

Subject : Mathematics (041)

M.M : 80

Date : 01– 09 –2025

Time : 3 Hours

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long answer (LA) – type questions carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study – based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in section B, 3 questions in section C, 2 questions in section D and one subpart each in 2 questions of section E.
9. Use of calculators is not allowed.

SECTION- A

(Multiple Choice Questions) Each question carries 1 mark

1. If a matrix $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A|^3 = 125$ then the value of $\alpha =$ _____ (1)
 (A) ± 3 (B) ± 2 (C) ± 5 (D) ± 9
2. Principal branch of $\tan^{-1} x$ is _____. (1)
 A) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ (B) $\left(0, \frac{\pi}{2}\right)$ (C) $(0, \pi)$ (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
3. If $E(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then $E(\alpha).E(\beta) =$ _____ (1)
 (A) (0°) (B) $E(\alpha\beta)$ (C) $E(\alpha + \beta)$ (D) $E(\alpha - \beta)$
4. If $f(x) = x \tan^{-1} x$, then $f'(1) =$ _____ (1)
 (A) $\frac{1}{2} - \frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{1}{2} + \frac{\pi}{4}$ (D) $\frac{1}{2}$
5. Let $f(x) = \begin{cases} 3x - 4, & 0 \leq x \leq 2 \\ 2x + \lambda, & 2 \leq x \leq 3 \end{cases}$, if f is continuous at $x = 2$ then find the value of λ . (1)
 (A) -2 (B) 2 (C) 4 (D) 0
6. Find the maximum and minimum values of the function $f(x) = -|x + 1| + 3$ (1)
 (A) no maximum, minimum 3 (B) no maximum, minimum 0
 (C) maximum 3, minimum -1 (D) maximum 3, no minimum
7. The side of an equilateral triangle is increasing at the rate of 0.5 cm/s. Find the rate of increase of its perimeter. (1)
 (A) 1.5 cm/sec (B) 1 cm/sec (C) 0.5 cm/sec (D) 3 cm/sec
8. Find $\int \frac{(5+3\sqrt{x})^2}{\sqrt{x}} dx$ (1)
 (A) $\frac{1}{9} (5 + 3\sqrt{x})^3 + C$ (B) $\frac{1}{3} (5 + 3\sqrt{x})^3 + C$
 (C) $\frac{2}{9} (5 + 3\sqrt{x})^2 + C$ (D) $\frac{2}{9} (5 + 3\sqrt{x})^3 + C$
9. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate. (1)

- (A) $(4, 11)$ and $(-4, \frac{31}{3})$ (B) $(4, -11)$ and $(-4, 4)$
 (C) $(4, 11)$ and $(-4, -\frac{31}{3})$ (D) $(4, -11)$ and $(-4, -\frac{31}{3})$

10. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}} dx$ (1)
 (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 1 (D) -1
11. If $y = \sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$ find $\frac{dy}{dx}$. (1)
 (A) $x + \frac{\pi}{4}$ (B) $x - \frac{\pi}{4}$ (C) 1 (D) -1
12. The relation “less than” in the set of natural numbers is _____. (1)
 (A) Only symmetric (B) Only transitive
 (C) Only reflexive (D) equivalence relation
13. Evaluate : $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx$ (1)
 (A) $x + C$ (B) $-x + C$ (C) $\sin x + C$ (D) 1
14. Let N be the set of natural numbers and relation R on N be defined by
 $R = \{(x, y) : x, y \in N, x + 4y = 10\}$. R is _____. (1)
 (A) reflexive (B) symmetric
 (C) not reflexive and not symmetric (D) reflexive but not symmetric
15. Evaluate: $\int \tan^{-1} \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} dx$ (1)
 (A) $\frac{x^2}{2} + c$ (B) $x + c$ (C) $\tan^{-1} x + c$ (D) $\frac{x}{2} + c$
16. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \sin x}$ (1)
 (A) 0 (B) 1 (C) 2 (D) -1
17. Evaluate : $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\}$ (1)
 (A) 1 (B) -1 (C) -1 (D) $-\frac{1}{4}$
18. Given a skew-symmetric matrix $\begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix}$, then the value of $(a + b - c)^2$ is _____. (1)
 (A) 2 (B) 0 (C) 1 (D) 4

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is not the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.
19. Assertion (A): In set $A = \{1, 2, 3\}$ a relation R defined as $R = \{(1, 1), (2, 2)\}$ is reflexive. (1)
 Reason (R) : A relation R is reflexive in set A if $(a, a) \in R$ for all $a \in A$
20. Assertion (A) : The value of determinant of a matrix and the value of determinant of its transpose are equal. (1)
 Reason (R) : The value of determinant remains unchanged if its rows and columns are interchanged.

SECTION - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21 Prove that: $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \frac{\pi}{4} + \frac{x}{2}$ (2)

22 Evaluate : $\int \sqrt{1+\sin x} dx$ (2)

OR

Evaluate : $\int \sec^4 x \cdot \tan x dx$

23 If $y = x^{\cos^{-1} x}$ then find $\frac{dy}{dx}$ (2)

24 Show that the function $\tan^{-1}(\cos x + \sin x)$ is strictly increasing on $\left(0, \frac{\pi}{4}\right)$. (2)

OR

Show that the function $f(x) = \log \sin x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

25 Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive. (2)

SECTION - C

This section comprises of short answer type-questions (SA) of 3 marks each.

26 If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k so that $A^2 = 8A + kI$. (3)

OR

Find equation of line joining $(1, 2)$ and $(3, 6)$ using determinants.

27 Evaluate : $\int \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$ (3)

28. If $A = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, verify that $A(\text{adj} A) = |A|I$. (3)

29 If $x = ae^{\theta}(\sin \theta - \cos \theta)$ and $y = ae^{\theta}(\sin \theta + \cos \theta)$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ (3)

OR

If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+\log x)^2}{\log y}$

30 Evaluate : $\int \frac{5x}{(x+1)(x^2+9)} dx$ (3)

OR

Evaluate using properties of integration: $\int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1} + \sqrt[3]{11-x}} dx$

31 Let N be the set of all natural numbers and let R be a relation on $N \times N$ defined by $(a, b)R(c, d) \Rightarrow ad = bc$ for all $(a, b), (c, d) \in N \times N$. Show that R is an equivalence relation on $N \times N$.

SECTION- D

This section comprises of Long Answer (LA) - type questions of 5 marks each

32 If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations: (5)

$2x + y - 3z = 13$; $3x + 2y + z = 4$; $x + 2y - z = 8$

33. If $x = \sin t$, $y = \sin pt$ prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

OR

If $x^{16}y^9 = (x^2 + y)^{17}$ Show that $\frac{dy}{dx} = \frac{2y}{x}$.

34 Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

35. Evaluate: $\int_2^5 \{ |x - 2| + |x - 3| + |x - 5| \} dx$.

OR

Evaluate : $\int \frac{\sqrt{x^2+1} [\log(x^2+1) - 2 \log x]}{x^4} dx$

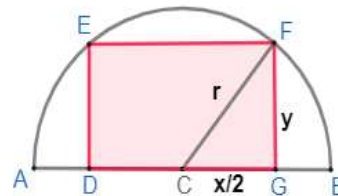
SECTION –E

This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts

36. A rectangle is inscribed in a semi- circle of radius r with one of its sides on the diameter of the semi- circle. Using the concept of maxima and minima, we need to find the dimensions of the rectangle, so that its area is maximum.

Use the figure to answer the following.

- Find the area of rectangle A in terms of r and x .
- The value of x in terms of r = ____.
- Find the length and breadth of the rectangle (x and y) in terms of r .



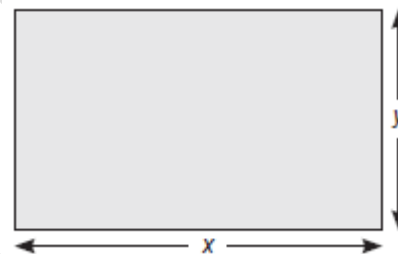
(1)
(1)
(2)

OR

- Maximum area = ____

37. Arav wants to donate a rectangular plot of land for a school in her village. When she was asked to give dimensions of the plot, she told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by $5300 m^2$.

Based on the information given above, answer the following questions:



- The equations in terms of x and y are ____ & ____.
- Which of the following matrix equation is represented by the given information?

- (A) $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -50 \\ -550 \end{bmatrix}$
- (C) $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

- The value of x (length of rectangular field) is ____.

OR

- How much is the area of rectangular field?

(1)
(1)

38. Mansi visited one Exhibition along with her family. The Exhibition had a huge swing, which attracted many children. Mansi found that the swing traced the path of a Parabola as given by $y = x^2$

Answer the following questions using the above information.

- Let $f : R \rightarrow R$ be defined by $f(x) = x^2$. Check whether f is bijective or not.
- Let $f : N \rightarrow N$ be defined by $f(x) = x^2$. Show that f one – one.

(2)
(2)

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