

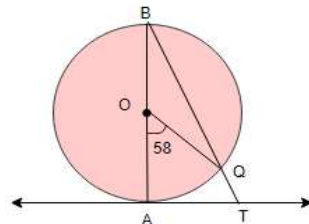
CIRCLES

CLASS X (2025-26)

SUJITHKUMAR KP 15-8-25

SECTION- A

- 1 In figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^\circ$, find $\angle ATQ$.
A) 51° B) 58° C) 71° D) 61°



ANS: $\angle ABQ = \frac{1}{2} \times 58^\circ = 29^\circ$

In ΔABT ,

$$\angle BAT + \angle ABT + \angle ATB = 180^\circ$$

$$90^\circ + 29^\circ + \angle ATB = 180^\circ$$

$$\angle ATB = 61^\circ \quad \text{as } \angle ATB = \angle ATQ \Rightarrow \angle ATQ = 61^\circ$$

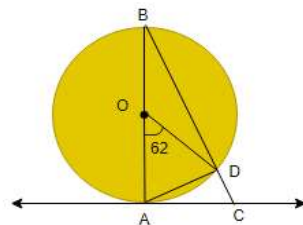
- 2 In the figure, AB is the diameter of a circle with centre O and AC is a tangent. If $\angle AOD = 62^\circ$, find $\angle ACD$.

(A) 51°

(B) 60°

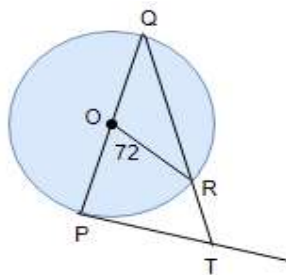
(C) 59°

(D) 61°



ANS: (C) 59°

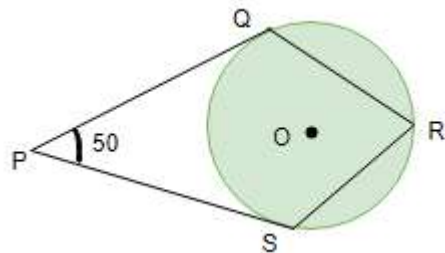
- 3 In the given figure, PQ is a diameter of a circle with centre O and PT is a tangent at P, QT meets the circle at R. If $\angle POR = 72^\circ$ then $\angle PTR =$ ____
A) 52° (B) 60°
(C) 54° (D) 64°



ANS: 54°

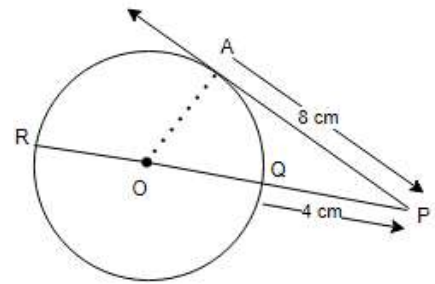
- 4 In the figure, O is the centre of the circle and PQ and PS are tangents to the circle at points Q and S respectively. $\angle QRS =$ ____

A) 65° B) 130° C) 55° D) 100°



ANS: A) 65°

- 5 In the figure, O is the centre of the circle and PA is tangent to the circle from the point P. PQR passes through the centre of the circle O. If PA = 8 cm, PQ = 4 cm, find the radius of the circle.
 A) 3 cm B) 6 cm C) 12 cm D) 10 cm



ANS: Let OA = r cm

$$OA^2 + AP^2 = OP^2$$

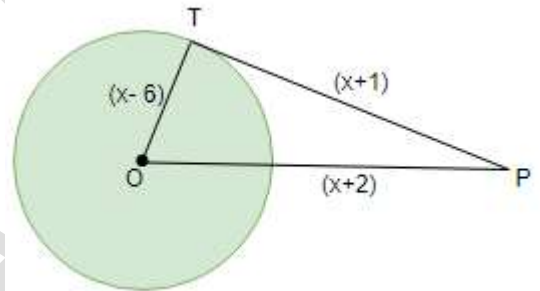
$$r^2 + 8^2 = (r + 4)^2$$

$$r^2 + 8^2 = r^2 + 4^2 + 8r$$

$$64 - 16 = 8r \Rightarrow r = 6 \text{ . The radius of the circle 6 cm.}$$

- 6 In the below figure, find the area of ΔOTP .

- A) 30 cm^2 (B) 60 cm^2
 (C) 15 cm^2 (D) 40 cm^2



ANS: $(x + 1)^2 + (x - 6)^2 = (x + 2)^2$

$$x^2 + 1 + 2x + x^2 + 36 - 12x = x^2 + 4 + 4x$$

$$x^2 - 14x + 33 = 0$$

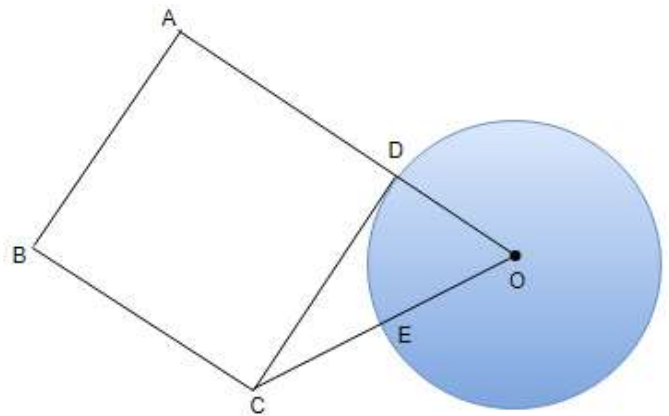
$$(x - 3)(x - 11) = 0$$

$$x = 11, \quad x \neq 3 \quad \text{sides are 5, 12, 13}$$

$$\text{Area} = \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$$

- 7 ABCD is a square, CD is a tangent to the circle with centre O. if OD = CE, the find the ratio of the area of circle to that of square.

- A) $\frac{2\pi}{3}$ B) $\frac{3}{\pi}$ C) $\frac{\pi}{9}$ D) $\frac{\pi}{3}$

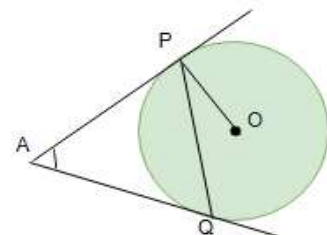


ANS: D) $\frac{\pi}{3}$

- 8 Tangents AP and AQ are drawn to a circle with centre O from external point A then _____

- A) $\angle PAQ = 2 \angle OPQ$
 B) $\angle PAQ = \angle OPQ$
 C) $\angle PQA = \angle OPA$
 D) $\angle PQA = 2 \angle OPA$

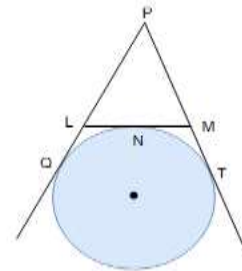
A) $\angle PAQ = 2 \angle OPQ$



- 9 The distance between two parallel tangents of a circle of radius 10 cm is ____
 A) 20 cm B) 10 cm C) 15 cm D) 5 cm

ANS: A) 20 cm

- 10 In the figure, If $PQ = 30\text{ cm}$, then find the perimeter of $\triangle PLM$.
 A) 30 cm B) 60 cm C) 40 cm D) 35 cm



ANS: $PQ = PT$

$PL + LQ = PM + MT$

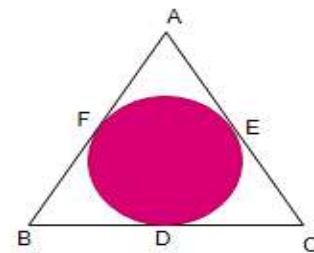
$\Rightarrow PL + LN = PM + MN$

perimeter of $\triangle PLM = PL + LN + MN + PM = PQ + PT = 2 \times PQ = 60\text{ cm}$.

- 11 The two tangents from an external point P to a circle with centre O are PA and PB. If $\angle APB = x^\circ$, what is the value of $\angle AOB$?
 A) x° B) $(180 - x)^\circ$ C) 90° D) $(90 - x)^\circ$

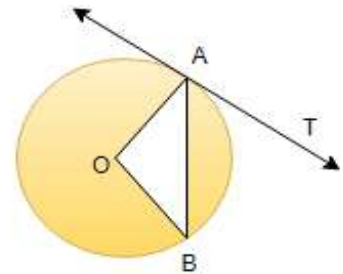
ANS: B) $(180 - x)^\circ$

- 12 A triangle ABC is drawn to circumscribe a circle. If $AB = 13\text{ cm}$, $BC = 14\text{ cm}$ and $AE = 7\text{ cm}$, then AC is equal to ____
 (A) 12 cm (B) 15 cm (C) 11 cm (D) 16 cm



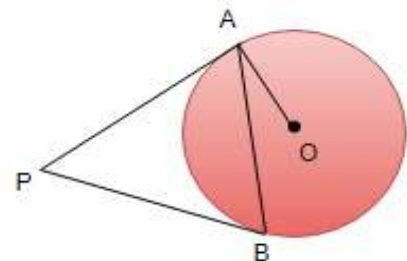
ANS: (B) 15 cm

- 13 In given figure, O is the centre of the circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 100^\circ$ then find $\angle BAT$.
 A) 100° B) 40° C) 50° D) 90°



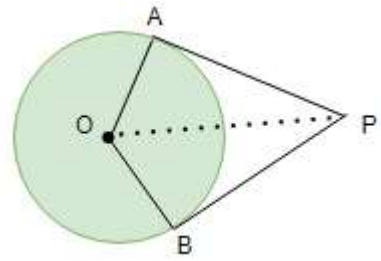
ANS: C) 50°

- 14 In the figure PA and PB are tangents to the circle with centre O. If $\angle APB = 70^\circ$, then $\angle OAB$ is ____
 A) 35° B) 70° C) 30° D) 15°



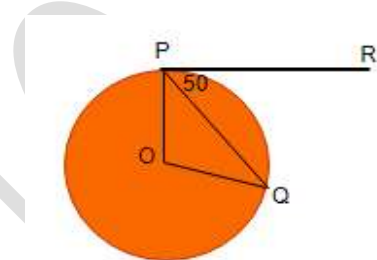
ANS: A) 35°

- 15 If angle between two tangents drawn from a point to a circle of radius a and centre O is 60° . then $OP =$ _____
- A) $\sqrt{3}a$ B) $\frac{a}{\sqrt{3}}$ C) $2a$ D) $\frac{a}{2}$



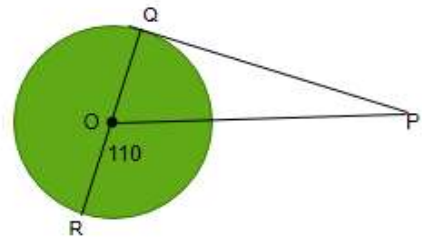
ANS: C) $2a$

- 16 In figure if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ , then $\angle POQ$ is equal _____
- A) 100° B) 80° C) 90° D) 75°



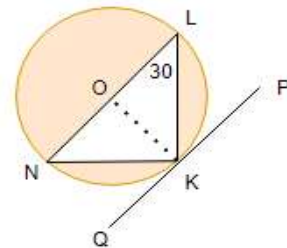
ANS: A) 100°

- 17 PQ is a tangent drawn from a point P to a circle with centre O and QR is a diameter of the circle such that $\angle POR = 110^\circ$. Find $\angle OPQ$.
- A) 10° B) 20° C) 30° D) 25°



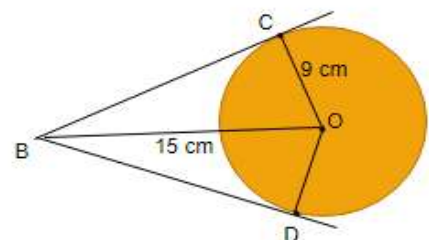
ANS: 20°

- 18 In Figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and $\angle KLN = 30^\circ$, find $\angle PKL$.
- A) 30° B) 20° C) 60° D) 25°



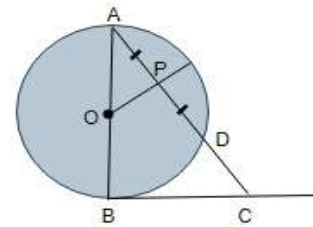
ANS: 60°

- 19 In the figure, BC and BD are tangents to the circle with centre O and radius 9 cm. If $OP = 15$ cm, then the length of $(BC + BD) =$ _____ cm
- (A) 18 (B) 12
(C) 24 (D) 21



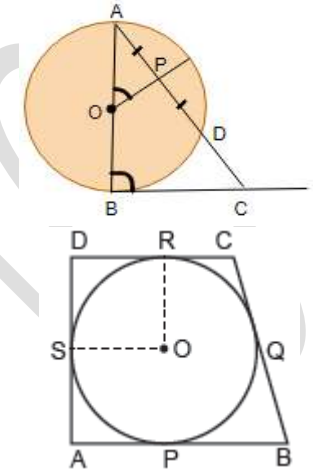
ANS: (C) 24

- 20 In the figure, O is the centre of the circle. BC is a tangent to the circle at B. If OP bisects the chord AD and $\angle AOP = 60^\circ$, Then find $\angle C$.
 A) 60° B) 30° C) 45° D) 25°



ANS: OP bisects the chord AD $\Rightarrow OP \perp AD$
 $\angle OPA = 90^\circ$ and $\angle AOP = 60^\circ \Rightarrow \angle A = 30^\circ$
 In $\triangle ABC$, $\angle C = 180^\circ - 120^\circ = 60^\circ$

- 21 A quadrilateral ABCD is drawn so that $\angle D = 90^\circ$, BC = 38 cm and CD = 25 cm. A circle is inscribed in the quadrilateral and it touches the side AB, BC, CD and DA at P, Q, R and S respectively. If BP = 27 cm, find the radius of the inscribed circle.
 A) 100° B) 105° C) 130° D) 125°



ANS: To find: $\angle 1 + \angle 2$

Solution: OQ is perpendicular to the tangent PQ. (radius is perpendicular to the tangent at the point of contact).

In $\triangle OPQ$, $\angle POR = \angle 1 + \angle RQP$ (external angle is equal to the sum of interior opposite angles)
 $\Rightarrow 130^\circ = \angle 1 + 90^\circ \Rightarrow \angle 1 = 40^\circ$ (i)

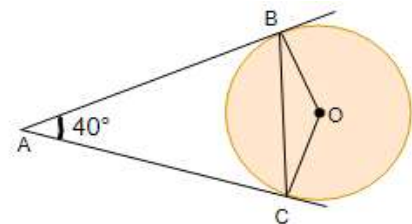
$\angle ROT$ is the angle subtended by arc RT at centre and $\angle RST$ is the angle subtended by same arc at point S on circumference.

$\angle 2 = \frac{1}{2} \times \angle ROT = \frac{1}{2} \times 130^\circ = 65^\circ$ (ii)

Adding (i) and (ii), we get $\angle 1 + \angle 2 = 40^\circ + 65^\circ = 105^\circ$

- 22 In the given figure, AB and AC are tangents to the circle with centre O such that $\angle BAC = 40^\circ$, then $\angle BOC$ is equal to ____.

A) 130° B) 120° C) 140° D) 150°



ANS: In quadrilateral ABOC

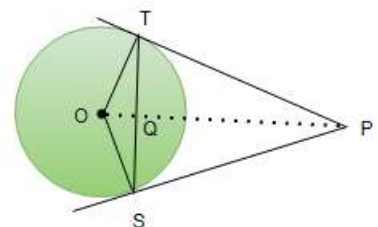
$\angle ABO + \angle BOC + \angle OCA + \angle BAC = 360^\circ$

$\Rightarrow 90^\circ + \angle BOC + 90^\circ + 40^\circ = 360^\circ$

$\Rightarrow \angle BOC = 360^\circ - 220^\circ = 140^\circ$

- 23 In figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r . If $OP = 2r$ $\angle OTS = \angle OST =$ _____

A) 20° B) 40° C) 30° D) 25°



ANS: In ΔOTS , $OT = OS$ [radii]

$$\Rightarrow \angle OTS = \angle OST \dots\dots(i)$$

In right ΔOTP , $\frac{OT}{OP} = \sin \angle TPO$

$$\Rightarrow \frac{r}{2r} = \sin \angle TPO$$

$$\Rightarrow \sin \angle TPO = \frac{1}{2} \Rightarrow \angle TPO = 30^\circ$$

Similarly $\angle OPS = 30^\circ$

$$\Rightarrow \angle TPS = 30^\circ + 30^\circ = 60^\circ$$

Also $\angle TPS + \angle SOT = 180^\circ$

$$\angle SOT = 120^\circ$$

In ΔSOT ,

$$\angle SOT + \angle OTS + \angle OST = 180^\circ$$

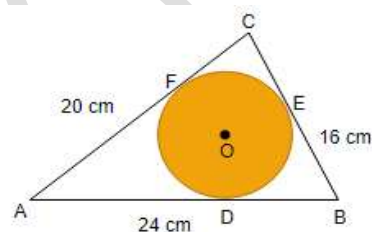
$$120^\circ + 2\angle OTS = 180^\circ$$

$$\Rightarrow \angle OTS = 30^\circ \dots(ii)$$

From (i) and (ii)

$$\angle OTS = \angle OST = 30^\circ$$

- 24 A circle is inscribed in a ΔABC having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF



ANS: Let $AD = AF = x$ [Tangents from external point are equal]

$$BD = BE = y \text{ and } CE = CF = z$$

According to the question,

$$AB = x + y = 24 \text{ cm} \dots (i)$$

$$BC = y + z = 16 \text{ cm} \dots (ii)$$

$$AC = x + z = 20 \text{ cm} \dots (iii)$$

Subtracting (iii) from (i), we get

$$y - z = 4 \dots (iv)$$

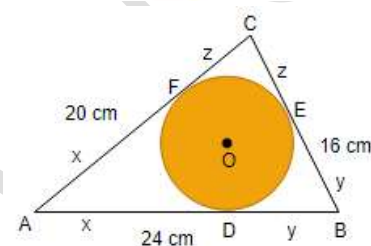
Adding (ii) and (iv), we get

$$2y = 20 \Rightarrow y = 10 \text{ cm.}$$

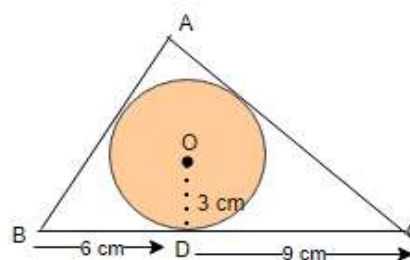
Substituting the value of y in (ii) and (i) we get

$$z = 6 \text{ cm; } x = 14 \text{ cm}$$

$$AD = 14 \text{ cm, } BE = 10 \text{ cm and } CF = 6 \text{ cm.}$$



- 25 In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2 , then find the lengths of sides AB and AC.



Let $AF = x \text{ cm}$

$$AF = AE = x \text{ [tangents from A]}$$

$$\text{Also } BD = BF = 6 \text{ cm and } CD = CE = 9 \text{ cm}$$

$$AB = (6 + x) \text{ cm and } AC = (9 + x) \text{ cm}$$

$$\text{Area } \Delta ABC = \text{Area } \Delta BOC + \text{Area } \Delta COA + \text{Area } \Delta AOB$$

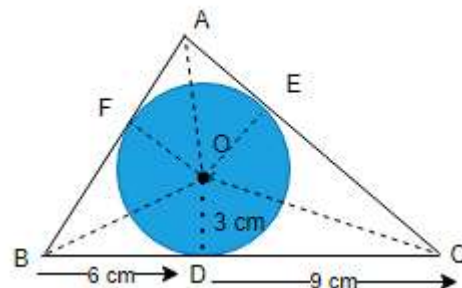
$$\Rightarrow 54 = \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE + \frac{1}{2} AB \times OF$$

$$\Rightarrow 54 \times 2 = 15 \times 3 + (6 + x) \times 3 + (9 + x) \times 3$$

$$108 = 45 + 18 + 3x + 27 + 3x$$

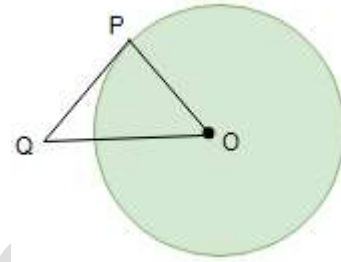
$$6x = 18 \Rightarrow x = 3$$

$$\Rightarrow AB = 6 + x = 6 + 3 = 9 \text{ cm and } AC = 9 + x = 9 + 3 =$$



12 cm

- 26 PQ is a tangent to a circle with centre O at point P. If ΔOPQ is isosceles triangle, then find $\angle OQP$.

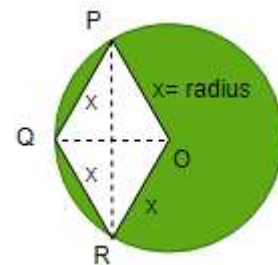
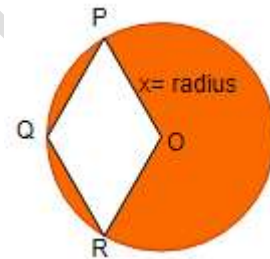


ANS: ΔOPQ is isosceles triangle, $\angle Q = \angle O$ and QP is tangent $\Rightarrow \angle P = 90^\circ$

$$\angle Q + \angle O + \angle P = 180^\circ$$

$$\Rightarrow 2\angle Q = 90^\circ \Rightarrow \angle OQP = 45^\circ$$

- 27 In the given figure, OPQR is a rhombus, three of whose vertices lie on a circle with centre O. If the area of the rhombus is $32\sqrt{3} \text{ cm}^2$, find the radius of the circle.



Side of the rhombus = radius of the circle

$$OP = OQ = PQ = x = OR = QR$$

ΔORQ and ΔOPQ are equilateral triangles

$$\text{Area of the equilateral triangle } OPQ = \frac{\sqrt{3}}{4} a^2$$

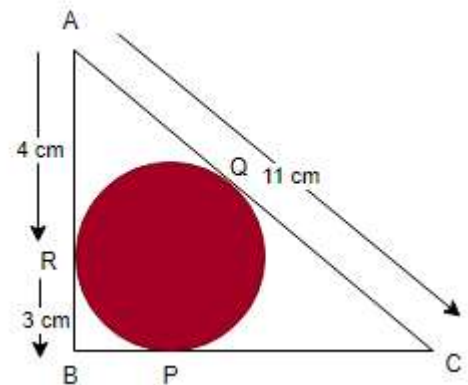
$$\text{Area of the equilateral } \Delta OPQ = \frac{\sqrt{3}}{4} x^2$$

$$\text{Area of the rhombus} = 2 \text{ ar } (\Delta OPQ)$$

$$2 \times \frac{\sqrt{3}}{4} x^2 = \frac{\sqrt{3}}{2} x^2 = 32\sqrt{3}$$

$$x^2 = 64 \text{ cm}^2 \quad x = 8 \text{ cm}$$

- 28 In figure, ΔABC is circumscribing a circle. Find the length of BC.



ANS: $AR = 4 \text{ cm}$

Also, $AR = AQ \Rightarrow AQ = 4 \text{ cm}$

Now, $QC = AC - AQ = 11 \text{ cm} - 4 \text{ cm} = 7 \text{ cm} \dots(i)$

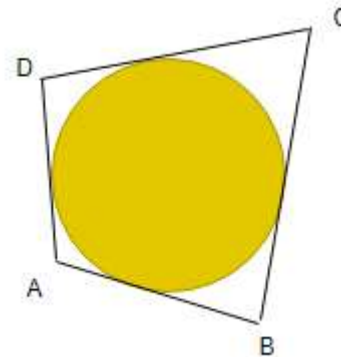
Also, $BP = BR$

$$BP = 3 \text{ cm and } PC = QC$$

$$PC = 7 \text{ cm [From (i)]}$$

$$BC = BP + PC = 3 \text{ cm} + 7 \text{ cm} = 10 \text{ cm}$$

- 29 In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are $AB = 6 \text{ cm}$, $BC = 9 \text{ cm}$ and $CD = 8 \text{ cm}$. Find the length of side AD.



ANS: If a circle touches all the four sides of quadrilateral ABCD, then

$$AB + CD = AD + BC$$

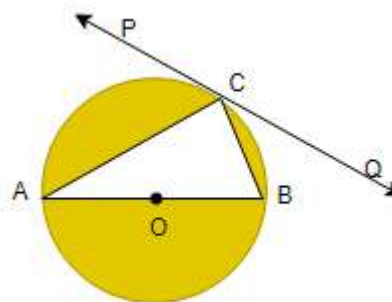
$$6 + 8 = AD + 9$$

$$\Rightarrow 14 = AD + 9$$

$$\Rightarrow 14 - 9 = AD$$

$$\Rightarrow AD = 5 \text{ cm}$$

- 30 In figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30^\circ$, find $\angle PCA$.



In $\triangle AOC$,

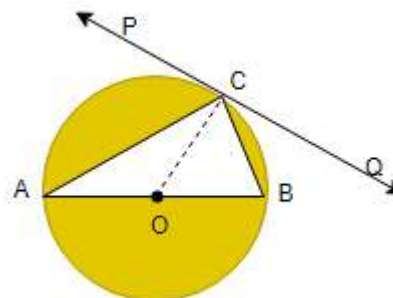
$$AO = CO \text{ [Radii]}$$

$$\angle OCA = \angle CAB = 30^\circ$$

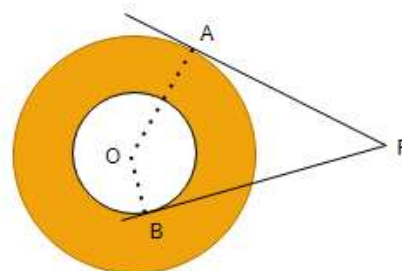
$$OC \perp PQ, \Rightarrow \angle OCP = 90^\circ$$

$$\Rightarrow \angle PCA + \angle OCA = 90^\circ$$

$$\Rightarrow \angle PCA + 30^\circ = 90^\circ \Rightarrow \angle PCA = 60^\circ$$



- 31 Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radius 8 cm and 5 cm respectively, as shown in the figure. If $AP = 15 \text{ cm}$, then $BP = \underline{\hspace{2cm}}$



A) $2\sqrt{66}$ B) $4\sqrt{66}$ C) $2\sqrt{33}$ D) $\sqrt{66}$

ANS: $OP = \sqrt{8^2 + 15^2} = \sqrt{289} = 17$

$$PB = \sqrt{17^2 - 5^2} = \sqrt{264} = 2\sqrt{66}$$

- 32 In figure, there are two concentric circles with centre of radii 5 cm and 3 cm. Tangents PA and PB are drawn from an external point P to these circles with centre O as shown in the figure. If AP = 12 cm, then BP = _____

A) $\sqrt{160}$ B) $\sqrt{150}$ C) 10 D) $2\sqrt{10}$

ANS: A) $\sqrt{160}$

- 33 In figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are tangents to the circle.

$$\text{ANS: } TQ = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$$

$$TQ = PT = 12 \text{ cm}$$

$$\text{Let } AE = AQ = x$$

$$AT = 12 - x, \quad ET = 13 - 5 = 8$$

$$AT^2 = AE^2 + ET^2$$

$$(12 - x)^2 = (x)^2 + 8^2$$

$$144 + x^2 - 24x = x^2 + 64$$

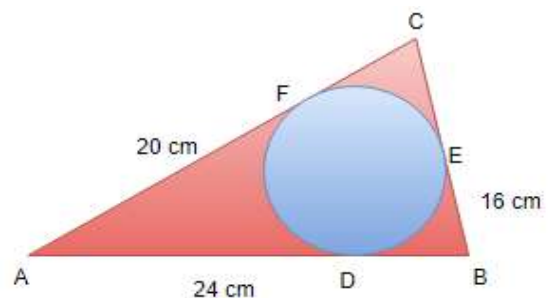
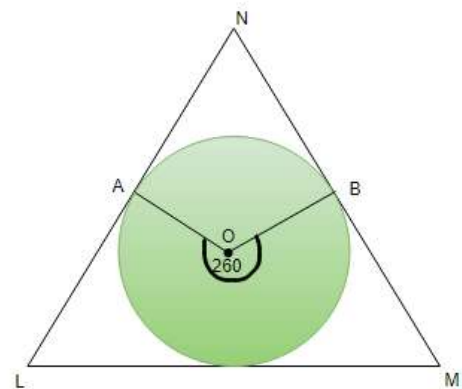
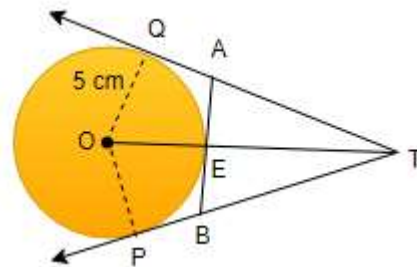
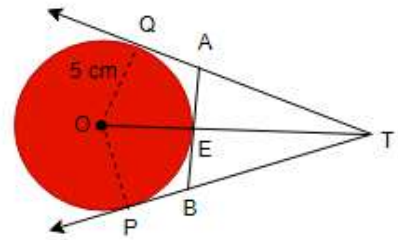
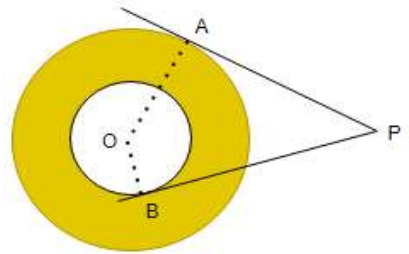
$$x = \frac{80}{24} = \frac{10}{3}$$

$$AB = 2 \times \frac{10}{3} = \frac{20}{3} \text{ cm}$$

- 34 A circle with centre O is inscribed in a triangle LMN. A and B are points of tangency. Reflex $\angle AOB = 260^\circ$, find $\angle ANB$.
A) 50° B) 100° C) 80° D) 130°

ANS: C) 80°

- 35 A circle is inscribed in a $\triangle ABC$ having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF.



ANS: Let $AD = AF = x$ [Tangents from external point are equal]

$BD = BE = y$ and $CE = CF = z$

According to the question,

$AB = x + y = 24 \text{ cm} \dots (i)$

$BC = y + z = 16 \text{ cm} \dots (ii)$

$AC = x + z = 20 \text{ cm} \dots (iii)$

Subtracting (iii) from (i), we get

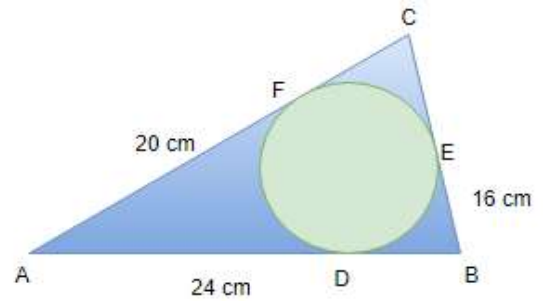
$y - z = 4 \dots (iv)$

Adding (ii) and (iv), we get

$2y = 20 \Rightarrow y = 10 \text{ cm}$.

Substituting the value of y in (ii) and (i) we get $z = 6 \text{ cm}$; $x = 14 \text{ cm}$

$AD = 14 \text{ cm}$, $BE = 10 \text{ cm}$ and $CF = 6 \text{ cm}$.



SECTION- B

- 1 Two parallel lines touch the circle at points A and B respectively. If area of the circle is $25 \pi \text{ cm}^2$, then AB is equal to _____

(A) 5 cm (B) 8 cm (C) 10 cm (D) 25 cm

ANS: (c) Let radius of circle = R

$\therefore \pi R^2 = 25 \pi$

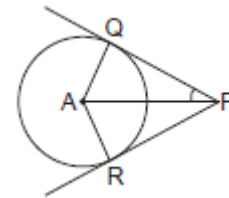
$\Rightarrow R = 5 \text{ cm}$

\therefore Distance between two parallel tangents = diameter = $2 \times 5 = 10 \text{ cm}$.

- 2 In figure, PQ and PR are tangents to a circle with centre

A. If $\angle QPA = 27^\circ$, then $\angle QAR$ equals to____

(A) 63° (B) 153° (C) 126° (D) 117°



ANS: (C) 126° $\angle QPA = \angle RPA$

[$\because \triangle AQP \cong \triangle ARP$ (RHS congruence rule)]

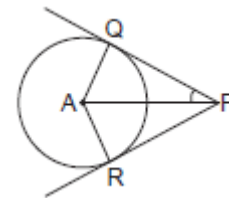
$\Rightarrow \angle RPA = 27^\circ$

$\therefore \angle QPR = \angle QPA + \angle RPA = 27^\circ + 27^\circ = 54^\circ$ Now,

$\angle QAR + \angle AQP + \angle ARP + \angle QPR = 360^\circ$

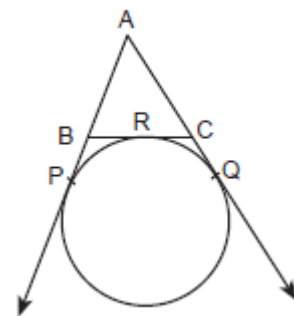
$\Rightarrow \angle QAR = 90^\circ + 90^\circ + 54^\circ = 234^\circ$

$\Rightarrow \angle QAR = 360^\circ - 234^\circ = 126^\circ$



- 3 In figure, AP, AQ and BC are tangents to the circle. If $AB = 5 \text{ cm}$, $AC = 6 \text{ cm}$ and $BC = 4 \text{ cm}$, then the length of AP (in cm) is _____

(A) 7.5 (B) 15 (C) 10 (D) 9



ANS: (A) $AP = AQ$

$$\Rightarrow AB + BP = AC + CQ$$

$$\Rightarrow 5 + BP = 6 + CQ$$

$$BP = 1 + CQ$$

$$BP = 1 + CR$$

$$(\because CQ = CR)$$

$$BP = 1 + (BC - BR)$$

$$BP = 1 + (4 - BP) (\because BR = BP)$$

$$2BP = 5 \Rightarrow BP = \frac{5}{2} = 2.5 \text{ cm}$$

$$\text{Now, } AP = AB + BP = 5 + 2.5 = 7.5 \text{ cm}$$

- 4 In the given figure, TP and TQ are two tangents to a circle with centre O, such that $\angle POQ = 110^\circ$. Then $\angle PTQ$ is equal to _____

- (a) 55° (b) 70° (c) 110° (d) 90°

ANS: (A) In quadrilateral POQT,

$$\angle PTQ + \angle TPO + \angle TQO + \angle POQ = 360^\circ$$

$$\Rightarrow \angle PTQ + 90^\circ + 90^\circ + 110^\circ = 360^\circ$$

$$\Rightarrow \angle PTQ + 290^\circ = 360^\circ$$

$$\Rightarrow \angle PTQ = 360^\circ - 290^\circ = 70^\circ$$

- 5 In the figure PA and PB are tangents to the circle with centre O. If $\angle APB = 60^\circ$, then $\angle OAB$ is _____

- (A) 30° (B) 60° (C) 90° (D) 15°

ANS: (A) Given $\angle APB = 60^\circ$

$$\because \angle APB + \angle PAB + \angle PBA = 180^\circ$$

$$\Rightarrow \angle APB + x + x = 180^\circ$$

$$[\because PA = PB \therefore \angle PAB = \angle PBA = x \text{ (say)}]$$

$$\Rightarrow 60^\circ + 2x = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 60^\circ$$

$$\Rightarrow 2x = 120^\circ$$

$$\Rightarrow x = 60^\circ$$

$$\text{Also, } \angle OAP = 90^\circ$$

$$\Rightarrow \angle OAB + \angle PAB = 90^\circ$$

$$\Rightarrow \angle OAB + 60^\circ = 90^\circ$$

$$\Rightarrow \angle OAB = 30^\circ$$

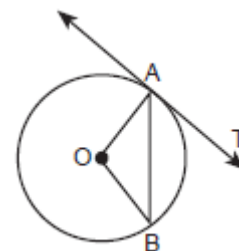
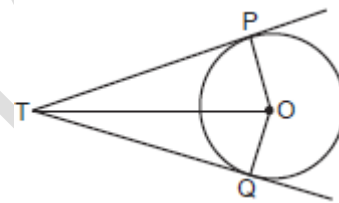
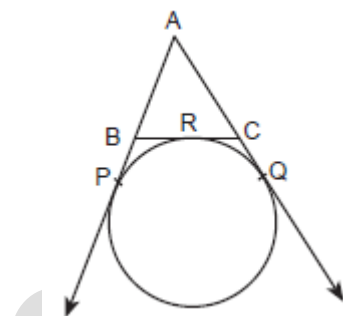
- 6 In figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 100^\circ$, then $\angle BAT$ is equal to _____

- (A) 100° (B) 40° (C) 50° (D) 90°

ANS: (C) $\angle AOB = 100^\circ$

$$\angle OAB = \angle OBA (\because OA \text{ and } OB \text{ are radii})$$

Now, in $\triangle AOB$,



$\angle AOB + \angle OAB + \angle OBA = 180^\circ$ (Angle sum property of Δ)

$\Rightarrow 100^\circ + x + x = 180^\circ$ [Let $\angle OAB = \angle OBA = x$]

$\Rightarrow 2x = 180^\circ - 100^\circ \Rightarrow 2x = 80^\circ \Rightarrow x = 40^\circ$

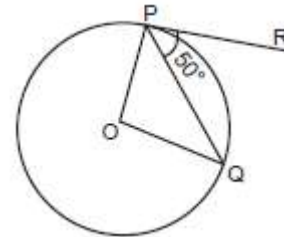
Also, $\angle OAB + \angle BAT = 90^\circ$

[\because OA is radius and TA is tangent at A]

$\Rightarrow 40^\circ + \angle BAT = 90^\circ \Rightarrow \angle BAT = 50^\circ$

- 7 In figure if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then $\angle POQ$ is equal to _____

(A) 100° (B) 80° (C) 90° (D) 75°



ANS: (A) $OP \perp PR$ [\because Tangent and radius are \perp to each other at the point of contact]

$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$

$OP = OQ$ [Radii]

$\therefore \angle OPQ = \angle OQP = 40^\circ$

In ΔOPQ ,

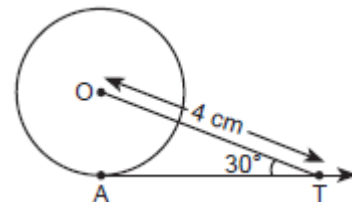
$\Rightarrow \angle POQ + \angle OPQ + \angle OQP = 180^\circ$

$\Rightarrow \angle POQ + 40^\circ + 40^\circ = 180^\circ$

$\angle POQ = 180^\circ - 80^\circ = 100^\circ$

- 8 In figure AT is a tangent to the circle with centre O such that $OT = 4$ cm and $\angle OTA = 30^\circ$. Then AT is equal to _____

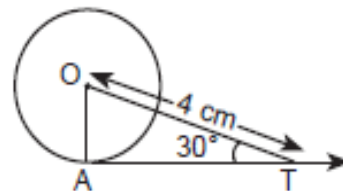
(A) 4 cm (B) 2 cm (C) $2\sqrt{3}$ cm (D) $4\sqrt{3}$ cm



ANS: $\angle OAT = 90^\circ$

In ΔOAT , $\frac{AT}{OT} = \cos 30^\circ$

$\frac{AT}{4} = \frac{\sqrt{3}}{2} \Rightarrow AT = 2\sqrt{3}$ cm



- 9 From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is _____

(A) 60 cm^2 (B) 65 cm^2 (C) 30 cm^2 (D) 32.5 cm^2

ANS: (A) 60 cm^2

$PQ = \sqrt{OP^2 - OQ^2}$

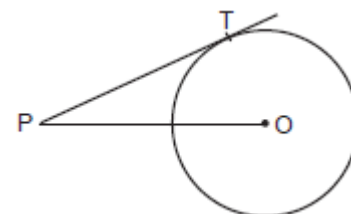
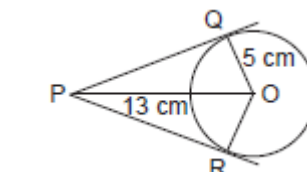
$PQ = 12$

area of the quadrilateral PQOR = ar ΔPOQ + ar of ΔPOR

$\frac{1}{2} \times 12 \times 5 + \frac{1}{2} \times 12 \times 5 = 60 \text{ cm}^2$

- 10 In the given figure, point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 24 cm. Then the radius of the circle is _____

(A) 25 cm (B) 26 cm (C) 24 cm (D) 10 cm



ANS: (D) \because OT is radius and PT is tangent \therefore OT \perp PT

Now, in $\triangle OTP$,

$$OP^2 = PT^2 + OT^2$$

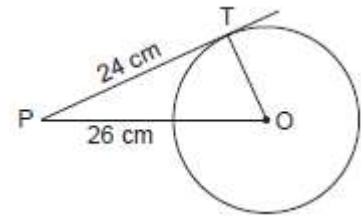
$$\Rightarrow 26^2 = 24^2 + OT^2$$

$$\Rightarrow 676 - 576 = OT^2$$

$$\Rightarrow 100 = OT^2 \Rightarrow OT = 10 \text{ cm}$$

- 11 In the given figure, AB and AC are tangents to the circle with centre O such that $\angle BAC = 40^\circ$, then $\angle BOC$ is equal to ____

(A) 40° (B) 50° (C) 140° (D) 150°



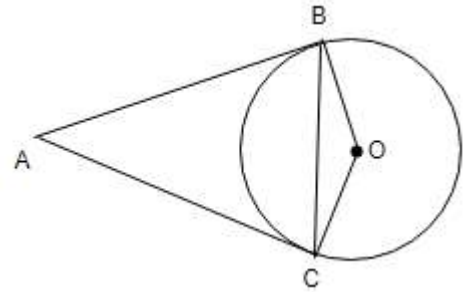
ANS: (C) In quadrilateral ABOC

$$\angle ABO + \angle BOC + \angle OCA + \angle BAC = 360^\circ$$

$$\Rightarrow 90^\circ + \angle BOC + 90^\circ + 40^\circ = 360^\circ$$

$$\Rightarrow \angle BOC = 360^\circ - 220^\circ = 140^\circ$$

- 12 In figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If $\angle PBT = 30^\circ$, prove that $BA : AT = 2 : 1$.



ANS: $\angle BPA = 90^\circ$ (Angle in semicircle)

In $\triangle BPA$,

$$\angle ABP + \angle BPA + \angle PAB = 180^\circ$$

$$30^\circ + 90^\circ + \angle PAB = 180^\circ$$

$$\angle PAB = 60^\circ$$

$$\text{Also } \angle POA = 2\angle PBA$$

$$\angle POA = 2 \times 30^\circ = 60^\circ$$

$$OP = AP \text{ (sides opposite to equal angles) ... (i)}$$

$$\text{In } \triangle OPT, \angle OPT = 90^\circ$$

$$\angle POT = 60^\circ \text{ and } \angle PTO = 30^\circ \text{ [angle sum property]}$$

$$\text{Also } \angle APT + \angle ATP = \angle PAO \text{ (exterior angle property)}$$

$$\angle APT + 30^\circ = 60^\circ$$

$$\angle APT = 30^\circ$$

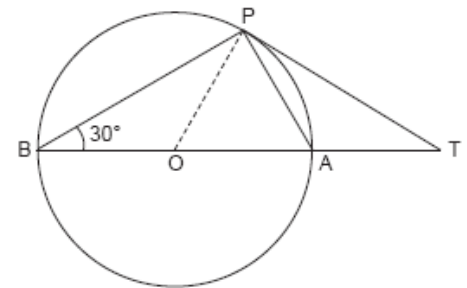
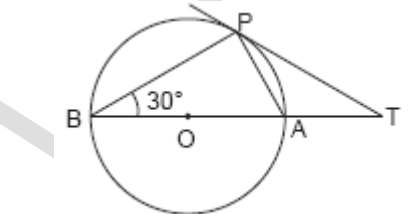
$$AP = AT \text{ (sides opposite to equal angles) ... (ii)}$$

From (i) and (ii) $AT = OP = \text{radius of the circle}$;

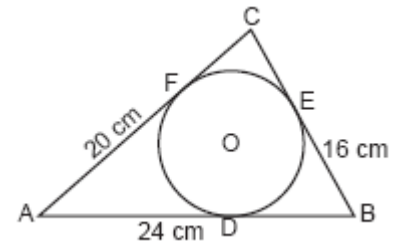
$$\text{and } AB = 2r$$

$$AB = 2AT \quad \frac{AB}{AT} = 2$$

$$AB : AT = 2 : 1$$



- 13 A circle is inscribed in a ΔABC having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF



ANS: Let $AD = AF = x$ [Tangents from external point are equal]

$BD = BE = y$ and $CE = CF = z$

According to the question,

$AB = x + y = 24$ cm ... (i)

$BC = y + z = 16$ cm ... (ii)

$AC = x + z = 20$ cm ... (iii)

Subtracting (iii) from (i), we get

$y - z = 4$... (iv)

Adding (ii) and (iv), we get

$2y = 20 \Rightarrow y = 10$ cm.

Substituting the value of y in (ii) and (i) we get $z = 6$ cm; $x = 14$ cm

$AD = 14$ cm, $BE = 10$ cm and $CF = 6$ cm.

- 14 A circle touches x-axis at A and y-axis at B. If O is origin and $OA = 5$ units, then diameter of the circle is

(A) 8 units

(B) 10 units

(C) $10\sqrt{2}$ units

(D) $8\sqrt{2}$ units

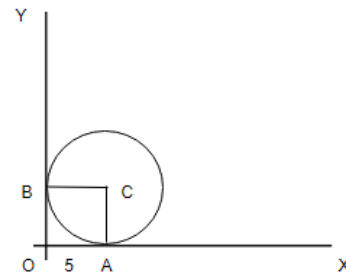
ANS: (B) $OA = OB \Rightarrow OB = 5$

$AC = BC$ [Radii]

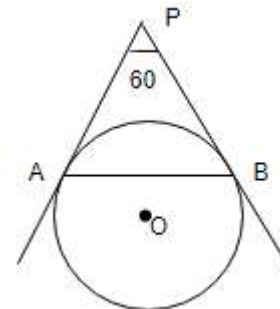
$\Rightarrow OACB$ is a square.

$\Rightarrow AC = OA = 5$

$\Rightarrow \text{Diameter} = 10$ units.



- 15 In figure, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60^\circ$. Find the length of chord AB.



ANS: $AP = BP$ [tangents from external point P]

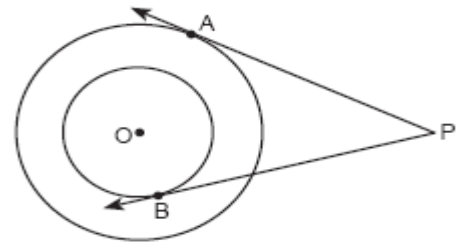
$\angle PAB = \angle PBA$ [Angles opposite to equal sides]

Now $\angle APB + \angle PAB + \angle PBA = 180^\circ$

$60^\circ + 2 \angle PAB = 180^\circ$

$\angle PAB = 60^\circ \Rightarrow \Delta APB$ is an equilateral $\Rightarrow AB = AP = 5$ cm

- 16 In figure, there are two concentric circles, with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.



$$PA = 12 \text{ cm}, OA = 5 \text{ cm}, OB = 3 \text{ cm}$$

$$OP^2 = OA^2 + AP^2 = OB^2 + BP^2$$

$$25 + 144 = 9 + BP^2$$

$$169 - 9 = BP^2$$

$$\Rightarrow BP = \sqrt{160} \text{ cm} = 12.65 \text{ cm. (Approx.)}$$

- 17 Find the length of the tangent drawn from a point whose distance from the centre of a circle is 25 cm. Given that radius of the circle is 7 cm.

Let O is the centre of the circle and P is a point such that OP = 25 cm and PQ is the tangent to the circle.

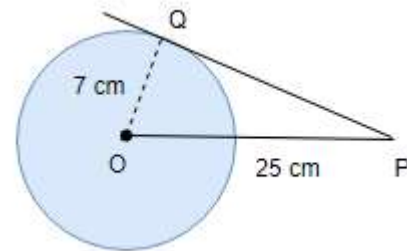
$$OQ = \text{radius} = 7 \text{ cm}$$

In ΔOQP , we have $\angle Q = 90^\circ$

$$OP^2 = OQ^2 + PQ^2$$

$$\Rightarrow (25)^2 = 7^2 + PQ^2 \Rightarrow PQ^2 = 625 - 49 = 576$$

$$\Rightarrow PQ = 24 \text{ cm} \text{ Hence, the length of the tangent} = 24 \text{ cm}$$



- 18 What is the angle between a tangent to a circle and the radius through the point of contact? Justify your answer.

ANS: 90° . Because radius through point of contact of tangent to a circle is perpendicular to the tangent.

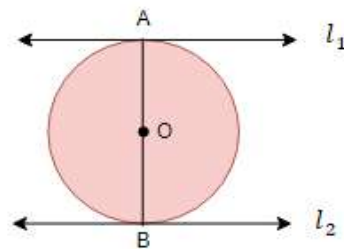
- 19 What is the distance between two parallel tangents of a circle of radius 7 cm?

Two parallel tangents of a circle can be drawn only at the end points of the diameter

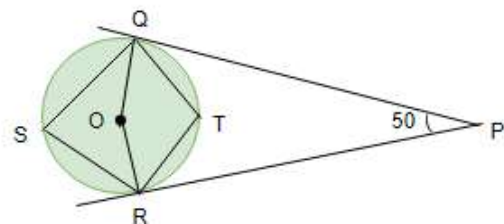
$$\Rightarrow l_1 \parallel l_2$$

$$\Rightarrow \text{Distance between } l_1 \text{ and } l_2 = AB = \text{Diameter of the circle}$$

$$= 2 \times r = 2 \times 7 \text{ cm} = 14 \text{ cm}$$



- 20 In the given figure, O is the centre. find $\angle QSR$



ANS: Given: PQ and PR are tangents to a circle with centre O and $\angle QPR = 50^\circ$.

To find: $\angle QSR$

$$\text{Sol. } \angle QOR + \angle QPR = 180^\circ$$

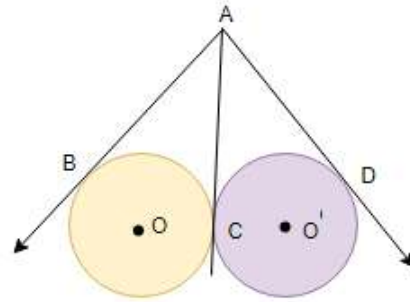
$$\angle QOR + 50^\circ = 180^\circ$$

$$\Rightarrow \angle QOR = 130^\circ$$

$$\Rightarrow \angle QSR = \frac{1}{2} \angle QOR \text{ [Degree measure theorem]}$$

$$\Rightarrow \angle QSR = \frac{1}{2} \times 130^\circ = 65^\circ$$

- 21 In the given figure, AB, AC and AD are tangents. If AB = 5 cm, find AD



ANS: Given: AB, AC and AD are tangents. AB = 5 cm.

To find: AD

Sol. AB and AC are tangents from the same point to the circle with centre O.

$$\Rightarrow AB = AC \dots(i)$$

(Length of the tangents from the same external point are equal).

AC and AD are tangents from the same point to the circle with centre O.

$$\Rightarrow AC = AD \dots(ii)$$

(Length of the tangents from the same external point are equal)

From (i) and (ii)

$$AB = AC = AD = 5 \text{ cm}$$

- 22 A point P is 26 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

Let O is the centre of the circle and PQ is the tangent from P. A.T.Q., OP = 26 cm and PQ = 24 cm

In ΔOQP , we have $\angle Q = 90^\circ$

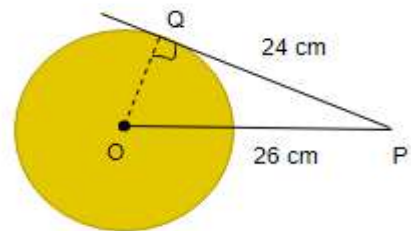
$$OP^2 = OQ^2 + PQ^2$$

$$\Rightarrow (26)^2 = OQ^2 + (24)^2$$

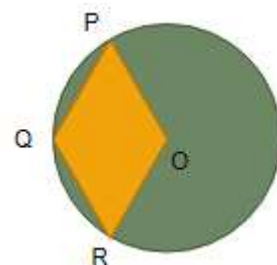
$$\Rightarrow OQ^2 = 676 - 576 = 100$$

$$\Rightarrow OQ = 10 \text{ cm}$$

Radius of the circle = 10 cm



- 23 In the given figure, OPQR is a rhombus, three of whose vertices lie on a circle with centre O. If the area of the rhombus is $32\sqrt{3} \text{ cm}^2$, find the radius of the circle.



Side of the rhombus = radius of the circle

$$OP = OQ = PQ = x = OR = QR$$

$\triangle ORQ$ and $\triangle OPQ$ are equilateral triangles

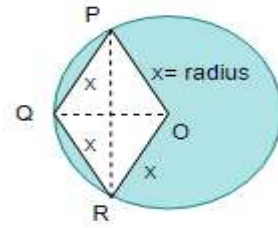
$$\text{Area of the equilateral triangle } OPQ = \frac{\sqrt{3}}{4} x^2$$

$$\text{Area of the equilateral } \triangle OPQ = \frac{\sqrt{3}}{4} x^2$$

Area of the rhombus = 2 ar ($\triangle OPQ$)

$$2 \times \frac{\sqrt{3}}{4} x^2 = \frac{\sqrt{3}}{2} x^2 = 32\sqrt{3}$$

$$x^2 = 64 \text{ cm}^2 \quad x = 8 \text{ cm}$$



- 24 From a point P, the length of the tangent to a circle is 15 cm and distance of P from the centre of the circle is 17 cm. Then what is the radius of the circle?

$$\angle OAP = 90^\circ$$

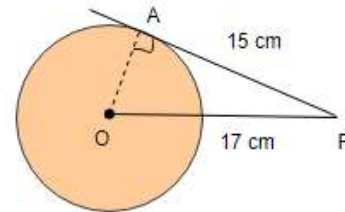
$$\Rightarrow 17^2 = r^2 + 15^2 \text{ [By Pythagoras theorem]}$$

$$\Rightarrow r^2 = 17^2 - 15^2 = (17 - 15)(17 + 15) = 2 \times 32$$

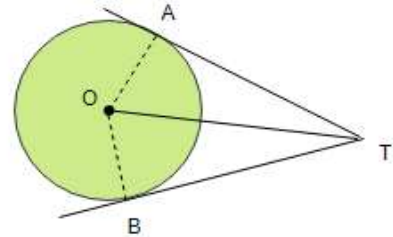
$$\Rightarrow r^2 = 64 \Rightarrow r = \pm 8 \text{ cm}$$

Ignoring negative value as length cannot be negative.

$$\Rightarrow r = 8 \text{ cm}$$



- 25 In figure, O is the centre of the circle, if $\angle ATO = 40^\circ$, find $\angle AOB$.



ANS: In $\triangle OAT$, $\angle ATO = 40^\circ$, $\angle OAT = 90^\circ$

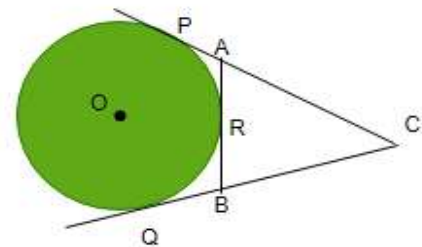
$$\angle AOT = 50^\circ \text{ [Angle sum property]}$$

Now $\angle BTO = 40^\circ$ as OT bisects $\angle ATB$

Similarly, $\angle BOT = 50^\circ$

$$\angle AOB = \angle AOT + \angle BOT = 50^\circ + 50^\circ = 100^\circ$$

- 26 In figure, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, and BC = 7 cm, then find the length of BR.



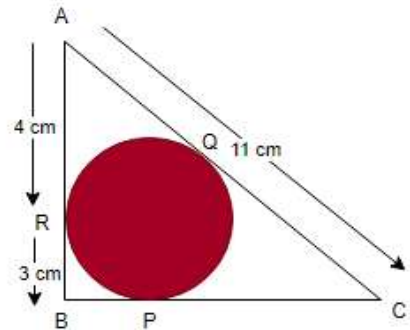
ANS: Since CP = CQ = 11 cm [Length of the two tangents from same external point]

$$CQ = CB + BQ$$

But BQ = BR

$$\text{Therefore, } 11 = 7 + BR \Rightarrow BR = 4 \text{ cm}$$

- 27 In figure, ΔABC is circumscribing a circle. Find the length of BC.



ANS: $AR = 4$ cm

Also, $AR = AQ \Rightarrow AQ = 4$ cm

Now, $QC = AC - AQ = 11 \text{ cm} - 4 \text{ cm} = 7 \text{ cm} \dots(i)$

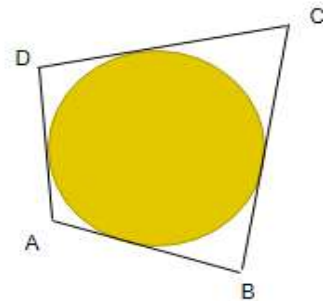
Also, $BP = BR$

$BP = 3$ cm and $PC = QC$

$PC = 7$ cm [From (i)]

$BC = BP + PC = 3 \text{ cm} + 7 \text{ cm} = 10 \text{ cm}$

- 28 In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are $AB = 6$ cm, $BC = 9$ cm and $CD = 8$ cm. Find the length of side AD.



ANS: If a circle touches all the four sides of quadrilateral ABCD, then

$AB + CD = AD + BC$

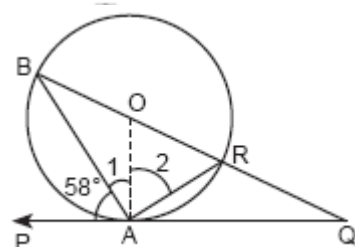
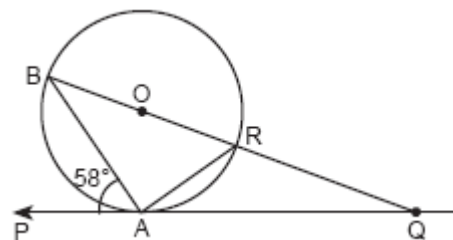
$6 + 8 = AD + 9$

$\Rightarrow 14 = AD + 9$

$\Rightarrow 14 - 9 = AD$

$\Rightarrow AD = 5$ cm

- 29 In figure, O is the centre of the circle, PQ is a tangent to the circle at A. If $\angle PAB = 58^\circ$, find $\angle ABQ$ and $\angle AQB$.



Join OA.

$OA \perp PAQ$.

$\angle OAP = 90^\circ$

$\Rightarrow \angle 1 + 58^\circ = 90^\circ$

$\Rightarrow \angle 1 = 90^\circ - 58^\circ = 32^\circ$

In ΔBOA , $OA = OB$.

Now $\angle 1 = \angle ABQ$

$$\Rightarrow \angle ABQ = 32^\circ$$

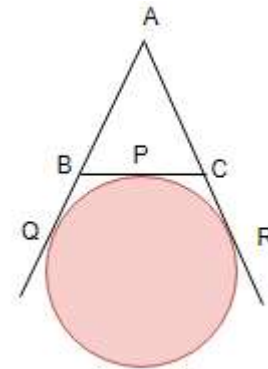
$$\angle PAB + \angle BAQ = 180^\circ \Rightarrow \angle BAQ = 180^\circ - 58^\circ = 122^\circ$$

In $\triangle ABQ$,

$$\angle ABQ + \angle BAQ + \angle AQB = 180^\circ$$

$$\angle AQB = 180^\circ - 122^\circ - 32^\circ = 26^\circ$$

- 30 In figure, a circle touches the side BC of $\triangle ABC$ at P and touches AB and AC produced at Q and R respectively. If $AQ = 5$ cm, find the perimeter of $\triangle ABC$.



ANS: Given: A circle touches the side BC of $\triangle ABC$ at P and touches AB and AC produced at Q and R respectively and $AQ = 5$ cm.

To find: Perimeter of $\triangle ABC$.

Sol.: AQ and AR are tangents from the same point

$$AQ = AR = 5 \text{ cm} \dots (i) \text{ [Tangents from the same external points are equal]}$$

BQ and BP are tangents from same point

$$\Rightarrow BQ = BP \dots (ii)$$

CP and CR are also tangents from the same point

$$\Rightarrow CP = CR \dots (iii)$$

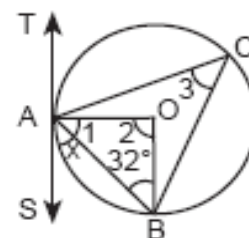
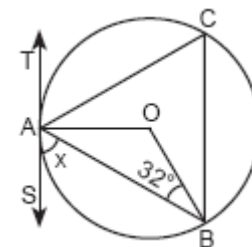
$$\text{In } \triangle ABC, \text{ Perimeter of } \triangle ABC = AB + BC + AC = AB + BP + CP + AC$$

$$AB + BQ + CR + AC = AQ + AR \text{ [From (ii) and (iii)]}$$

$$= 5 \text{ cm} + 5 \text{ cm} = 10 \text{ cm [From (i)]}$$

$$\text{Perimeter of } \triangle ABC = 10 \text{ cm}$$

- 31 In the given figure, TAS is a tangent to the circle, with centre O, at the point A. If $\angle OBA = 32^\circ$, find the value of x .



Given: TAS is tangent to the circle with centre O at point A.

$$\angle OBA = 32^\circ$$

To find: x

Sol. In $\triangle OAB$, $OA = OB$ [Radii of the same circle]

$$\Rightarrow \angle 1 = 32^\circ \text{ [Angle opposite to equal sides of a triangle are equal]}$$

In $\triangle OAB$,

$$\angle 1 + \angle 2 + 32^\circ = 180^\circ \text{ [Angle sum property of a triangle]}$$

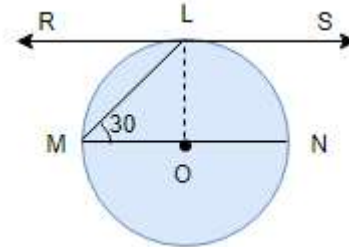
$$\Rightarrow \angle 2 + 32^\circ + 32^\circ = 180^\circ \Rightarrow \angle 2 = 180^\circ - 64^\circ = 116^\circ$$

$$\angle 3 = \frac{1}{2} \angle 2 = \frac{1}{2} \times 116^\circ = 58^\circ \text{ [Degree measure theorem]}$$

TAS is tangent to the circle (Given)

$x = \angle ACB = 58^\circ$ [Angles in the alternate segments are equal]

- 32 In the given figure, RS is the tangent to the circle at L and MN is a diameter. If $\angle NML = 30^\circ$, determine $\angle RLM$.



Join OL

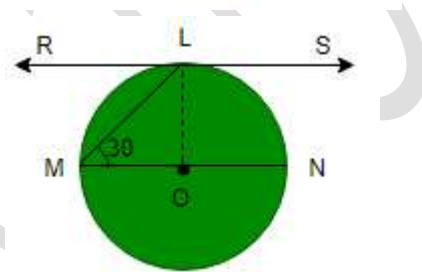
$OL \perp RS$.

Also $OL = OM$ [Radii of the same circle]

$$\angle OML = \angle OLM$$

$$\Rightarrow \angle OLM = 30^\circ$$

$$\Rightarrow \angle RLM = 90^\circ - 30^\circ = 60^\circ$$



- 33 Two tangents PA and PB are drawn to the circle with centre O, such that $\angle APB = 120^\circ$. Prove that $OP = 2AP$.

Given. A circle $C(O, r)$. PA and PB are tangents to the circle from point P, outside the circle such that

$$\angle APB =$$

120° . OP is joined.

To Prove. $OP = 2AP$.

Construction. Join OA and OB.

Proof. Consider Δs PAO and PBO

$PA = PB$ [Tangents to a circle, from a point outside it, are equal.]

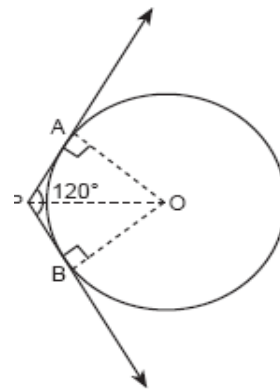
$OP = OP$ [Common]

$$\angle OAP = \angle OBP = 90^\circ$$

$$\Delta OAP \cong \Delta OBP \text{ [RHS]}$$

$$\angle OPA = \angle OPB = \frac{1}{2} \angle APB = \frac{1}{2} \times 120^\circ = 60^\circ.$$

In right angled ΔOAP , $\frac{AP}{OP} = \cos 60^\circ = \frac{1}{2} \Rightarrow OP = 2AP$.



- 34 The two tangents from an external point P to a circle with centre O are PA and PB. If $\angle APB = 70^\circ$, what is the value of $\angle AOB$?

PA and PB are tangents to the circle.

$\angle A = \angle B = 90^\circ$ [Tangent makes 90° angle with the radius at the point of contact]

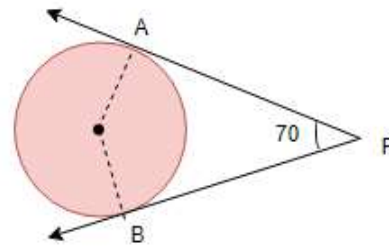
In quadrilateral OAPB

$\angle AOB + \angle A + \angle P + \angle B = 360^\circ$ [Angle sum property of a quadrilateral].

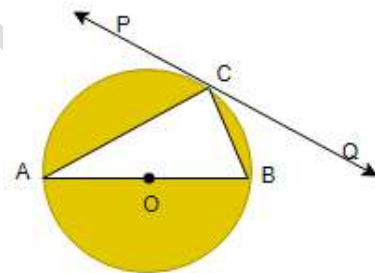
$$\Rightarrow \angle AOB + 90^\circ + 70^\circ + 90^\circ = 360^\circ$$

$$\Rightarrow \angle AOB + 250^\circ = 360^\circ$$

$$\Rightarrow \angle AOB = 360^\circ - 250^\circ = 110^\circ$$



- 35 In figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30^\circ$, find $\angle PCA$.



In $\triangle AOC$,

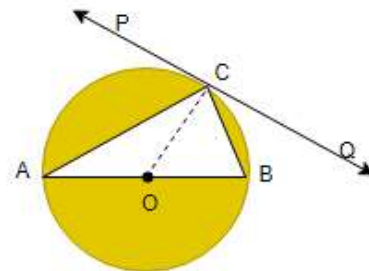
$AO = CO$ [Radii]

$$\angle OCA = \angle CAB = 30^\circ$$

$$OC \perp PQ, \Rightarrow \angle OCP = 90^\circ$$

$$\Rightarrow \angle PCA + \angle OCA = 90^\circ$$

$$\Rightarrow \angle PCA + 30^\circ = 90^\circ \Rightarrow \angle PCA = 60^\circ$$



- 36 In figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130^\circ$, then find $\angle ACO$

$$\angle AOC = 180^\circ - 130^\circ = 50^\circ$$

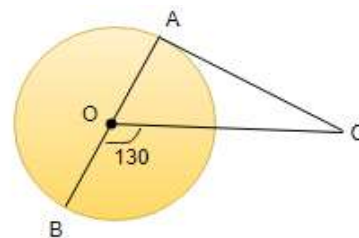
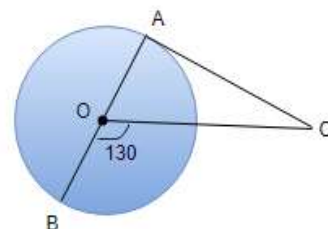
$$\angle OAC = 90^\circ [OA \perp AC]$$

In $\triangle OAC$,

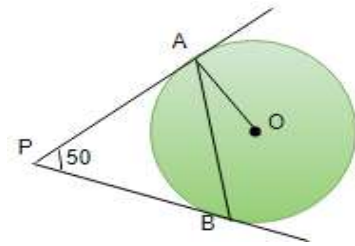
$$\angle OAC + \angle AOC + \angle ACO = 180^\circ$$

$$90^\circ + 50^\circ + \angle ACO = 180^\circ$$

$$\Rightarrow \angle ACO = 40^\circ$$



- 37 In figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$. Write the measure of $\angle OAB$



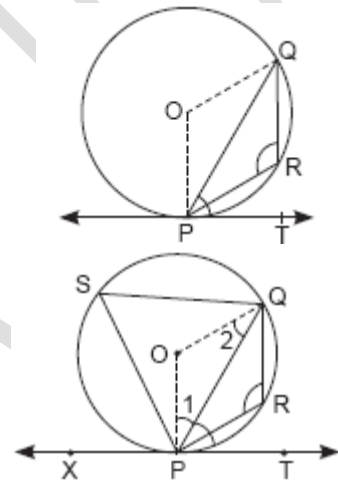
ANS: In $\triangle APB$, $\angle BAP = \angle ABP$ (angle opp. to equal side)

$$\angle BAP = \frac{1}{2} (180^\circ - \angle APB) = \frac{1}{2} (130^\circ) = 65^\circ$$

$$\angle OAP = 90^\circ$$

$$\angle OAB = 90^\circ - \angle BAP = 90^\circ - 65^\circ = 25^\circ$$

- 38 In figure, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$



$$\angle OPQ + \angle QPT = 90^\circ$$

$$\angle OPQ = 90^\circ - \angle QPT = 90^\circ - 60^\circ = 30^\circ$$

In $\triangle OPQ$,

$$\angle POQ + \angle 1 + \angle 2 = 180^\circ$$

$$\angle POQ + 2 \angle 1 = 180^\circ \quad (\angle 1 = \angle 2)$$

$$\angle POQ + 2(30^\circ) = 180^\circ$$

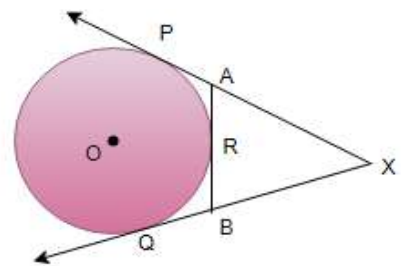
$$\angle POQ = 120^\circ$$

$$\angle PSQ = \frac{1}{2} \angle POQ = \frac{1}{2} \times 120^\circ = 60^\circ$$

$\angle PSQ + \angle PRQ = 180^\circ$ (sum of opp. angles of cyclic quadrilateral)

$$\angle PRQ = 180^\circ - \angle PSQ = 180^\circ - 60^\circ = 120^\circ$$

- 39 In figure, XP and XQ are two tangents to a circle with centre O from a point X outside the circle. ARB is tangent to circle at R. Prove that $XA + AR = XB + BR$.



Given: XP, XQ and ARB are three tangents.

To prove: $XA + AR = XB + BR$

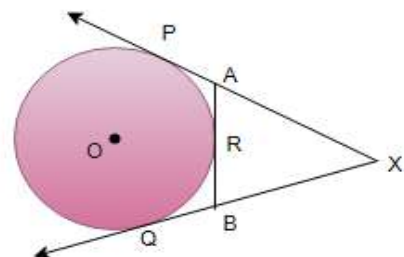
Proof: $AP = AR$... (i)

and $BQ = BR$... (ii)

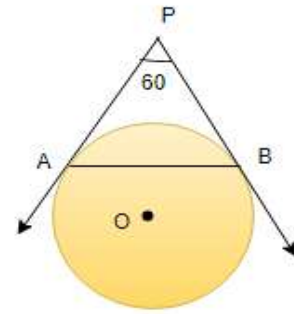
Also $XQ = XP$ [Tangents drawn from an external point]

$$XA + AP = XB + BQ$$

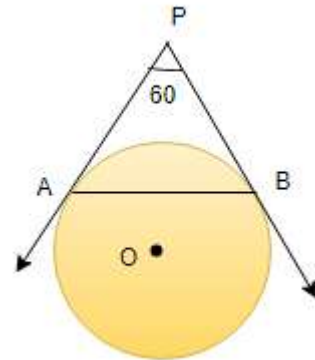
$$XA + AR = XB + BR \text{ [From (i) and (ii)]}$$



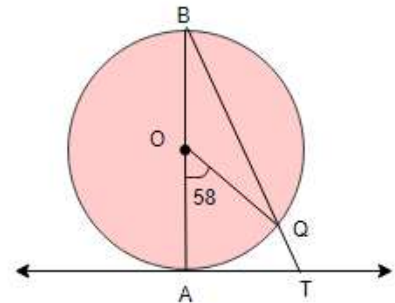
- 40 In figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and $\angle APB = 60^\circ$. Find the length of chord AB.



ANS: $AP = BP$ [tangents from external point P]
 $\angle PAB = \angle PBA$ [Angles opposite to equal sides]
 Now $\angle APB + \angle PAB + \angle PBA = 180^\circ$
 $60^\circ + 2 \angle PAB = 180^\circ$
 $\angle PAB = 60^\circ$
 ΔAPB is an equilateral Δ
 $AB = AP = 5$ cm



- 41 In figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^\circ$, find $\angle ATQ$.



$$\angle ABQ = \frac{1}{2} \times 58^\circ = 29^\circ$$

In ΔABT ,

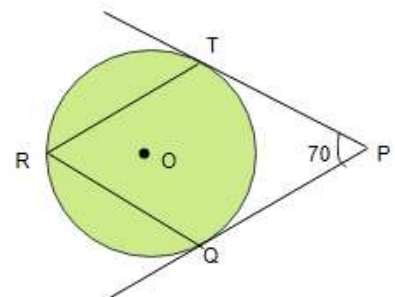
$$\angle BAT + \angle ABT + \angle ATB = 180^\circ$$

$$90^\circ + 29^\circ + \angle ATB = 180^\circ$$

$$\angle ATB = 61^\circ$$

$$\text{as } \angle ATB = \angle ATQ \Rightarrow \angle ATQ = 61^\circ$$

- 42 In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ = 70^\circ$, find $\angle TRQ$



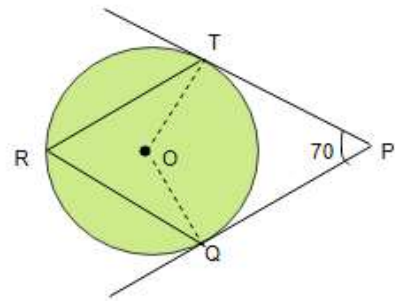
$$\angle TOQ + \angle TPQ = 180^\circ$$

$$\Rightarrow \angle TOQ = 110^\circ$$

$$\text{Also } \angle TOQ = 2 \angle TRQ$$

[angle subtended by an arc at centre of the circle is twice the angle subtended by it in alternate segment]

$$\Rightarrow 110^\circ = 2 \angle TRQ \Rightarrow \angle TRQ = 55^\circ$$



- 43 ABC is an isosceles triangle, in which $AB = AC$, circumscribed about a circle. Show that BC is bisected at the point of contact

Here, $AB = AC$ (Given) ...(i)

$AF = AE$ (Tangents from A) ...(ii)

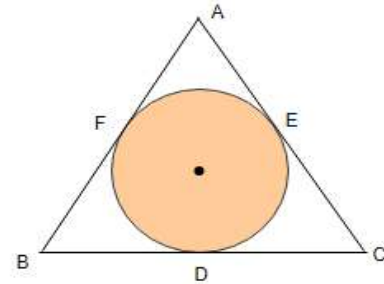
$$AB - AF = AC - AE$$

$$\Rightarrow BF = CE \text{ ...(iii)}$$

Now, $BF = BD$ (Tangents from B)

Also, $CE = CD$ (Tangents from C)

$$\Rightarrow BD = CD$$



- 44 Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre

Given. AB and CD are two tangents to a circle and

$AB \parallel CD$.

Tangent BD intercepts an angle BOD at the centre.

To Prove. $\angle BOD = 90^\circ$.

Construction. Join OQ, OB, OD and OR.

Proof. $OP \perp BD$.

[A tangent at any point of a circle is perpendicular to the radius through the point of contact.]

In right angled Δ s OQB and OPB,

$$\angle 1 = \angle 2,$$

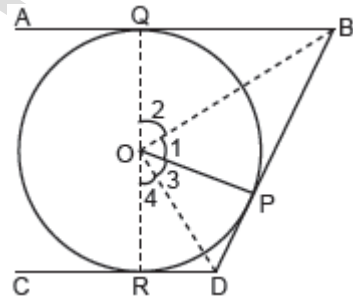
Similarly in right angled Δ s OPD and ORD

$$\angle 3 = \angle 4$$

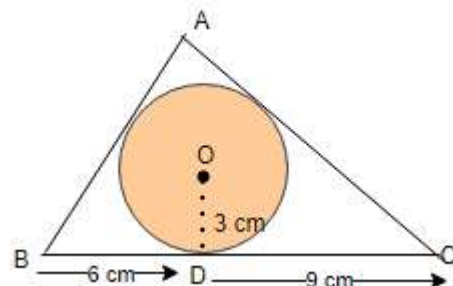
$$\angle BOD = \angle 1 + \angle 3 = \frac{1}{2} [2\angle 1 + 2\angle 3] = \frac{1}{2} (\angle 1 +$$

$$\angle 1 + \angle 3 + \angle 3)$$

$$= \frac{1}{2} (\angle 1 + \angle 2 + \angle 3 + \angle 4) = \frac{1}{2} (180^\circ) = 90^\circ.$$



- 45 In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2 , then find the lengths of sides AB and AC.



Let $AF = x$ cm

$AF = AE = x$ [tangents from A]

Also $BD = BF = 6$ cm and $CD = CE = 9$ cm

$AB = (6 + x)$ cm and $AC = (9 + x)$ cm

Area $\triangle ABC = \text{Area } \triangle BOC + \text{Area } \triangle COA + \text{Area } \triangle AOB$

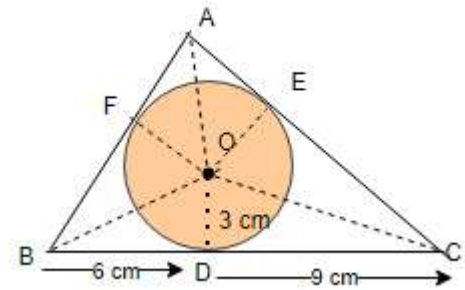
$$\Rightarrow 54 = \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE + \frac{1}{2} AB \times OF$$

$$\Rightarrow 54 \times 2 = 15 \times 3 + (6 + x) \times 3 + (9 + x) \times 3$$

$$108 = 45 + 18 + 3x + 27 + 3x$$

$$6x = 18 \Rightarrow x = 3$$

$$\Rightarrow AB = 6 + x = 6 + 3 = 9 \text{ cm and } AC = 9 + x = 9 + 3 = 12 \text{ cm}$$



- 46 AB is diameter and AC is a chord of a circle such that $\angle BAC = 30^\circ$. If tangent at C intersects AB produced in D, prove that $BC = BD$.

Join OC

$$OC \perp CD \therefore \angle 2 + \angle 3 = 90^\circ$$

$$OC = OA \therefore \angle 1 = 30^\circ$$

Now $\angle 1 + \angle 2 = 90^\circ$ [Angles in a semicircle]

$$\therefore \angle 2 = 90^\circ - 30^\circ = 60^\circ$$

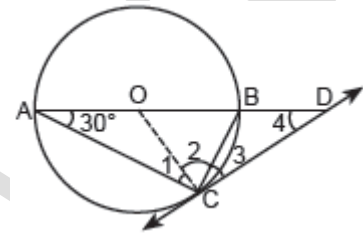
$$\Rightarrow \angle 3 = 30^\circ$$

$$\text{In } \triangle ACD, \angle ACD + \angle CAD + \angle 4 = 180^\circ$$

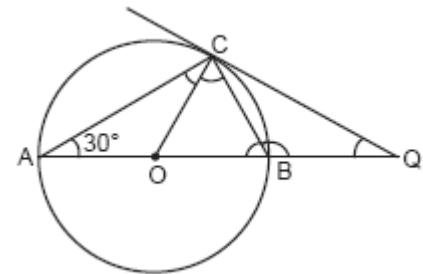
$$\Rightarrow 120^\circ + 30^\circ + \angle 4 = 180^\circ \Rightarrow \angle 4 = 30^\circ$$

$$\text{In } \triangle BCD, \angle 3 = \angle 4$$

$$\therefore BC = BD.$$



- 47 In the figure, AB is diameter of a circle with centre O and QC is a tangent to the circle at C. If $\angle CAB = 30^\circ$, find $\angle CQA$ and $\angle CBA$.



ANS: In $\triangle AOC$, $OA = OC$ [Radii of the same circle]

$$\angle ACO = \angle CAO = 30^\circ \text{ [Opp. angles of equal sides are equal.]}$$

Also $\angle ACB = 90^\circ$ [Angle in semicircle]

$$\angle OCB = 90^\circ - 30^\circ = 60^\circ$$

In $\triangle COB$, $OC = OB$ [Radii of the semicircle]

$$\angle OCB = \angle OBC = 60^\circ \text{ [Opposite angles of equal sides]}$$

Now $OC \perp CQ$

$$\angle OCQ = 90^\circ \Rightarrow \angle BCQ = 90^\circ - 60^\circ = 30^\circ$$

$$\text{Also } \angle OBC + \angle CBQ = 180^\circ$$

$$\Rightarrow 60^\circ + \angle CBQ = 180^\circ \Rightarrow \angle CBQ = 120^\circ$$

In $\triangle CBQ$,

$$\begin{aligned}\angle BCQ + \angle CBQ + \angle CQB &= 180^\circ \\ \Rightarrow 30^\circ + 120^\circ + \angle CQB &= 180^\circ \Rightarrow \angle CQB = 30^\circ \\ \angle CQA &= 30^\circ,\end{aligned}$$

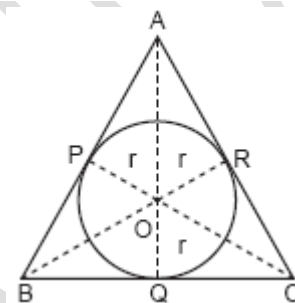
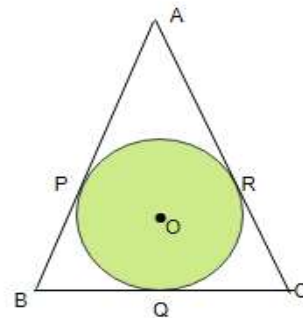
$$\text{Now } \angle CBA = 180^\circ - \angle CBQ = 180^\circ - 120^\circ = 60^\circ.$$

- 48 In figure, the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P, Q and R respectively.

Prove that

(i) $AB + CQ = AC + BQ$

(ii) $\text{Area}(\Delta ABC) = \frac{1}{2} (\text{perimeter of } \Delta ABC) \times r$



i) $AP = AR$ [Tangents from A] ...(i)

Similarly, $BP = BQ$...(ii)

$CR = CQ$...(iii)

Now, $AP = AR$

$$\Rightarrow (AB - BP) = (AC - CR)$$

$$\Rightarrow AB + CR = AC + BP$$

$$\Rightarrow AB + CQ = AC + BQ \text{ [Using eq. (ii) and (iii)]}$$

(ii) Let $AB = x$, $BC = y$, $AC = z$

Perimeter of $\Delta ABC = x + y + z$...(iv)

$$\text{Area of } \Delta ABC = \frac{1}{2} [\text{area of } \Delta AOB + \text{area of } \Delta BOC + \text{area of } \Delta AOC]$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2} AB \times OP + \frac{1}{2} \times BC \times OQ + \frac{1}{2} \times AC \times OR$$

$$\text{Area of } \Delta ABC = \frac{1}{2} x \times r + \frac{1}{2} y \times r + \frac{1}{2} z \times r$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2} (x + y + z) \times r$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2} (\text{Perimeter of } \Delta ABC) \times r \text{ [Using (iv)]}$$

- 49 PQR is a right angled triangle right angled at Q. $PQ = 5$ cm, $QR = 12$ cm. A circle with centre O is inscribed in ΔPQR , touching its all sides. Find the radius of the circle.

Let $QS = x$; $SR = 12 - x$

$PT = 5 - x$; $PM = PT$

$PM = 5 - x$

Also $SR = MR \Rightarrow MR = 12 - x$

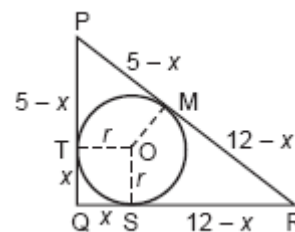
Also $PQ^2 + QR^2 = PR^2$

$$\Rightarrow PR = 13 \Rightarrow PM + MR = 13$$

$$\Rightarrow 5 - x + 12 - x = 13 \Rightarrow 2x = 4 \Rightarrow x = 2$$

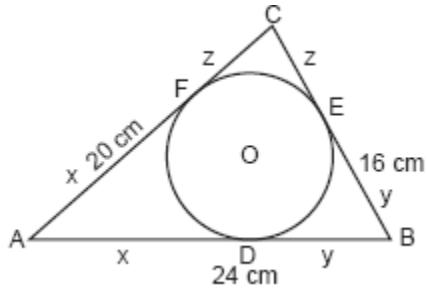
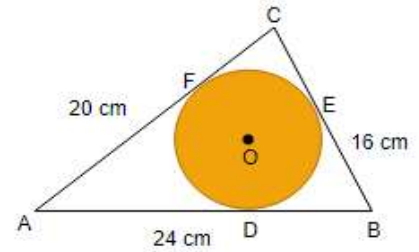
Also OSQT is a square

$\therefore OS = QS \Rightarrow OS = 2$ cm



\therefore Radius of incircle = 2 cm.

- 50 A circle is inscribed in a ΔABC having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF.



Let $AD = AF = x$ [Tangents from external point are equal]

$BD = BE = y$ and $CE = CF = z$

According to the question,

$$AB = x + y = 24 \text{ cm} \dots (i)$$

$$BC = y + z = 16 \text{ cm} \dots (ii)$$

$$AC = x + z = 20 \text{ cm} \dots (iii)$$

Subtracting (iii) from (i), we get $y - z = 4 \dots (iv)$

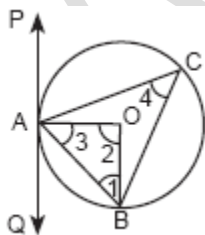
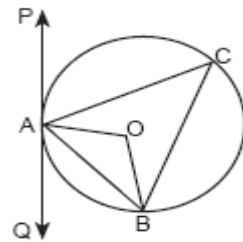
Adding (ii) and (iv), we get

$$2y = 20 \Rightarrow y = 10 \text{ cm.}$$

Substituting the value of y in (ii) and (i) we get $z = 6 \text{ cm}; x = 14 \text{ cm}$

$AD = 14 \text{ cm}$, $BE = 10 \text{ cm}$ and $CF = 6 \text{ cm}$.

- 51 PAQ is a tangent to the circle with centre O at a point A as shown in figure. If $\angle OBA = 35^\circ$, find the value of $\angle BAQ$ and $\angle ACB$.



Given: PAQ is a tangent to the circle with centre O at a point A as shown in figure $\angle OBA = 35^\circ$.

To find: $\angle BAQ$ and $\angle ACB$

Proof: $OA = OB$ [Radii of the same circle]

$\Rightarrow \angle 3 = 35^\circ$ [Angles opposite to equal sides of a triangle are equal]

But, $\angle 1 + \angle 2 + \angle 3 = 180^\circ$ [Angle sum property]

$$\Rightarrow 35^\circ + 35^\circ + \angle 2 = 180^\circ \Rightarrow \angle 2 = 180^\circ - 70^\circ = 110^\circ$$

$$4 = \frac{1}{2} \angle 2 = \frac{1}{2} \times 110^\circ = 55^\circ$$

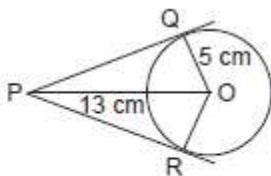
$\Rightarrow \angle ACB = 55^\circ$ [Degree measure theorem]

$\angle BAQ = \angle ACB = 55^\circ$ [Angles in the same segment]

- 52 From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is

(a) 60 cm^2 (b) 65 cm^2 (c) 30 cm^2 (d) 32.5 cm^2

(a) Here, $PQ = \sqrt{OP^2 - OQ^2}$
 $= \sqrt{13^2 - 5^2} = 12 \text{ cm}$



Area of quadrilateral PQOR

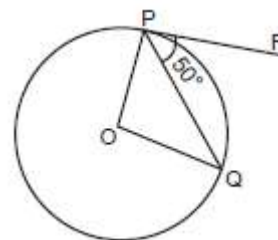
$= \text{ar. of } \triangle POQ + \text{ar. of } \triangle POR$

$$= \frac{1}{2} \times 12 \times 5 + \frac{1}{2} \times 12 \times 5 = 30 + 30 = 60 \text{ cm}^2.$$

- 53 In figure if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then

$\angle POQ$ is equal to

(a) 100° (b) 80° (c) 90° (d) 75°



ANS: (a) $OP \perp PR$ [\because Tangent and radius are \perp to each other at the point of contact]

$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

$OP = OQ$ [Radii]

$$\therefore \angle OPQ = \angle OQP = 40^\circ$$

$$\text{In } \triangle OPQ, \Rightarrow \angle POQ + \angle OPQ + \angle OQP = 180^\circ$$

$$\Rightarrow \angle POQ + 40^\circ + 40^\circ = 180^\circ \quad \angle POQ = 180^\circ - 80^\circ = 100^\circ.$$

- 54 Two concentric circles are of radii 13 cm and 5 cm. The length of the chord of larger circle which touches the smaller circle is _____.

\because AB touches the smaller circle

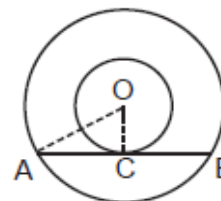
$\therefore OC \perp AB$ and hence $AC = BC$

$$\text{In right } \triangle OCA, OA^2 = OC^2 + AC^2$$

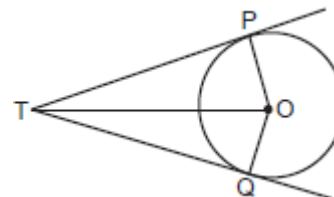
$$\Rightarrow AC^2 = 13^2 - 5^2$$

$$\Rightarrow AC = 12$$

$$\therefore AB = 2 \times 12 = 24 \text{ cm.}$$



- 55 In the given figure, TP and TQ are two tangents to a circle with centre O, such that $\angle POQ = 110^\circ$. Then $\angle PTQ$ is equal to (a) 55° (b) 70° (c) 110° (d) 90°



ANS: (b) In quadrilateral POQT,

$$\angle PTQ + \angle TPO + \angle TQO + \angle POQ = 360^\circ$$

$$\Rightarrow \angle PTQ + 90^\circ + 90^\circ + 110^\circ = 360^\circ$$

$$\Rightarrow \angle PTQ + 290^\circ = 360^\circ$$

$$\Rightarrow \angle PTQ = 360^\circ - 290^\circ = 70^\circ$$

- 56 In figure, PQ and PR are tangents to a circle with centre A. If $\angle QPA = 27^\circ$, then $\angle QAR$ equals to _____

- (a) 63° (b) 153° (c) 126° (d) 117°

ANS: (c) $\angle QPA = \angle RPA$

[$\because \triangle AQP \cong \triangle ARP$ (RHS congruence rule)]

$$\Rightarrow \angle RPA = 27^\circ$$

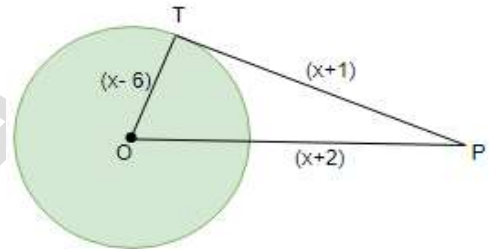
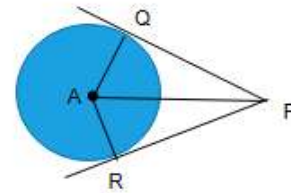
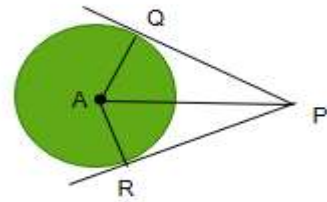
$$\therefore \angle QPR = \angle QPA + \angle RPA = 27^\circ + 27^\circ = 54^\circ \text{ Now,}$$

$$\angle QAR + \angle AQP + \angle ARP + \angle QPR = 360^\circ$$

$$\Rightarrow \angle QAR = 90^\circ + 90^\circ + 54^\circ = 234^\circ$$

$$\Rightarrow \angle QAR = 360^\circ - 234^\circ = 126^\circ$$

- 57 In the below figure, find the actual length of sides of $\triangle OTP$.



In the figure, find the value of x.

