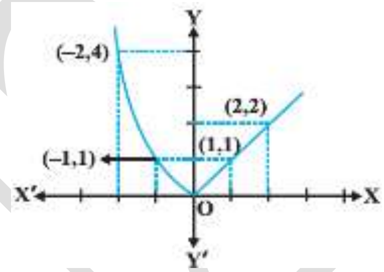
CONTINUITY AND DIFFERENTIATION-2025

CLASS XII SUJITHKUMAR KP

1 Discuss the continuity of the function f given by

$$f(x) = \begin{cases} x, & x \geq 0 \\ x^2, & x < 0 \end{cases} \quad \text{at } x = 0$$

ANS: Clearly the function is defined at every real number. Graph of the function is given.



$RHL = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = 0$

$LHL = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 = 0$

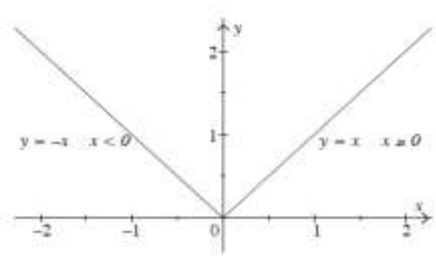
$f(0) = 0$

$\lim_{x\to 0} f(x) = f(0) = 0.$

2 If $f(x) = |x|$, Check the continuity at x = 0

ANS: Re - define the function

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



The graph of $y = |x|$.

$RHL = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = 0$

$LHL = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} -x = 0$

$LHL = RHL$

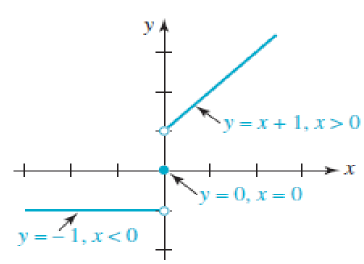
$f(0) = 0$, $\lim_{x\to 0} f(x) = f(0) = 0$

so the function is continuous at $x = 0$.

3 Examine the continuity of the function $f(x)$ at x= 0

$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ x+1, & x > 0 \end{cases}$$

ANS: $RHL = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x + 1 = 1$



$LHL = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(-1) = -1$

$LHL \neq RHL$

$\lim_{x\to 0} f(x)$ does not exist

Also, $f(0) = 0$

$f(x)$ is discontinuous at x= 0.

4 If $f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$, show that $f(x)$ is continuous at $x = 1$

ANS:

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x\to 1} (x+1) = 2$

at $x = 1$, $f(x) = 2$

$f(1) = 2$

$\lim_{x\to 1} f(x) = f(1)$ ∴ $f(x)$ is continuous at $x = 1$

5 Examine the continuity of the following function at $x = 2$

$$f(x) = \begin{cases} 1+x, x \le 2 \\ 5-x, x > 2 \end{cases}$$

ANS:

$RHL = \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 5 - x$ = 5- 2 = 3

$LHL = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 1 + x = 1 + 2 = 3$

$\lim_{x\to 2} f(x) = 3$

$f(2) = 1 + 2 = 3$

$\lim_{x\to 2} f(x) = f(2)$ = 3 ,

so the function is continuous at $x = 2$.

6 Prove that

$f(x) = \begin{cases} 3x-2, x \le 0 \\ x+1, x > 0 \end{cases}$ is discontinuous at x = 0.

ANS: RHL = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x + 1 = 1$

LHL = $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 3x - 2 = -2$

LHL ≠ RHL

$\lim_{x\to 0} f(x)$ does not exist

Also, $f(0) = -2$

so, $f(x)$ is discontinuous at x= 0.

7 If $f(x) = \begin{cases} \frac{|x-4|}{4-x}, & x \ne 4 \\ 0, & x = 4 \end{cases}$, show that $f(x)$ *is not continuous* at x=4

ANS: $x < 4$, $\frac{|x-4|}{4-x} = \frac{4-x}{4-x} = 1$

$x > 4$, $\frac{|x-4|}{4-x} = \frac{x-4}{4-x} = -1$

$$RHL = \lim_{x\to 4^+} \frac{|x-4|}{4-x} = -1 \quad LHL = \lim_{x\to 4^-} \frac{|x-4|}{4-x} = 1$$

$RHL \ne LHL$

$\lim_{x\to 4} f(x)$ does not exist

Also, $f(4) = 0$

8 Show that the function f(x) is defined as

f(x) = $\begin{cases} \frac{x^2-x-6}{x-3}, & if\ x \ne 3 \\ 5, & if\ x = 3 \end{cases}$ continuous at x = 3

ANS:

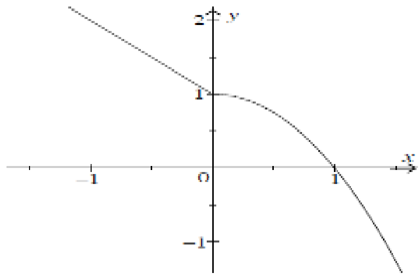
$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-x-6}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+2)}{x-3} = 5$

$f(3) = 5$ $\quad f$ is continuous at x = 3

9 Examine the continuity of the function $f(x)$ at x= 0

$$f(x) = \begin{cases} 1 - x^2 & , x \geq 0 \\ 1 - x , & x < 0 \end{cases}$$

ANS: $RHL = \lim_{x \to 0^+} f(x) = \lim_{x\to 0^+} 1 - x^2 = 1$



$LHL = \lim_{x \to 0^-} f(x) \ = \lim_{x \to 0^-}(1 - x) \ = 1$

$f(0) = 1 - 0 = 1$

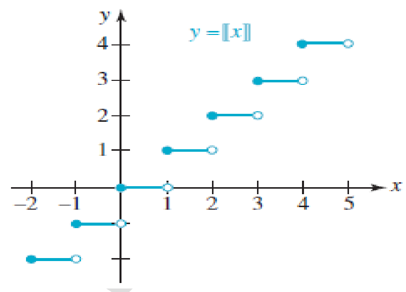
$\lim_{x\to 0} f(x) = f(0)$ = 1 ,

so the function is continuous at $x = 0$.

10 Examine the continuity of the function at $x = 1$, $f(x) = [x]$

ANS:

$RHL = \lim_{x \to 1^+} f(x) = \lim_{x\to 1^+}[x] \ = 1$



$LHL = \lim_{x \to 1^-} f(x) \ = \ \lim_{x \to 1^-}[x] = 0$

$LHL \ \neq RHL$

Discontinuous at x = 1

11 For what value of k is the function

f(x) = $\begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ k \quad , & x = 3 \end{cases}$ is continuous at x= 3

ANS: $\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3} = \ \lim_{x\to 3} \frac{(x-3)(x+3)}{x-3}$

= $\lim_{x\to 3} \ x + 3$ = 6

$f(3) = k$

given that f(x) is continuous at x = 3

$\lim_{x \to 3} f(x) = f(3)$

$k = 6$

12 For what value of k is the following function continuous at x = 0?

$$f(x) = \begin{cases} \frac{\sin 5x}{3x} & x \neq 0 \\ k & x = 0 \end{cases}$$

ANS: If the function is continuous at x = 0,

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin 5x}{3x} = \lim_{x\to 0} \frac{\sin 5x}{5x} \times \frac{5}{3}$$

$= \frac{5}{3} \lim_{x\to 0} \frac{\sin 5x}{5x} = k$

so, k = $\frac{5}{3}$

13 Discuss the continuity of the function f given by $f(x) = x^3 + x^2 - 1$.

ANS: Clearly f is defined at every real number c and its value at c is $c^3 + c^2 - 1$. We also know that

$\lim_{x \to c} f(x) = \lim_{x \to c} x^3 + x^2 - 1$

$= c^3 + c^2 - 1$

$\lim_{x \to c} f(x) = f(c)$

Hence f is continuous at every real number

14 Let $f(x) = \begin{cases} 3x - 2, & x \le 1 \\ 3x - 4, & x > 1 \end{cases}$. Is it continuous function? Justify.

ANS: domain of f is R. so we have to examine f for continuity at all $x \in R$. Let c be any real number.

Case 1 : c > 1

$RHL = \lim_{x \to c^+} f(x) = \lim_{x \to c^+} 3x - 4 = 3c - 4 = f(c)$

Case 2 : c < 1

$LHL = \lim_{x \to c^-} f(x) = \lim_{x \to c^-} 3x - 2 = 3c - 2 = f(c)$

case 3 : c = 1 , $f(1) = 3 - 2 = 1$

now, $\lim_{x \to 1^+} 3 - 4 = -1 \ne f(1)$ so, f is discontinuous at x = 1 and continuous at all other points.

15 Find all the points of discontinuity of the function f defined by

$f(x) = \begin{cases} x + 2, & x < 1 \\ 0, & x = 1 \\ x - 2, & x > 1 \end{cases}$

ANS: case1 $x < 1$

Clearly f is defined at every real number c and its value at c is c + 2, $x < 1$

case2 : $x > 1$

f is defined at every real number c and its value at c is

c $- 2$, $x > 1$

at x =1

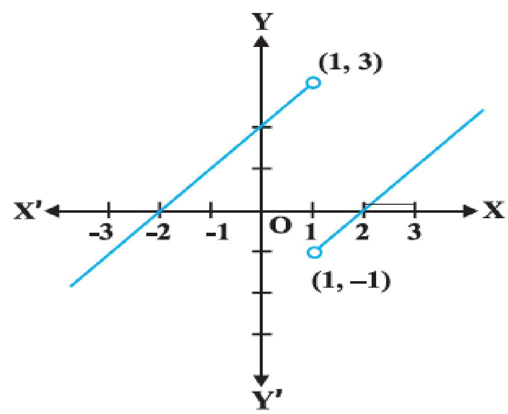
LHL = $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x + 2$ = 3

RHL = $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x - 2 = -1$

LHL ≠ RHL

f is not continuous at x = 1.

x = 1 is the only point of discontinuity of f.



16 Determine the given function continuous at x= 0 or not

$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & if\ x \ne 0 \\ 0, & if\ x = 0 \end{cases}$

ANS: $\lim_{x \to 0} x^2 \sin\frac{1}{x} = 0$ ×a finite number lies between -1 and 1 = 0

$f(0) = 0$

Note : As $x \to 0$ $\sin\frac{1}{x}$ *is a* a finite number lies between -1 and 1.

17 If the function $f(x)$ defined by

$$f(x) = \begin{cases} a \sin\frac{\pi}{2}(x+1) \quad , & x \le 0 \\ \frac{\tan x - \sin x}{x^3} \quad , & x > 0 \end{cases}$$ is continuous at x = 0, find a.

Ans: $LHL = \lim_{x \to 0} a \sin\frac{\pi}{2}(x+1) = a$

$$RHL = \lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \lim_{x \to 0} \frac{\sin x(1 - \cos x)}{\cos x .\ x^3}$$

Simplify $RHL = \frac{1}{2}$

a= ½

18 If the function $f(x)$ defined by

$$f(x) = \begin{cases} a \sin\frac{\pi}{2}(x+1) , & x \le 0 \\ \frac{\tan x - \sin x}{x^3} , & x > 0 \end{cases}$$ is continuous at x = 0, find a.

$LHL = \lim_{x \to 0} a \sin\frac{\pi}{2}(x+1) = a$

$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \lim_{x \to 0} \frac{\sin x(1 - \cos x)}{\cos x\, x^3}$ $\quad RHL = \frac{1}{2}$

19 Find the value of k so that $f(x) = \begin{cases} \frac{x^2+3x-10}{x-2} , & x \ne 0 \\ k , & x = 0 \end{cases}$ is continuous at 0.

ANS: 5

20 For what value of *k is* the function defined by

$f(x) = \begin{cases} \frac{\sin x + x\cos x}{x} , & x \ne 0 \\ k , & x = 0 \end{cases}$ continuous at $x = 0$?

ANS: 2

21 Find *k*, so that the function

$f(x) = \begin{cases} \frac{x^2-25}{x-5} , & x \ne 5 \\ k , & x = 5 \end{cases}$ is continuous at $x = 5$

ANS: 10

22 Find the value of k so that $f(x) = \begin{cases} \frac{x^2+3x-10}{x-2} , & x \ne 2 \\ k , & x = 2 \end{cases}$ is continuous at 2.

ANS: 7

23 Find λ, so that the function

$$f(x) = \begin{cases} x+\lambda, & x<3 \\ 4, & x=3 \\ 3x-5, & x>3 \end{cases}$$ is continuous at $x = 3$

ANS: 1

24 Examine the following functions for continuity

i) f(x) = $x-5$

ii) f(x) $= \frac{1}{x-5}$, $x \neq 5$

iii) f(x) $= \frac{x^2-25}{x+5}$, $x \neq -5$

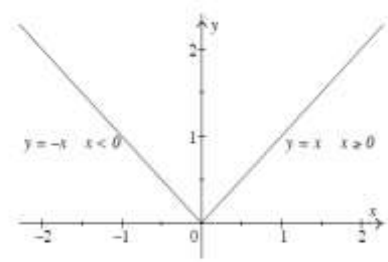
iv) f(x) = $|x-5|$

25 If a function *f* is differentiable at a point *c*, then it is also continuous at that point.
(Every differentiable function is continuous.)
The converse of the above theorem may not be true. ie. The function may be continuous at a point but may not be derivable at that point. Discuss

Ex. $y = |x|$ let us examine the continuity and differentiability at $x = 0$

Given, $y = |x|$

Check the continuity at x = 0



The graph of $y = |x|$.

ANS: Re - define the function

$$RHL = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = 0$$

$$LHL = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} -x = 0$$

$$LHL = RHL$$

$f(0) = 0$, $\lim_{x\to 0} f(x) = f(0) = 0$

so the function is continuous at $x = 0$.

For differentiability of $y = |x|$ *ie.*

$$f'(c) = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$$

$$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{|h|-0}{h}$$

$$= \lim_{h\to 0^+} \frac{|h|}{h} = \frac{h}{h} = 1 \text{ (RHD)}$$

$$= \lim_{h\to 0^-} \frac{|h|}{h} = \lim_{h\to 0^-} \frac{-h}{h} = -1$$

(LHD)

RHD ≠ LHD $f'(0)$ does not exist.

26 Examine the function f for continuity and differentiability at x=0

f(x) = $\begin{cases} 1-x^2 & ,x \le 0 \\ 1+x^2 & ,x > 0 \end{cases}$

ANS: $LHL = \lim_{x\to 0^-} f(x) \;= \lim_{x\to 0^-}(1-x^2) = 1$

$RHL = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 1+x^2 = 1$

$f(0) = 1 - 0 = 1$

$\lim_{x\to 0} f(x) = f(0) = 1$

$f(x)$ is continuous at $x = 0$

f(x) = $\begin{cases} 1-x^2 & ,x \le 0 \\ 1+x^2 & ,x > 0 \end{cases}$

$f'(c) = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$

LHD = $f'(0) = \lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-} \frac{(1-h^2)-1}{h}$

$\lim_{h\to 0^-} \frac{-h^2}{h} = 0$

RHD= $\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{(1+h^2)-1}{h} = 0$

f(x) is differentiable at x = 0

27 Show that the function

$f(x) = |x-3|$ is not differentiable at $x = 3$

$|x-3| = x-3,\ x \ge 3$

$= 3 - x\ ,\ x < 3$

$$f'(c) = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$$

RHD = $\lim_{h\to 0^+} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0^+} \frac{|3+h-3|-0}{h} = \frac{h}{h} = 1$

LHD = $\lim_{h\to 0^-} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0^-} \frac{|3+h-3|-0}{h} = \frac{-h}{h} = -1$

LHD≠ RHD

$|x-3|$ is not differentiable at $x = 3$

<u>Alternate method</u>

Show that the function

$f(x) = |x-3|$ is not differentiable at $x = 3$

$$f'(c) = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$$

RHD = $\lim_{h\to 0^+} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0^+} \frac{|3+h-3|-|3-3|}{h} = \lim_{h\to 0^+} \frac{h}{h} = 1$

LHD = $\lim_{h\to 0^-} \frac{f(3-h)-f(3)}{-h} = \lim_{h\to 0^-} \frac{|3-h-3|-|3-3|}{-h} = -1$

LHD$\neq$ RHD

$|x-3|$ is not differentiable at $x=3$

28 Differentiate : i) $y=\sin^3 x$ ii) $y=tan(x^2)$

$y=\sin^3 x$ then $y=u^3$, $u=sinx$

$y=tan(x^2)$ then $y=tanu$, $u=x^2$

29 Find the derivative of the function given by $y=sin(x^2)$

ANS: $y=sin(x^2)$

then , $y=\sin(u)$, $u=x^2$

$\frac{dy}{du}=cosu$, $\frac{du}{dx}=2x$

$$\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}\frac{dy}{dx}=cosu.2x$$

$$\frac{dy}{dx}=\cos(x^2).2x$$

30 Find the derivative of the function given by

$$y=\sqrt{tanx}$$

$$y=\sqrt{u}, \qquad u=tanx$$

$$\frac{dy}{du}=\frac{1}{2\sqrt{u}} \quad , \quad \frac{du}{dx}=sec^2x$$

$$\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$$

$\frac{dy}{dx}=\frac{1}{2\sqrt{u}}.sec^2x$

$=\frac{1}{2\sqrt{tanx}}.sec^2x=\frac{sec^2x}{2\sqrt{tanx}}$

31 Differentiate the functions with respect to *x*.

i) cos^3x

ii) $tan\sqrt{x}$

iii) $(ax+b)^{10}$

iv) $\sqrt{3x^2+5x-3}$

v) $2\,sinx+3\,cosx$

vi) $\frac{1}{3}\,e^x-5x^3$

vii) $\frac{3cosx+5}{sinx}$

viii) $tanx+2sinx+3\,cosx\,-\frac{1}{2}logx\,-e^{2x}$

ANS: i)

$y=\ cos^3x$

$\frac{dy}{dx}=3cos^2x\,.\,(-sinx)=-3\,cos^2x\,.\,sinx$

ANS: ii)

$$\text{y}=tan\sqrt{x}\frac{dy}{dx}=sec^2\sqrt{x}.\ \left(\frac{1}{2\sqrt{x}}\right)=\frac{1}{2}.\ \frac{sec^2\sqrt{x}}{\sqrt{x}}$$

ANS: iii)

$$y = (ax+b)^{10}$$

$\frac{dy}{dx} = 10\,(ax+b)^9 . a = 10a\,(ax+b)^9$

ANS: iv)

$$\text{y} = \sqrt{3x^2+5x-3}$$

$\frac{dy}{dx} = \frac{1}{2\sqrt{3x^2+5x-3}} . 6x+5 = \frac{6x+5}{2\sqrt{3x^2+5x-3}}$

ANS: v)

$$\text{y} = 2\,sinx + 3\,cosx \frac{dy}{dx} = 2cosx - 3sinx$$

ANS: vi)

$$\text{y} = \frac{1}{3}\,e^x - 5x^3 \frac{dy}{dx} = \frac{1}{3}\,e^x - 15x^2$$

ANS: vii)

$$\text{y} = \frac{3cosx+5}{sinx} = 3cotx + 5cosecx \frac{dy}{dx} = -3cosec^2x - 5cosecx\,cotx$$

ANS: viii)

$$\text{y} = tanx + 2sinx - cosx - \frac{1}{2}logx - e^{2x} \frac{dy}{dx} = sec^2x + 2cosx + sinx - \frac{1}{2x} - 2\,e^{2x}$$

32 Differentiate the functions with respect to x

1) $y = sinx^5$ 2) $y = \tan(sinx^5)$ 3) $y = log\,\{\tan(sinx^5)$

Differentiate the functions with respect to x

1) $y = sinx^5$

$\frac{dy}{dx} = cosx^5 .\ 5x^4 = 5x^4cosx^5.$

2) $y = \tan(sinx^5)$

$\frac{dy}{dx} = sec^2(sinx^5).\ cosx^5.\ 5x^4$

3) $y = log\,\{\tan(sinx^5)\}$

$\frac{dy}{dx} = \frac{1}{\tan(sinx^5)}\ sec^2(sinx^5).\ cosx^5.\ 5x^4$

TRY YOURSELF

1) $\tan(2x+3)$
2) $\cos(sinx)$
3) $(x^2+cosx)^4$
4) $\cos(sinx^2)$
5) sin^3x

33 If $y = cos(sinx^2)$, find $\frac{dy}{dx}\ at\ x = \sqrt{\frac{\pi}{2}}$

ANS: $y = cos(sinx^2)$

$$\frac{dy}{dx} = -\sin(sinx^2).\ cosx^2.2x$$

$= -2\,x\sin(sinx^2).\ cosx^2$

$\frac{dy}{dx} at\ \ x = \sqrt{\frac{\pi}{2}} = -2 \times \sqrt{\frac{\pi}{2}}\ \sin\left(sin\left(\sqrt{\frac{\pi}{2}}\ \right)^2\right).\ cos\left(\sqrt{\frac{\pi}{2}}\ \right)^2$

$= 0$ $\quad (\cos\frac{\pi}{2} = 0)$

34 Find $\frac{dy}{dx}$ if $y = (x^2 + cosx)^4$

$\frac{dy}{dx} = 4\,(x^2 + cosx)^3 \times (2x - sinx)$

35 If $y = logtan\frac{x}{2}$, find $\frac{dy}{dx}$

ANS:

$y = logtan\frac{x}{2}$

$\frac{dy}{dx} = \frac{1}{tan\frac{x}{2}}.\ sec^2\frac{x}{2}.\frac{1}{2}$

$= \frac{1}{2}\ .\ \frac{cos\frac{x}{2}}{sin\frac{x}{2}}.\frac{1}{cos^2\frac{x}{2}} = \frac{1}{2sin\frac{x}{2}cos\frac{x}{2}}$

$= \frac{1}{sinx} = cosecx$

36 Find $\frac{dy}{dx}$ if $y = sec\left(tan\sqrt{x}\right)$

ANS: $y = sec\left(tan\sqrt{x}\right)$

$\frac{dy}{dx} = sec\left(tan\sqrt{x}\right)tan\left(tan\sqrt{x}\right).sec^2\sqrt{x}.\left(\frac{1}{2\sqrt{x}}\right)$

37 If $y = e^x log(sin2x),\ find\ \frac{dy}{dx}$

ANS : $y = e^x log(sin2x)$

Apply product rule.

$\frac{dy}{dx} = e^x \times \frac{d}{dx}(log(sin2x))+ log(sin2x) \times \frac{d}{dx}(e^x)$

$= e^x.\ \frac{1}{sin2x}.cos2x.2 + log(sin2x).\ e^x$

$= e^x(2cot2x) + log(sin2x)$

38 Find the derivative of the function given by :

i) $y = cos(1 - x^2)^2$

ii) $y = cos(sinx^2)$

iii) $y = \sqrt{sinx^3}$

iv) $y = cosec(1 + x^2)$

v) $y = sec^2\left(\frac{x}{a}\right)$

vi) $y = tan^4(x^4)$

vii) $y = e^{2x} sin3x$

viii) $y = e^x \log(1 + x^2)$

ix) $y = logtan\sqrt{x}$

x) $y = \cos(logsinx)$

xi) $y = \frac{1}{logcosx}$

xii) $y = e^x \text{logx}$

39 If $y = x^2 sin\frac{1}{x}$, find $\frac{dy}{dx}$

$y = x^2 sin\frac{1}{x}$

Apply product rule.

$\frac{dy}{dx} = x^2 . cos\left(\frac{1}{x}\right)\frac{-1}{x^2} + sin\left(\frac{1}{x}\right). 2x$

$= -cos\left(\frac{1}{x}\right) + 2x\ sin\left(\frac{1}{x}\right)$

40 If $y = \frac{x}{sin3x}$, find $\frac{dy}{dx}$

ANS: $y = \frac{x}{sin3x}$

Apply quotient rule

$\frac{dy}{dx} = \frac{sin3x \times \frac{d}{dx}(x) - x\frac{d}{dx}(sin3x)}{sin^2 3x}$

$= \frac{sin3x \times 1 \ - x\ (cos3x) \times 3}{sin^2 3x}$

$= \frac{sin3x \ \ -3x\ (cos3x)}{sin^2 3x}$

41 Differentiate $log\sqrt{\frac{1 - cosx}{1 + cosx}}$, w.r.t. x.

ANS: y = $log\sqrt{\frac{1 - cosx}{1 + cosx}}$

y = $log\sqrt{\frac{2sin^2\frac{x}{2}}{2cos^2\frac{x}{2}}} = log \tan\frac{x}{2}$

$\frac{dy}{dx} = \frac{1}{\tan\frac{x}{2}} .\ sec^2\frac{x}{2} . \frac{1}{2}$

$\frac{dy}{dx} = \frac{cos\frac{x}{2}}{\sin\frac{x}{2}} . \frac{1}{cos^2\frac{x}{2}} . \frac{1}{2} \quad = \frac{1}{2\sin\frac{x}{2} . cos\frac{x}{2}} = \frac{1}{sinx} = cosecx$

42 Differentiate $e^{5\ log\ x}$ w.r.t. x.

ANS: $let\ y = e^{5\ log\ x} = e^{logx^5} = x^5$, $\frac{dy}{dx} = 5x^4$

$\{ e^{mlogx} = e^{\log x^m} = x^m \}$ why?

let $e^{\log n} = y$
Take log on both sides
log($e^{\log n}$)= logy
$\log n$ (loge) = logy
$\log n$ = logy , n = y

43 Differentiate $y = e^x \log(\sin 2x)$ with respect to x.

ANS: $y = e^x \log(\sin 2x)$

$\frac{dy}{dx} = e^x \cdot \frac{d}{dx}(\log \sin 2x) + \log \sin 2x \cdot \frac{d}{dx}(e^x)$

$= e^x \frac{1}{sin2x} \cos 2x \cdot 2 + \log(\sin 2x) \cdot e^x$

$= e^x[2 \cot 2x + \log(\sin 2x)]$

44 Find $\frac{dy}{dx}$ if $y = log\left(sec\frac{x}{2} + tan\frac{x}{2}\right)$

ANS: $\frac{1}{2}sec\frac{x}{2}$

45 If $y = \frac{x-1}{sin3x}$, find $\frac{dy}{dx}$

ANS: $y = \frac{x-1}{sin3x}$

Apply quotient rule

$$\frac{dy}{dx} = \frac{sin3x \times \frac{d}{dx}(x-1) - (x-1)\frac{d}{dx}(sin3x)}{sin^2 3x}$$

$= \frac{sin3x \times 1 \ -(x-1)(cos3x)\times 3}{sin^2 3x}$

$= \frac{sin3x \ \ -3(x-1)(cos3x)}{sin^2 3x}$

46 Find $\frac{dy}{dx}$ if $y = \sqrt{\cos(1+x^2)}$

ANS: $\frac{x\sin(1+x^2)}{\sqrt{\cos(1+x^2)}}$

47 If $f(x) = \sqrt{1 + cos^2(x^2)}$ find $f'\left(\frac{\sqrt{\pi}}{2}\right)$

ANS: $-\sqrt{\frac{\pi}{6}}$

48 Find $\frac{dy}{dx}$ in the following $2x + 3y = sinx$

ANS: differentiating both sides

$$2 + 3\frac{dy}{dx} = cosx$$

$\frac{dy}{dx} = \frac{1}{3}(cosx - 2)$

49 Find $\frac{dy}{dx}$, $x^2 + xy + y^2 = 100$

ANS: differentiating both sides

$$2x + x\frac{dy}{dx} + y.1 + 2y\frac{dy}{dx} = 0\,\frac{dy}{dx}(2y + x) = -(2x + y)\,\frac{dy}{dx} = \frac{-(2x + y)}{(2y + x)}$$

50 Find $\frac{dy}{dx}$ if $x^2 + y^2 = r^2$

ANS: given, $x^2 + y^2 = r^2$

differentiating both sides

$2x + 2y\,\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-x}{y}$$

51 Find $\frac{dy}{dx}$ if $xy = 100(x + y)$

Find $\frac{dy}{dx}$ if $xy = 100(x + y)$

Given $xy = 100(x + y)$

differentiating both sides

$$x.\ \frac{dy}{dx} + \ y.1 = 100(1 + \frac{dy}{dx})$$

$x.\ \frac{dy}{dx} - 100\,\frac{dy}{dx} = 100 - \mathrm{y}$

$\frac{dy}{dx}(x - 100) = 100 - \mathrm{y}$

$$\frac{dy}{dx} = \frac{(100 - \mathrm{y})}{(x - 100)}$$

52 Find $\frac{dy}{dx}$ if $(x^2 + y^2)^2 = xy$

Find $\frac{dy}{dx}$ if $(x^2 + y^2)^2 = xy$.

differentiating both sides

$\frac{d}{dx}(x2\ +\ y2)2 = \ \frac{d}{dx}(xy)$

$$2(x2\ +\ y2)\left(2x + 2y.\frac{dy}{dx}\right) = x.\frac{dy}{dx} + y.1$$

$$4x\left(x2\ +\ y2\right) + 4y\left(x2\ +\ y2\right)\frac{dy}{dx} = \ x.\frac{dy}{dx} + y$$

$$\frac{dy}{dx} = \ \frac{y - 4x(x^2 + y^2)}{4y(x^2 + y^2) - x}$$

53 Find $\frac{dy}{dx}$ if $xy^2 - x^2y = 4$

Find $\frac{dy}{dx}$ if $xy^2 - x^2y = 4$

ANS: differentiating both sides

$x.2y\frac{dy}{dx} + y^2.1 - \left(x^2.\frac{dy}{dx} + y.2x\right) = 0$

$$(2xy - x^2)\frac{dy}{dx} = 2xy - y^2$$

$$\frac{dy}{dx} = \frac{2xy - y^2}{2xy - x^2} = \frac{y(2x-y)}{x(2y-x)}$$

54 1. If $\frac{x}{a} + \frac{y}{b} = 1$ $find\ \frac{dy}{dx}$ ANS: $-\frac{b}{a}$

2. Find $\frac{dy}{dx}$ at (4, 9) if $\sqrt{x} + \sqrt{y} = 5$

3. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2022$ ANS: $\frac{dy}{dx} = -\frac{b^2x}{a^2y}$

4. $(x^2 - y^2)^2 = 4xy$ ANS: $\frac{dy}{dx} = \frac{-x^3+xy^2+y}{y^3-yx^2-x}$

55 If $y = \tan^{-1}\left(\frac{7x}{1-12x^2}\right)$. Find $\frac{dy}{dx}$

ANS: $y = \tan^{-1}\left(\frac{7x}{1-12x^2}\right)$.

$$y = \tan^{-1}\left(\frac{4x+3x}{1-4x.\,3x}\right).$$

$y = \tan^{-1} 4x + \tan^{-1} 3x$ $\frac{dy}{dx} = \frac{1}{1+(4x)^2} \times 4 + \frac{1}{1+(3x)^2} \times 3$

$$\frac{dy}{dx} = \frac{4}{1+16x^2} + \frac{3}{1+9x^2}$$

56 Similar qns:

1. If $y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right)$. Find $\frac{dy}{dx}$

2. If $y = \tan^{-1}\left(\frac{4x}{1+5x^2}\right)$. Find $\frac{dy}{dx}$

57 Find $\frac{dy}{dx}$ $if\ y = \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$

ANS: let $x = asin\theta \Rightarrow \theta = \sin^{-1}\frac{x}{a}$

$y = \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) = \tan^{-1}\left(\frac{asin\theta}{\sqrt{a^2-(asin\theta)^2}}\right)$

$y = \tan^{-1}\left(\frac{asin\theta}{\sqrt{a^2(1-sin^2\theta)}}\right)$

$y = \tan^{-1}\left(\frac{asin\theta}{a\,cos\theta}\right) = \tan^{-1}\tan\theta = \theta$

$y = \sin^{-1}\frac{x}{a}$

$\frac{dy}{dx} = \frac{1}{\sqrt{a^2-x^2}}$

58 If $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ find $\frac{dy}{dx}$

ANS: putting $x = \cos\theta$, $\theta = \cos^{-1} x$

LHS= $\tan^{-1}\left(\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right)$

$y = \tan^{-1}\left(\frac{\sqrt{2cos^2\theta/_2} - \sqrt{2sin^2\theta/_2}}{\sqrt{2cos^2\theta/_2} + \sqrt{2sin^2\theta/_2}}\right)$

$y = \tan^{-1}\left(\frac{\cos\theta/_2 - \sin\theta/_2}{\cos\theta/_2 + \sin\theta/_2}\right)$

Dividing Nr. And Dr. by $\cos\theta/_2$ inside

$y = \tan^{-1}\left(\frac{1 - \tan\theta/_2}{1 + \tan\theta/_2}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right) = \frac{\pi}{4} - \frac{\theta}{2}$

$y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$

$$\frac{dy}{dx} = \frac{\mathbf{1}}{2\sqrt{\mathbf{1} - \boldsymbol{x}^{\mathbf{2}}}}$$

59 Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{1 - \cos x}{sinx}\right)$

60 Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{\sqrt{x} + \sqrt{a}}{1 - \sqrt{ax}}\right)$

61 If $y = \sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$ find $\frac{dy}{dx}$

ANS:

$$y = \sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$$

$$y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) y = \sin^{-1}\left(cos\frac{\pi}{4}\sin x + sin\frac{\pi}{4}\cos x\right)$$

$$y = \sin^{-1}\left(\sin\left(x + \frac{\pi}{4}\right)\right) = x + \frac{\pi}{4}\frac{dy}{dx} = 1$$

62 Differentiate the given function with respect to x. $y = \tan^{-1}\left\{\frac{\sqrt{1+sinx} + \sqrt{1-sinx}}{\sqrt{1+sinx} - \sqrt{1-sinx}}\right\}$

$$y = \tan^{-1}\left\{\frac{\sqrt{1 + sinx} + \sqrt{1 - sinx}}{\sqrt{1 + sinx} - \sqrt{1 - sinx}}\right\}$$

$$1 + sinx = sin^2\frac{x}{2} + cos^2\frac{x}{2} + 2\sin\frac{x}{2}cos\frac{x}{2} = \left(\sin\frac{x}{2} + cos\frac{x}{2}\right)^2$$

$$1 - sinx = sin^2\frac{x}{2} + cos^2\frac{x}{2} - 2\sin\frac{x}{2}cos\frac{x}{2} = \left(cos\frac{x}{2} - \sin\frac{x}{2}\right)^2$$

$$y = \tan^{-1}\left\{\frac{\sqrt{\left(\sin\frac{x}{2} + cos\frac{x}{2}\right)^2} + \sqrt{\left(cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2} + cos\frac{x}{2}\right)^2} - \sqrt{\left(cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}\right\}$$

$$y = \tan^{-1}\left\{\frac{\sin\frac{x}{2} + cos\frac{x}{2} + cos\frac{x}{2} - sin\frac{x}{2}}{\sin\frac{x}{2} + cos\frac{x}{2} - \left(cos\frac{x}{2} - sin\frac{x}{2}\right)}\right\}$$

$y = tan^{-1}\left\{\frac{cos\frac{x}{2}}{sin\frac{x}{2}}\right\} = \tan^{-1}\ \cot\frac{x}{2} = \tan^{-1}\tan\left(\frac{\pi}{2}-\frac{x}{2}\right) = \frac{\pi}{2}-\frac{x}{2}$

$\frac{dy}{dx} = -\frac{1}{2}$

63 if $y = \tan^{-1}\left(\frac{4x}{1+5x^2}\right) + \tan^{-1}\left(\frac{2+3x}{3-2x}\right)$

$$y = \tan^{-1}\left(\frac{5x-x}{1+5x.x}\right) + \tan^{-1}\left(\frac{\frac{2}{3}+x}{1-\frac{2}{3}x}\right)$$

$$y = \tan^{-1}5x - \tan^{-1}x + \tan^{-1}\frac{2}{3} + \tan^{-1}x$$

$$\frac{dy}{dx} = \frac{1}{1+25x^2}\times 5 = \frac{5}{1+25x^2}$$

64 If $y = cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$, Find $\frac{dy}{dx}$

$y = cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right) = \cos^{-1}\left(\frac{2.2^x}{1+(2^x)^2}\right)$

$$\theta = \tan^{-1}2^x$$

65 $If\ \ sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$ find $\frac{dy}{dx}$

66 Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right]$

67 If $y = \tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$, Find $\frac{dy}{dx}$

68 Differentiate x^x, $x > 0$ w.r.t. x.

ANS: let $y = x^x$

Taking logarithm on both sides, we have

$logy = x\ logx$ *differentiating both sides*

$\frac{1}{y}\ \frac{dy}{dx} = x.\frac{1}{x} + logx.1$

$$\frac{dy}{dx} = y(1+logx).\frac{dy}{dx} = x^x(1+logx).$$

Similarly, Differentiate a^x = ?

69 If $x^x = y^y$, then find $\frac{dy}{dx}$

ANS: $x^x = y^y$

taking log on both sides

$$xlogx = ylogyx.\frac{1}{x} + logx.1 = y.\frac{1}{y}\ \frac{dy}{dx} + logy.\frac{dy}{dx}$$

$1 + logx = \frac{dy}{dx}(1+logy)$

$$\frac{dy}{dx} = \frac{1+logx}{1+logy}$$

70 If $y = x^{\cos^{-1}x}$ then find $\frac{dy}{dx}$

ANS: y = $x^{\cos^{-1}x}$

$$\log y = \cos^{-1}x.\ logx$$

$\frac{1}{y}\frac{dy}{dx} = \cos^{-1}x\ .\ \frac{1}{x} + logx\,\frac{-1}{\sqrt{1-x^2}}.$

$$\frac{dy}{dx} = y\left(\cos^{-1}x\ .\ \frac{1}{x} + logx\,\frac{-1}{\sqrt{1-x^2}}.\ \right)$$

$= x^{\cos^{-1}x}\left(\frac{1}{x}\ \cos^{-1}x - \frac{logx}{\sqrt{1-x^2}}\right)$

71 Differentiate x^{sinx}, $x > 0$ w.r.t. x.

ANS: let $y = x^{sinx}$

Taking logarithm on both sides, we have

$logy = sinx\ logx$ *differentiating both sides*

$\frac{1}{y}\ \frac{dy}{dx} = sinx.\frac{1}{x} + logx\ .\ cosx$

$$\frac{dy}{dx} = y\left(\frac{sinx}{x} + logx.\ cosx\right)$$

$$\frac{dy}{dx} = x^{sinx}\left(\frac{sinx}{x} + logx.\ cosx\right)$$

72 If $(\cos x)^y = (cos\ y)^x$ find $\frac{dy}{dx}$

Ans: $(\cos x)^y = (cos\ y)^x$

Taking log on both sides

$$y\log cosx = x\log cosy\ y.\frac{-sinx}{cosx} + \log cosx\ .\ \frac{dy}{dx} = x\ \frac{-siny}{cosy}\ .\ \frac{dy}{dx} + logcosy$$

$$\frac{dy}{dx} = \frac{\log cos\,y + y\tan x}{\log cosx +\ xtany}$$

73 If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+logx)^2}{logy}$

ANS: Consider, $y^x = e^{y-x}$

Taking log on both sides, we get

$x\log y = (y - x)\ loge$

$x\log y = y - x$

$x(1 + logy) = y \Rightarrow x = \frac{y}{1+logy}$ *differentiate w:r:t y*

$\frac{dx}{dy} = \frac{(1+logy).1-y.\frac{1}{y}}{(1+logy)^2} = \frac{(1+logy)-1}{(1+logy)^2}$

$$\frac{dy}{dx} = \frac{(1+logx)^2}{logy}$$

74 y = $x^{x^{x^{x^{\dots\dots\infty}}}}$ find $\frac{dy}{dx}$

ANS: y = $x^{x^{x^{x^{\dots\dots\infty}}}}$

y = x^y

taking log both sides, log y = log (x^y)

log y = ylog (x) , differentiating,

$$\frac{1}{y}\ \frac{dy}{dx} = y.\frac{1}{x} + logx.\ \frac{dy}{dx}$$

$$\left(\frac{1}{y} - logx\right)\frac{dy}{dx} = \frac{y}{x} \Rightarrow \quad \left(\frac{1 - ylogx}{y}\right)\frac{dy}{dx} = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{y^2}{x(1 - ylogx)}$$

75 If y = $(sinx)^{(sinx)^{(sinnx)^{\dots\dots to\infty}}}$ prove that $\frac{dy}{dx} = \frac{y^2 cotx}{1 - y\, logsinx}$

ANS: $y = (\sin x)^y$ Taking log on both sides, we get

log y = y log(sin x) On differentiating both sides,

76 i) $y = x^{(x^2)}$ Find $\frac{dy}{dx}$ ANS: $x^{(x^2)}$[x + 2xlogx] = $x^{(x^2+1)}$[1 + 2logx]

ii) $y = x^{(sin2x+cos2x)}$ Find $\frac{dy}{dx}$

ANS: $x^{(sin2x+cos2x)}\{\frac{sin2x+cos2x}{x} + 2(cos2x - sin2x).logx\}$

iii) $y = \left(\sqrt{x}\right)^{(\sqrt{x})^{(\sqrt{x})^{\sqrt{x}\dots\dots}}}$ Find $\frac{dy}{dx}$ ANS: $\frac{y^2}{x(2-y\ logx)}$

77 Find $\frac{dy}{dx}$ If $(\cos x)^y = (\sin y)^x$

ANS: taking log on both sides

y log$(cosx)$ = $x\ log(siny)$

Differentiating both sides w.r.t. x, we get

$$y.\ \frac{1}{cosx}.(-sinx) + \log(cosx).\frac{dy}{dx} = x.\frac{1}{siny}.(cosy)\frac{dy}{dx} + \log(siny).1$$

$$\log(cosx).\frac{dy}{dx} - xcoty.\frac{dy}{dx} = \text{ytanx} + \log(siny).$$

$$\frac{dy}{dx} = \frac{\text{ytanx} + \log(siny).}{\log(cosx) - xcoty.}$$

78 If $x^p\ y^q\ =\ (x+y)^{p\,+\,q}\ prove\ that\ \frac{dy}{dx} = \frac{y}{x}$

Taking log on both sides, we get p log x + q log y = (p + q) log (x + y)

Differentiate both sides w.r.t. x, we get

79 If $x\sin(a+y)+\sin a\cos(a+y)=0$, prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

80 If $y=\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$, find $\frac{dy}{dx}$

ANS: $y=\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)=\tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)=\tan^{-1}\tan\frac{x}{2}=\frac{\theta}{2}=\frac{x}{2}$

$\frac{dy}{dx}=\frac{1}{2}$

81 Find $\frac{dy}{dx}$ if $y=(\log x)^x+(x)^{\log x}$

82 TRY YOURSELF

1. *If* $y=(\sin x)^{\tan x}+(\cos x)^{\sec x}$, find $\frac{dy}{dx}$.

ANS: $(\sin x)^{\tan x}\left(1+\log\sin x.\ \sec^2 x\right)+$
$(\cos x)^{\sec x}\sec x.\tan x(\log\cos x-1)$

2. *If* $y=(\cos x)^{\log x}+(\log x)^x$, find $\frac{dy}{dx}$.

ANS: $(\cos x)^{\log x}\left[\frac{\log\cos x}{x}-\log x.\ \tan x\right]+$
$(\log x)^x\left[\log(\log x)+\frac{1}{\log x}\right]$

83 i) $y=x^{\sin x}+\sin x^{\cos x}$ *find* $\frac{dy}{dx}$

ii) $y=(\sin x)^x+\sin^{-1}\sqrt{x}$ *find* $\frac{dy}{dx}$

ANS: $\frac{dy}{dx}=(\sin x)^x[x\cot x+\log\sin x]+\frac{1}{2\sqrt{x-x^2}}$

iii) $y=(\sin x)^{\tan x}+(\cos x)^{\sec x}$ *find* $\frac{dy}{dx}$

ANS: $(\sin x)^{\tan x}$[1+ log(sinx) .$\sec^2 x$] + $(\cos x)^{\sec x}$ sec $x\tan x$[logcosx-1]

84 Find $\frac{dy}{dx}$ if $y=(x)^{x\cos x}+\frac{x^2+1}{x^2-1}$

85 $If\ x^{16}y^9=(x^2+y)^{17}$

Similar (Do as home work)

If $x^{13}y^7=(x+y)^{20}$, *prove that* $\frac{dy}{dx}=\frac{y}{x}$

If $x^4y^5=(x+y)^9$, *prove that* $\frac{dy}{dx}=\frac{y}{x}$

If $x^{\frac{1}{2}}y^{\frac{3}{2}}=(x+y)^2$, *prove that* $\frac{dy}{dx}=\frac{y}{x}$

86 HOME WORK

Find $\frac{dy}{dx}$

1. $x^{\frac{1}{x}}$ ANS: $x^{\frac{1}{x}}\left(\frac{1-\log x}{x^2}\right)$

2. $x^{\sqrt{x}}$ ANS: $x^{\sqrt{x}}\left(\frac{2+\log x}{2\sqrt{x}}\right)$

3. $(\sin x)^{\tan x}$

ANS: $(\sin x)^{\tan x}(1+\sec^2 x.\log\sin x)$

4. $(\sin^{-1}x)^x$

5. $(x)^{\sin^{-1}x}$

6. $x^x \ \sin^{-1}\sqrt{x}$

87 If $y=\sqrt{sinx+\sqrt{sinx+\sqrt{sinx+\sqrt{sinx+\cdots..}}}}$ prove that $\frac{dy}{dx}=\frac{cosx}{2y-1}$

88 *If* $y=(sinx)^{tanx}+(cosx)^{secx}$, find $\frac{dy}{dx}$.

ANS: $(sinx)^{tanx}\left(1+logsinx.\ sec^2x\right)+$

$$(cosx)^{secx}secx.tanx(logcosx-1)$$

Similar

If $y=(cosx)^{logx}+(logx)^x$, find $\frac{dy}{dx}$.

ANS: $(cosx)^{logx}\left[\frac{logcosx}{x}-logx.\ tanx\right]+$

$$(logx)^x\left[\log(logx)+\frac{1}{logx}\right]$$

89 $x=acos\theta,\ \ y=b\ sin\theta$, find $\frac{dy}{dx}$

ANS: $x=acos\theta,$

$$\frac{dx}{d\theta}=-asin\theta\quad y=b\ sin\theta\ \frac{dy}{d\theta}=b\ cos\theta$$

$$\frac{dy}{dx}=\frac{b\ cos\theta}{-asin\theta}=-\frac{b}{a}\ cot\theta$$

90 If $x=ae^\theta(sin\theta-cos\theta) and\ y=ae^\theta(sin\theta+cos\theta)$, find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$

ANS: $x=ae^\theta(sin\theta-cos\theta)\ \Longrightarrow\ \frac{dx}{d\theta}=2ae^\theta(sin\theta)$

$$y=ae^\theta(sin\theta+cos\theta)\Longrightarrow\frac{dy}{d\theta}=2ae^\theta(cos\theta)$$

$$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=cot\theta$$

at $\theta=\frac{\pi}{4}$ $\frac{dy}{dx}=1$

91 If $x=sint,\quad y=cos2t$, find $\frac{dy}{dx}$

92 If $x=cos\theta-cos2\theta,\ y=sin\theta-sin2\theta,$ find $\frac{dy}{dx}$

93 If $x=sint,\quad y=cos2t$, find $\frac{dy}{dx}$ ANS: $-4\ sint$

94 If $x=a(\theta-sin\theta),\ y=a(1+cos\theta),$ find $\frac{dy}{dx}$

95 If $x=\ a(cost+tsint),\ y=a(sint-tcost),$ find $\frac{dy}{dx}$

96 If $x=10(t-sint),\ y=12\ (1-cost)$ find $\frac{dy}{dx}$

97 If $x=e^\theta\left(\theta+\frac{1}{\theta}\right) and\ y=e^{-\theta}\left(\theta-\frac{1}{\theta}\right)$ find $\frac{dy}{dx}$.

98 If y $= a\,sint$ and x $= a\left(\cos t + \log\tan\frac{t}{2}\right)$ $find\ \frac{dy}{dx}$

99 If $x = asec^3\theta\ \ and\ y = atan^3\theta$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$

100 If $x = asec\theta$, y = b $\tan\theta$, find $\frac{dy}{dx}$

101 If $x = 2cos^2\theta,\ y = 2\ sin^2\theta$, find $\frac{dy}{dx}$

102 Find the second derivative $\frac{d^2y}{dx^2}$ of i) $y = x^2 + 3x + 2$ ii) $y = logx$ iii) $y = x^3logx$

ANS: i) $y = x^2 + 3x + 2$

$\frac{dy}{dx} = 2x + 3$, $\frac{d^2y}{dx^2} = 2$

ANS: ii) $y = logx$

$\frac{dy}{dx} = \frac{1}{x}$, $\frac{d^2y}{dx^2} = -\frac{1}{x^2}$

103 $y = e^x sin5x$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

ANS: $y = e^x sin5x$

$\frac{dy}{dx} = e^x cos5x.5 + sin5x.e^x$

$= e^x(5\ cos5x + sin5x)$

$\frac{d^2y}{dx^2} = e^x(-25\ sin5x + 5\cos5x) + e^x(5\ cos5x + sin5x)$

$$\frac{d^2y}{dx^2} = e^x(-24sin5x + 10cos5x)$$

104 $If\ y = \tan^{-1}x,\ show\ that$ $(1 + x^2)y_2 + 2xy_1 = 0$

ANS: $\frac{dy}{dx} = \frac{1}{1+x^2}$

$(1 + x^2)\ \frac{dy}{dx} = 1$

$(1 + x^2).\frac{d^2y}{dx^2} + \frac{dy}{dx}.2x = 0$

$(1 + x^2)y_2 + 2xy_1 = 0$

105 i) $y = \sin^{-1}x$, $show\ that$ $(1 - x^2)y_2 - xy_1 = 0$

ii) If $y = P\,e^{mx} + Qe^{nx}$, $then\ prove\ that$ $y_2 - (m + n)y_1 + mn\,y = 0.$

iii) $y = 3\,e^{2x} + 2e^{3x}$ Prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

106 $i)$ $e^y(x + 1) = 1,\ show\ that$ $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

$ii)\ If\ y = e^{\tan^{-1}x},\ show\ that$ $(1 + x^2)y_2 + (2x - 1)y_1 = 0$

107 $If\ y = e^{\tan^{-1}x},\ show\ that$ $(1 + x^2)y_2 + (2x - 1)y_1 = 0$

ANS: $\frac{dy}{dx} = e^{\tan^{-1}x}\frac{1}{1+x^2}$

$(1 + x^2)\ \frac{dy}{dx} = e^{\tan^{-1}x}(= y)$

$(1+x^2).\frac{d^2y}{dx^2}+\frac{dy}{dx}.2x = \frac{dy}{dx}$

$(1+x^2)y_2+(2x-1)y_1 = 0$

108 If x = $tan\left(\frac{1}{a}logy\right)$, $show\ that\ \ (1+x^2)y_2+(2x-a)y_1 = 0$

Hint : $\tan^{-1}x = \frac{1}{a}logy \qquad atan^{-1}x = logy$

$e^{atan^{-1}x} = y$, y = $e^{atan^{-1}x}$

now differentiate….

109 If $y = (\tan^{-1}x)^2\ show\ that \qquad (x^2+1)^2y_2+2x(x^2+1)y_1 = 2$

110 If $x = a(\theta - sin\theta)$, $y = a(1+cos\theta)$, find $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$

$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(-cot\frac{\theta}{2}\right). \ \frac{1}{a(1-cos\theta)}$

$= \ -\left(-cosec^2\frac{\theta}{2}.\frac{1}{2}\right).\frac{1}{a(2sin^2\frac{\theta}{2})} = \frac{1}{4a}\ cosec^4\frac{\theta}{2}$

111 Find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$, $\quad x = a(\theta + sin\theta)\ , y = a(1-cos\theta)$

ANS: $\frac{dx}{d\theta} = a(1+cos\theta)$, $\frac{dy}{d\theta} = a(sin\theta)$

$\frac{dy}{dx} = \frac{a(sin\theta)}{a(1+cos\theta)} = \frac{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}.}{2cos^2\frac{\theta}{2}} = tan\frac{\theta}{2}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}\left(\frac{dy}{dx}\right).\frac{d\theta}{dx} = \frac{d}{d\theta}\left(tan\frac{\theta}{2}\right).\frac{1}{a(1+cos\theta)}$

$=\frac{1}{2}sec^2\frac{\theta}{2}.\frac{1}{a(2cos^2\frac{\theta}{2})} = \frac{1}{4acos^4\frac{\theta}{2}}$

$\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ is $\frac{4}{4a} = \frac{1}{a}$

112 If $x = ae^{\theta}(sin\theta - cos\theta) and\ y = ae^{\theta}(sin\theta + cos\theta)$ find $\frac{d^2y}{dx^2}$

ANS: $x = ae^{\theta}(sin\theta - cos\theta) \implies \frac{dx}{d\theta} = 2ae^{\theta}(sin\theta)$

$y = ae^{\theta}(sin\theta + cos\theta) \implies \frac{dy}{d\theta} = 2ae^{\theta}(cos\theta)$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = cot\theta$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}\left(\frac{dy}{dx}\right).\frac{d\theta}{dx}$

$= \frac{d}{d\theta}(cot\theta).\frac{1}{2ae^{\theta}(sin\theta)} = -cosec^2\theta.\ cosec\theta.\frac{1}{2ae^{\theta}}$

$\frac{d^2y}{dx^2} = \frac{-cosec^3\theta}{2ae^{\theta}}$

113 If y = $a\,(1+sint)$ and x = $a\left(\cos t + \log\tan\frac{t}{2}\right)\ find\ \frac{d^2y}{dx^2}$.

114 If $x = acos^3\theta\ \ and\ y = asin^3\theta\ \ find \ \ \frac{d^2y}{dx^2}$ at x = $\frac{\pi}{6}$

115 Differentiate x^2 with respect to x^3.

$Let\ u = x^2\ ,\ and\ v = x^3$

We have to find $\frac{du}{dv}$

$\frac{du}{dx} = 2x, \quad \frac{dv}{dx} = 3x^2$

$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{3x^2} = \frac{2}{3x}$

116 Differentiate : $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ w.r.t $\sin\left(2\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$

ANS: $u = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$, $v = \sin\left(2\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$

Sub. $x = \sin\theta$

$u = \tan^{-1}\left(\frac{\sin\theta}{1+\sqrt{1-(\sin\theta)^2}}\right) = \tan^{-1}\left(\frac{\sin\theta}{1+cos\theta}\right)$

$\tan^{-1}\left(\frac{\sin\theta}{1+cos\theta}\right) = \tan^{-1}\left(\frac{2\sin\theta/2\cos\theta/2}{2cos^2\theta/2}\right) = \tan^{-1}(\tan\theta/2)$

$u = \theta/2$

$u = \frac{1}{2}\sin^{-1}x$, $\qquad \frac{du}{dx} = \frac{1}{2}\ \frac{1}{\sqrt{1-x^2}}$

$v = \sin\left(2\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$

$x = cos\theta$

$v = \sin\left(2\cot^{-1}\sqrt{\frac{1+\cos\theta}{1-cos\theta}}\right) = \sin\left(2\cot^{-1}\sqrt{\frac{2\,cos^2\theta/2}{2\,sin^2\theta/2}}\right)$

$= \sin\left(2\cot^{-1}\sqrt{\frac{2\,cos^2\theta/2}{2\,sin^2\theta/2}}\right) = \sin(2\cot^{-1} cot\,\theta/2) = \sin\theta$

$\sqrt{1-cos^2\theta} = \sqrt{1-x^2}\ \frac{dv}{dx} = \frac{1}{2\sqrt{1-x^2}} \times -2x = \frac{-x}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = \frac{du}{dx} \div \frac{dv}{dx} = \frac{1}{2}\ \frac{1}{\sqrt{1-x^2}} \div \frac{-x}{\sqrt{1-x^2}} = -\frac{1}{2x}$

117 1. Differentiate: $sin^2x\ w{:}\,r{:}\,t\ e^{cosx}$

2. Differentiate : $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ w.r.t $\sin\left(2\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$

3 . Differentiate : $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t $\cos^{-1}(2x\sqrt{1-x^2})$

4 Differentiate : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w : r: t $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

5 . Differentiate: $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ w:r:t $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

6. Differentiate : $log(1+x^2)$ with respect to $\tan^{-1}x$. ANS: $2x$

8. Differentiate $(logx)^x$ with respect to $log\,x$**.**

9. Differentiate : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w . r . t $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at x= 0 is ¼

118 If $y = \cot x + \operatorname{cosec} x$, show that $\sin x \frac{d^2y}{dx^2} = y^2$