x + y) = 60 \Rightarrow x + y = 60$$

Algebraic representation is $x - y = 12; x + y = 60$

$$x = 36, y = 24, \text{ length of the garden} = 36 \text{ m}$$

12. If the opposite angular points of a square are $(4, 3)$ and $(2, -3)$ then the side of the square is _____ (1)
- (A) 40 (B) 20 (C) $\sqrt{20}$ (D) $\sqrt{40}$

ANS: C) $\sqrt{20}$

$$\text{Diagonal} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(4 - 2)^2 + (3 + 3)^2} = \sqrt{40} \quad \text{Side} = \frac{\sqrt{40}}{\sqrt{2}} = \sqrt{20}$$

13. Determine k for which the system of equations has infinite solutions: $4x + y = 3$ and $8x + 2y = 5k$. (1)

(A) $\frac{5}{6}$ (B) 1 (C) $\frac{6}{5}$ (D) $\frac{3}{5}$

ANS: (C) $\frac{6}{5}$ For infinite many solutions $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{4}{8} = \frac{1}{2} = \frac{3}{5k} \Rightarrow k = \frac{6}{5}$$

14. There are 576 boys and 448 girls in a school that are to be divided into equal sections of either boys or girls alone. The total number of sections thus formed are: (1)

(A) 15 (B) 16 (C) 17 (D) 20

ANS: (B) 16

15. If the prime factorisation of a natural number N is $2^4 \times 3^4 \times 5^3 \times 7$ find the number of zeros in N is _____. (1)

(A) 1 (B) 2 (C) 3 (D) 4

ANS: (C) 3

16. If $px^2 + 3x + q = 0$ has two roots $x = -1$ and $x = -2$, then $q - p =$ ____ (1)

(A) 1 (B) -1 (C) 3 (D) 2

ANS: (A) 1

17. If the distance between the points $(4, p)$ and $(1, 0)$ is 5 units, then the value of p is _____. (1)

(A) 4 only (B) ± 4 (C) -4 only (D) 0

ANS: (b) $\sqrt{(4-1)^2 + (p-0)^2} = 5$

$$\Rightarrow 3^2 + p^2 = 5^2 \Rightarrow p^2 = 25 - 9 = 16 \Rightarrow p = \pm 4$$

18. The quadratic equation $4x^2 + 6x + 3 = 0$ has _____. (1)

(A) two distinct real roots (B) two equal real roots
(C) no real roots (D) more than 2 real roots

ANS: (C) no real roots

In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

19. **Assertion (A):** The coordinates of a point P which divides the line segment joining the points (1)

$A(-2, 3)$ and $B(4, 7)$ internally in the ratio $\frac{4}{7}$ is $\left(\frac{49}{11}, \frac{2}{11}\right)$.

Reason (R): The coordinates of the point $P(x, y)$ which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m:n$ is $x = \frac{mx_2 + ny_1}{m+n}$, $y = \frac{my_2 + nx_1}{m+n}$.

ANS: (D) A is false but R is true.

20. **Assertion (A):** The graph of the linear equations $x + 3y = 6$, $2x - 3y = 12$ gives a pair of intersecting lines. (1)

Reason (R): A pair of linear equations in two variables in x and y , $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ gives a pair of intersecting lines.

ANS: (A) Both A and R are true and R is the correct explanation of A.

SECTION - B

21. If $\sec \theta - \tan \theta = \frac{1}{2}$, find the value of $(\sec \theta + \tan \theta)$. (2)

OR

Prove that: $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$

ANS:

$$\sec \theta - \tan \theta = \frac{1}{2} \quad \text{Also, } \sec^2 \theta - \tan^2 \theta = 1$$

$$\Rightarrow (\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$$

$$\Rightarrow (\sec \theta + \tan \theta) \times \frac{1}{2} = 1 \Rightarrow \sec \theta + \tan \theta = 2$$

OR

$$\begin{aligned} \text{ANS: } \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} &= \frac{(\sin^2 A - \sin^2 B) + (\cos^2 A - \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)} \\ &= \frac{\sin^2 A + \cos^2 A - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)} = \frac{1 - 1}{(\cos A + \cos B)(\sin A + \sin B)} \\ &= 0 \end{aligned}$$

22. Determine the set of values of k for which the given quadratic equation has real roots: (2)
- $$2x^2 + 3x + k = 0.$$

OR

Had Ajita scored 10 more marks in her mathematics test out of 30 marks, 9 times these marks would have been the square of her actual marks. How many marks did she get in the test?

ANS: Given,

$$2x^2 + 3x + k = 0$$

It's of the form of $ax^2 + bx + c = 0$

Where, $a = 2, b = 3, c = k$

For the given quadratic equation to have real roots $D = b^2 - 4ac \geq 0$

$$D = 9 - 4(2)(k) \geq 0$$

$$\Rightarrow 9 - 8k \geq 0$$

$$\Rightarrow k \leq 9/8$$

The value of k should not exceed $9/8$ to have real roots

OR

ANS: Let her actual marks be x Therefore, $9(x + 10) = x^2$

$$\text{i.e., } x^2 - 9x - 90 = 0 \text{ i.e., } x^2 - 15x + 6x - 90 = 0$$

$$\text{i.e., } x(x - 15) + 6(x - 15) = 0 \text{ i.e., } (x + 6)(x - 15) = 0$$

Therefore, $x = -6$ or $x = 15$ Since x is the marks obtained, $x \neq -6$.

Therefore, $x = 15$. So,

Ajita got 15 marks in her mathematics test.

23. Find the 12^{th} term from the end of the following arithmetic progression: 3, 5, 7, 9, 201 (2)

ANS: Given A.P = 3, 5, 7, 9, 201

Re - write the AP in the reverse order.

201, 199, 197, , 7, 5, 3.

12^{th} term from the end of AP 3, 5, 7, 9, 201

is same as 12^{th} term of 201, 199, 197, 9, 7, 5, 3

Here, $a = 201$ and $d = -2$

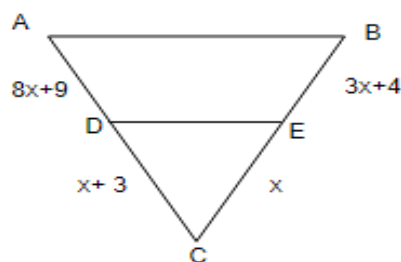
$$a_n = a + (n - 1)d \quad a_{12} = 201 + (12 - 1)(-2)$$

$$a_{12} = 201 + (11)(-2)$$

$$a_{12} = 179$$

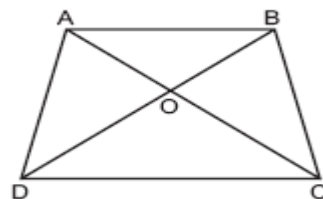
12^{th} term from the end = 179

24. What value(s) of x will make $DE \parallel AB$ in the given figure? (2)



OR

In the given figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and $AB = 4$ cm. Find the value of DC .



ANS: Given : $\triangle ABC$

Proof : DE will be parallel to AB

Only, if $\frac{CD}{AD} = \frac{CE}{BE}$ [Converse of BPT]

$$\Rightarrow \frac{x+3}{8x+9} = \frac{x}{3x+4}$$

$$\begin{aligned}
 (x+3)(3x+4) &= x(8x+9) \\
 3x^2 + 9x + 4x + 12 &= 8x^2 + 9x \\
 \Rightarrow 5x^2 - 4x - 12 &= 0 \\
 \Rightarrow 5x^2 - 10x + 6x - 12 &= 0 \\
 \Rightarrow 5x(x-2) + 6(x-2) &= 0 \\
 \Rightarrow (x-2)(5x+6) &= 0 \Rightarrow \text{either } x=2 \text{ or } 5x=-6 \\
 \Rightarrow x = -\frac{6}{5} \quad (\text{Impossible}) \Rightarrow x=2 &\quad \text{if } x=2 \text{ then } DE \parallel AB.
 \end{aligned}$$

OR

ANS: Given : $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and $AB = 4$ cm

To find : DC

Proof : In $\triangle AOB$ and $\triangle COD$,

$$\frac{AO}{OC} = \frac{BO}{OD} \quad \text{and } \angle AOB = \angle COD$$

$\triangle AOB \sim \triangle COD$ (SAS similarity)

$$\frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{CD} \Rightarrow \frac{1}{2} = \frac{4}{CD}$$

$$CD = 8 \text{ cm}$$

25. Find the ratio in which the point $(2, y)$ divides the line segment joining the point $A(-2, 2)$ and $B(3, 7)$. Also find the value of y . (2)

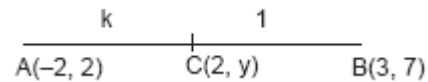
ANS: Let C divides AB in the ratio $k : 1$

$$x \text{ coordinate of C} = \frac{3k+1 \times (-2)}{k+1}$$

$$2 = \frac{3k-2}{k+1} \Rightarrow 2k+2 = 3k-2 \Rightarrow k=4$$

C divides AB in the ratio 4 : 1

$$\text{Now } y \text{ coordinate of C} = \frac{4 \times 7 + 1 \times (2)}{4+1} = 6 \quad [k=4]$$



SECTION -C

26. Find the values of α and β for which the following system of linear equations has infinite solutions (3)
- $$2x + 3y = 7, \quad 2ax + (\alpha + \beta)y = 28.$$

ANS: For infinite solutions, the condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{2\alpha} = \frac{3}{\alpha+\beta} = \frac{7}{28}$

(i) (ii) (iii)

Take (i) and (iii) $\frac{1}{\alpha} = \frac{1}{4} \Rightarrow \alpha = 4$

Take (ii) and (iii) $\frac{3}{\alpha+\beta} = \frac{7}{28} \Rightarrow \frac{3}{4+\beta} = \frac{1}{4}$

$$4 + \beta = 12 \Rightarrow \beta = 8$$

$$\text{so } \alpha = 4, \beta = 8.$$

27. If a and b are roots of the equation $2x^2 + 7x + 5 = 0$ then write a quadratic equation whose roots are $2a + 3$ and $2b + 3$. (3)

OR

In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and time increased by 30 minutes. Find the original duration of the flight.

ANS:

Here given quadratic equation is $2x^2 + 7x + 5 = 0$

a and b are roots $a + b = -\frac{7}{2} \dots(i)$ and $ab = \frac{5}{2} \dots(ii)$

Now, quadratic equation whose roots are $2a + 3$ and $2b + 3$ is

$$x^2 - [2a + 3 + 2b + 3]x + (2a + 3)(2b + 3) = 0$$

$$x^2 - [2(a + b) + 6]x + (4ab + 6(a + b) + 9) = 0$$

$$x^2 - \left[2\left(-\frac{7}{2}\right) + 6\right]x + \left[4 \times \frac{5}{2} + 6 \times \left(-\frac{7}{2}\right) + 9\right] = 0 \quad [\text{using eq. (i) and (ii)}]$$

$$x^2 + x - 2 = 0$$

OR

Let original duration of the flight be x hours.

Distance = 2800 km

$$\text{Usual speed} = \frac{2800}{x} \text{ km/h}$$

$$\text{When time} = \left(h + \frac{1}{2}\right) \text{ hrs}$$

$$\text{New speed} = \frac{2800}{x + \frac{1}{2}} = \frac{5600}{2x+1} \text{ km/hr}$$

$$\text{ATQ } \frac{2800}{x} - \frac{5600}{2x+1} = 100$$

$$\Rightarrow 2800 = 100 (2x^2 + x)$$

$$\Rightarrow 2x^2 + x - 28 = 0$$

$$\Rightarrow 2x^2 + 8x - 7x - 28 = 0$$

$$\Rightarrow 2x(x+4) - 7(x+4) = 0$$

$$\Rightarrow (x+4)(2x-7) = 0$$

$$\Rightarrow x = -4 \text{ (rejected) or } x = \frac{7}{2} = 3\frac{1}{2}$$

Original duration = $3\frac{1}{2}$ hours.

28. The sum of three numbers of an AP is 27 and their product is 405. Find the numbers. (3)

ANS: Let three numbers in AP are $a - d$, a and $a + d$

$$(a - d) + a + (a + d) = 27$$

$$\Rightarrow 3a = 27 \Rightarrow a = 9$$

$$\text{Also } (a - d)(a)(a + d) = 405 \Rightarrow (9 - d)(9)(9 + d) = 405$$

$$\Rightarrow (9 - d)(9 + d) = 45 \Rightarrow 81 - d^2 = 45$$

$$\Rightarrow d^2 = 36 \Rightarrow d = 6, -6$$

When $d = 6$, numbers are 3, 9, 15

When $d = -6$, numbers are 15, 9, 3

29. Show that 21^n cannot end with the digits 0, 2, 4, 6 and 8 for any natural number n . (3)

ANS:

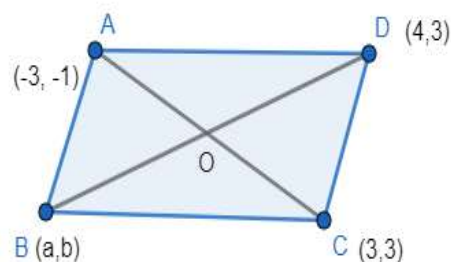
Prime factorisation of $21^n = (3 \times 7)^n = 3^n \times 7^n$ Prime factorisation of 21^n contains only prime numbers 3 and 7

21^n may end with the digit 0, 2, 4, 6 or 8 for some natural number n if its prime factorisation contains 2 and 5, but these are not present.

So, there is no natural number ' n ' for which 21^n ends with the digits 0, 2, 4, 6 and 8.

30. If four vertices of a parallelogram taken in order (3)

are $(-3, -1)$, (a, b) , $(3, 3)$ and $(4, 3)$, then find the ratio $a : b$



OR

Find the point on y-axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$.

ANS: Coordinates of mid-point of AC = coordinate of mid-point of BD

$$\left(\frac{-3+3}{2}, \frac{-1+3}{2}\right) = \left(\frac{4+a}{2}, \frac{3+b}{2}\right)$$

$$(0, 1) = \left(\frac{4+a}{2}, \frac{3+b}{2}\right)$$

$$\frac{4+a}{2} = 0, a = -4$$

$$\frac{3+b}{2} = 1, b = -1$$

$$a : b = 4 : 1$$

OR

ANS: Let point on y-axis be $(0, a)$

Now distance of this point from $(5, -2)$ is equal to distance from point $(-3, 2)$

$$\text{i.e., } \sqrt{5^2 + (-2 - a)^2} = \sqrt{3^2 + (a - 2)^2}$$

Squaring and simplifying, we get

$$25 + 4 + a^2 + 4a = 9 + a^2 + 4 - 4a \Rightarrow 8a = -16 \Rightarrow a = -2$$

Required point $(0, -2)$

31. If the roots of the equation $12x^2 + mx + 5 = 0$ are in the ratio $3 : 2$, then m (3)

ANS: Let roots are 3α and 2α .

$$\text{Sum of roots} = 3\alpha + 2\alpha = 5\alpha = -\frac{m}{12} \Rightarrow \alpha = -\frac{m}{60} = \dots(i)$$

$$\text{Product of roots} = 3\alpha \times 2\alpha = 6\alpha^2 = \frac{5}{12} \Rightarrow \alpha^2 = \frac{5}{72} \dots(ii)$$

$$\text{From (i) and (ii), } \frac{m^2}{3600} = \frac{5}{72} \Rightarrow m^2 = 5 \times 50$$

$$\Rightarrow m^2 = 5 \times 5 \times 10 \Rightarrow m = \pm 5\sqrt{10}$$

SECTION -D

32. If α and β are zeroes of $3x^2 - 6x + 4$, then find the value of: $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$ (5)

OR

If α and β are roots of $ax^2 + bx + c = 0$, then find the value of $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{c}{a}}$.

ANS:

$$\alpha + \beta = 2, \quad \alpha\beta = \frac{4}{3}$$

$$\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$$

$$\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta =$$

$$\frac{(2)^2 - 2 \times \frac{4}{3}}{\frac{4}{3}} + 2\left(\frac{2}{\frac{4}{3}}\right) + 3 \times \frac{4}{3} = 1 + 2 \times \frac{3}{2} + 3 \times \frac{4}{3} = 8$$

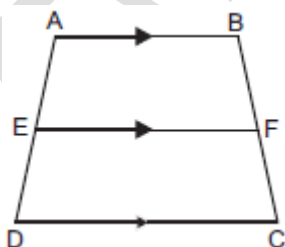
OR

$$\text{ANS: } \alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{c}{a}} = \frac{\alpha + \beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{c}{a}} = \frac{-\frac{b}{a}}{\sqrt{\frac{c}{a}}} + \sqrt{\frac{c}{a}} = 0 \Rightarrow -\sqrt{\frac{b}{a}} + \sqrt{\frac{b}{a}} = 0$$

33. If a line is drawn parallel to one side of a triangle, the other two sides are divided in the same ratio, prove it. Use this result to prove the following: (5)

In the given figure, if ABCD is a trapezium in which $AB \parallel DC \parallel EF$, then $\frac{AE}{ED} = \frac{BF}{FC}$



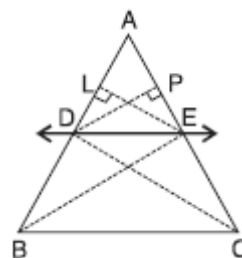
ANS:

Given : A triangle ABC, $DE \parallel BC$, meeting AB at D and AC at E.

$$\text{To Prove : } \frac{AD}{BD} = \frac{AE}{EC}$$

Construction : Join BE, CD and draw $EL \perp AD$.

Proof : $\triangle BDE$ and $\triangle CDE$ are on the same base and between the same parallel BC and DE, hence equal in area,



i.e.,

$$\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE) \dots (i)$$

$$\frac{\text{area}(\triangle ADE)}{\text{area}(\triangle BDE)} = \frac{\frac{1}{2}AD \cdot EL}{\frac{1}{2}BD \cdot EL} = \frac{AD}{BD}$$

$$\frac{\text{area}(\triangle ADE)}{\text{area}(\triangle CDE)} = \frac{\frac{1}{2}AE \cdot DL}{\frac{1}{2}EC \cdot DP} = \frac{AE}{EC}$$

$$\frac{\text{area}(\triangle ADE)}{\text{area}(\triangle BDE)} = \frac{\text{area}(\triangle ADE)}{\text{area}(\triangle CDE)}$$

$$\frac{AD}{BD} = \frac{AE}{EC}$$

Second part

Join BD intersecting EF at G.

In $\triangle DAB$, $EG \parallel AB$

$$\frac{AE}{DE} = \frac{BG}{GD}, \quad \frac{BG}{GD} = \frac{BF}{FC}$$

$$\frac{AE}{DE} = \frac{BF}{FC}$$

34. Solve for x and y , $\sqrt{2}x + \sqrt{3}y = 5$, $\sqrt{3}x - \sqrt{8}y = -\sqrt{6}$ (5)

ANS: $\sqrt{2}x + \sqrt{3}y = 5 \Rightarrow \sqrt{2} \cdot \sqrt{3}x + \sqrt{3} \cdot \sqrt{3}y = 5 \cdot \sqrt{3}$

$$\sqrt{6}x + 3y = 5\sqrt{3} \dots \dots (i)$$

$$\sqrt{3}x - \sqrt{8}y = -\sqrt{6} \Rightarrow \sqrt{3} \cdot \sqrt{2}x - \sqrt{8} \cdot \sqrt{2}y = -\sqrt{6} \cdot \sqrt{2}$$

$$\sqrt{6}x - 4y = -\sqrt{12} \dots \dots \dots (ii)$$

$$(i) - (ii) \Rightarrow 7y = 5\sqrt{3} - (-\sqrt{12})$$

$$\Rightarrow 7y = 5\sqrt{3} + 2\sqrt{3} = 7\sqrt{3}$$

$$\Rightarrow y = \sqrt{3} \text{ substitute and we get } x = \sqrt{2}$$

Solution $x = \sqrt{2}$, $y = \sqrt{3}$

35. Prove that: $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}$ (5)

OR

If $\sin\theta + \cos\theta = p$ and $\sec\theta + \csc\theta = q$, show that $q(p^2 - 1) = 2p$.

ANS: $LHS = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$

$$= \frac{\tan\theta + \sec\theta - (\tan\theta + \sec\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1} = \frac{\tan\theta + \sec\theta(1 + \tan\theta - \sec\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \tan\theta + \sec\theta = \frac{\sin\theta}{\csc\theta} + \frac{1}{\cos\theta} = \frac{1 + \sin\theta}{\cos\theta} = RHS$$

OR

ANS: $\sin\theta + \cos\theta = p$, $\sec\theta + \csc\theta = q$

$$LHS = q(p^2 - 1) = (\sec\theta + \csc\theta)[(\sin\theta + \cos\theta)^2 - 1]$$

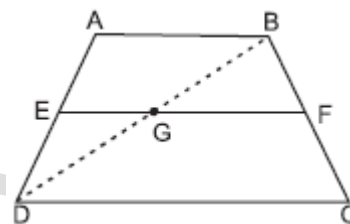
$$= \left[\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} \right] [\sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta - 1]$$

$$= \left[\frac{\cos\theta + \sin\theta}{\cos\theta} \right] [1 + 2\cos\theta\sin\theta - 1] \quad (\sin^2\theta + \cos^2\theta = 1)$$

$$= \left[\frac{\cos\theta + \sin\theta}{\cos\theta} \right] \times 2\cos\theta\sin\theta$$

$$= 2(\sin\theta + \cos\theta) = 2p \quad (\sin\theta + \cos\theta = p)$$

LHS = RHS. Hence proved



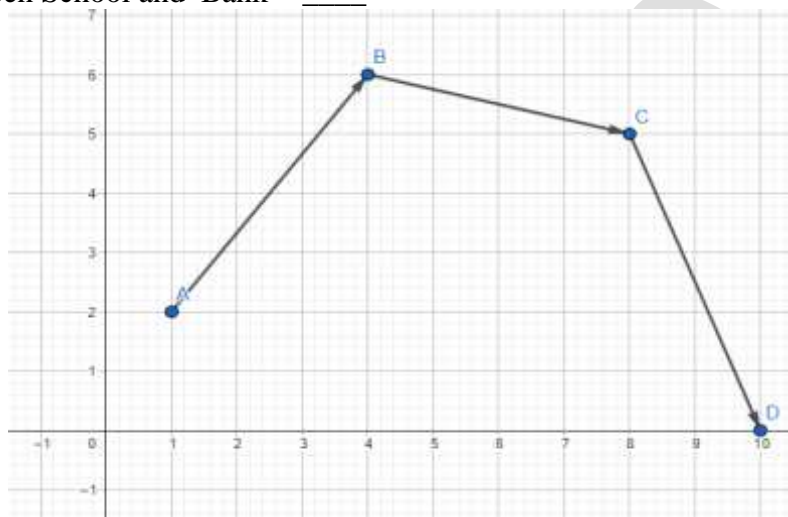
SECTION – E

36. Aditya starts walking from his house to office. Instead of going to the office directly, he goes to the school drop his daughter first and there to a bank and reaches the office. Assume that all distances covered are in straight line. If the house is situated at A (1, 2) . School at B (4, 6), The bank at C(8,5) and the office at D (0,10). Distances are in km.

- i) Distance between House and School = _____ (1)
 ii) Actual distance between House and office = _____ (1)
 iii) The mid point of AC = _____ (2)

OR

- iii) Distance between School and Bank = _____



ANS: i) 5m ii) $\sqrt{65}$ km iii) $(\frac{9}{2}, \frac{7}{2})$, OR $\sqrt{17}km$

37. Raj and Ajay are very close friends. Both the families decide to go to Rann of Kutch by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete his journey of 400 km.

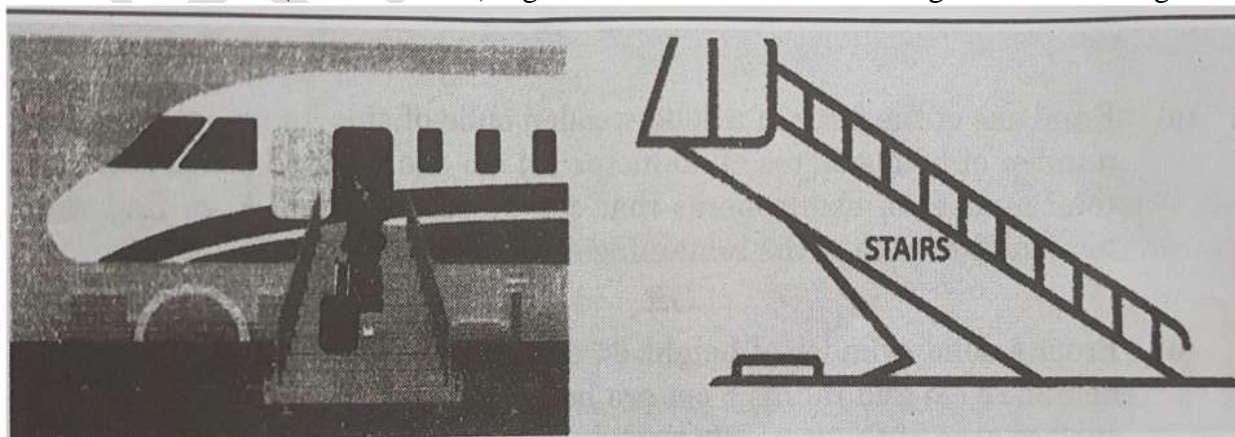
- i) What will be the distance covered by Ajay's car in two hours? (1)
 ii) Find the quadratic equation which describes the speed of Raj's car? (1)
 iii) How much time took Ajay to travel 400 km?.

OR

- iii) What is the speed of Raj's car?

ANS: i) 50 km ie $[2(x + 5) km]$ ii) $x^2 + 5x - 500 = 0$ iii) 16 hour OR 20 km/hour

38. Passengers boarding stairs, sometimes referred to as boarding ramps, stair cars or air craft steps, provide a mobile means to travel between the air craft doors and the ground. Larger air craft have door sills 5 to 20 feet (1 foot = 30 cm) high. Stairs facilitate safe boarding and de- boarding



An air craft has a door sill at a height of 15 feet above the ground. A stair car is placed at a horizontal distance of 15 feet from the plane.

Based on the given information, answer the following questions given in part (i) and (ii).

- (i) Find the angle at which the stairs are inclined to reach the door sill 15 feet high above the ground. (1)
- (ii) Find the length of the stairs used to reach the door sill. (1)

Further, answer any **one** of the following questions

- (iii) (a) If the 20 feet long stairs is inclined at an angle of 60° to reach the door sill, then find the height of the door sill above the ground ($\sqrt{3} = 1.732$) (2)
- OR**
- (b) What should be the shortest possible length of the stairs to reach the door sill of the plane 20 feet above the ground, if the angle of elevation cannot exceed 30° ? Also find the horizontal distance of base of the stair car from the plane.

- ANS: (i) 45°
- (ii) $15\sqrt{2} \text{ m} = 21.21 \text{ feet}$
- (iii) (a) $10\sqrt{3} \text{ m} = 10 \times 1.732 = 17.32 \text{ m}$
- (iii) (b) 40 ft, 34.64 ft.

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