CLASS XI - PERMUTATIONS AND COMBINATIONS

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1. $If\ \ (m+\ n)P_2 = 90,\ \ (m-\ n)P_2 = 30$ find the value of m and n (Ans: m = 8 , n = 2)
2. Find n if $2n\ C_3 : n\ C_3 = 11 : 1$. (Ans: 6)
3. $(n+1)C_{r+1} : n\ C_r : (n-1)C_{r-1}$ = 11 : 6 : 3 (Ans: n = 10, r = 5)
4. If $nP_r = 272\ and\ \ nC_r = 136$ find n and r. (Ans: r = 2 n = 15)
5. $15P_{r-1} : 16\ P_{r-2} = 3:4\ find\ r$ (Ans: 14)
6. How many diagonals has a polygon of n sides? ($Ans: nC_2 - n$
7. How many diagonals has an octagon? (Ans: 20)
8. A polygon has 35 diagonals, find the number of sides of the polygon. (Ans: 10)
9. How many i) lines ii) triangles can be drawn through n points on a circle . ($Ans: n\ C_2\ \ , n\ C_3$)
10. How many i) lines ii) triangles can be formed by joining 12 points in a plane of which 5 are collinear.
($Ans:\ 12\ C_2 - 5\ C_2 + 1 = 57, 12\ C_3 - 5\ C_3 = 210$)
11. If all the permutations of the letters of the word INDIA are arranged as in a dictionary , what are the 1^{st} , 14^{th} , 49^{th} ,50^{th} and 60^{th} words. (ADIIN, DAINI, NADII, NAIDI, NIIDA)
12. Suppose a number plate of a vehicle contains two letters followed by three digits with first digit is not 0 . how many different number plates can be made if i) no letter or digit can be repeated ii) repetition of letters or digits are allowed. (Ans : 26 x 25 x 9 x 9 x 8 = 421200, 26 x 26 x 9 x 10 x 10 = 608400.)
13. How many 6 digits telephone numbers can be constructed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 if each starts with 35 and no digits appears more than once. (Ans: 1680)
14. In how many ways 6 boys and 4 girls can be seated in a row so that no girls are together? (604800)
15. Find the number of ways in which 3 boys and 5girls can be arranged in a row so that i) no two boys are together. ii) All the girls are together (Ans:14400, 2880)
16. How many permutations can be made out of the letters of the word TRIANGLE ? How many of these i) begin with T ? ii) end with E iii) T and E occupy end places (40320 ; 5040 ;5040; 1440)
17. How many different words can be formed with the letters of the word H A R Y A N A? How many of these : (i) have H and N together, (ii) begin with H and end with N and (iii) have three vowels together.
18. Find the number of ways in which 5 cards can be selected out of 52 cards, if at least one of the cards is an ace. (Ans: 778320 + 103776 + 4512 + 48 = 886656)
19. How many words can be made out of the letters of the word 'MALENKOV' so that i) first letter is a vowel. ii) no two vowels come together iii) vowels may occupy odd places iv) vowels being always together. (15120; 120 x 6 x 5 x 4 = 14400 ; 2880 ; 4320)

20. How many permutations can be made out of the letters of the word DAUGHTER?
    i) taking all the letters together ii) beginning with D and ending with R iii) vowels being always together. iv) Vowels occupying even places. (40320; 720; 4320; 120)
21. If all the permutations of the letters of the word BHARAT are arranged as in a dictionary, what is the 121^{st} word? (BAAHRT)
22. How many numbers are there between 100 and 1000 (including 100 but excluding 1000) such that
23. every digit is either 2 or 5 ii) there is no restriction iii) the digits in hundred's place is 5
24. How many diagonals has a polygon of n sides? How many triangles can be formed by joining the vertices of the polygon?
25. Out of 6 men and 3 women, a cabinet of 5 ministers is to be formed. In how many ways can this be done if the cabinet is to include at least 3 men? (111)
26. In how many ways can a team of 11 players be chosen from 15 players? How many of them
    i) will include a particular player A? ii) Will exclude a particular player B iii) will include A and exclude B? (Ans : 1365,1001,364,286)
27. It was found at a certain meeting that after every member has shaken hands with every other member, 45 handshakes were interchanged. How many members were present at the meeting? (10)
28. From a set of 7 batsman, 6 bowlers and 2 wicket keepers, a team of 11 has to be chosen so as to include at least 5 batsmen, 4 bowlers and 1 wicket keeper. Find the number of selection? (777)
29. From a class of 12 boys and 10 girls, 10 students are to be chosen for a competition, including at least 4 boys and 4 girls. The two girls who won the prizes last year should be included. In how many ways can the selection be made? (34650 + 44352 + 25872 = 104874).
30. A committee of 12 is to be formed from 9 women and 8 men. In how many ways this can be done if at least five women have to be included in the committee? In how many of these committees, women are in majority.
31. In a class, there are 27 boys and 14 girls. The teacher wants to select 1 boy and 1 girl to represent the class for a function. In how many ways can the teacher make this selection? (Ans: $27 \times 14 = 378$.)
32. How many numbers are there between 99 and 1000 having 7 in the units place? (Ans: $10 \times 9 = 90$)
33. How many numbers are there between 99 and 1000 having at least one of their digits 7?
    (Ans: Total number of 3 digit numbers having at least one of their digits as 7 = (Total numbers of three digit numbers) – (Total number of 3 digit numbers in which 7 does not appear at all). = $(9 \times 10 \times 10) – (8 \times 9 \times 9) = 900 – 648 = 252$.)

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34. In how many ways can 5 children be arranged in a line such that (i) two particular children of them are always together (ii) two particular children of them are never together?

    (We consider the arrangements by taking 2 particular children together as one and hence the remaining 4 can be arranged in 4! = 24 ways. Again two particular children taken together can be arranged in two ways. Therefore, there are 24 × 2 = 48 total ways of arrangement.)

    (Among the 5! = 120 permutations of 5 children, there are 48 in which two children are together. In the remaining 120 – 48 = 72 permutations, two particular children are never together.)

35. If all permutations of the letters of the word AGAIN are arranged in the order as in a dictionary. What is the 49th word?

    (Starting with letter A, and arranging the other four letters, there are 4! = 24 words. These are the first 24 words. Then starting with G, and arranging A, A, I and N in different ways, there are $\frac{4!}{2!1!1!} = 12$ words. Next the 37th word starts with I. There are again 12 words starting with I. This accounts up to the 48th word. The 49th word is NAAGI.)

36. 6 In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together.

    (First we take books of a particular subject as one unit. Thus there are 4 units which can be arranged in 4! = 24 ways. Now in each of arrangements, mathematics books can be arranged in 3! ways, history books in 4! ways, chemistry books in 3! ways and biology books in 2! ways. Thus the total number of ways = 4! × 3! × 4! × 3! × 2! = 41472.)

37. A student has to answer 10 questions, choosing at least 4 from each of Parts A and B. If there are 6 questions in Part A and 7 in Part B, in how many ways can the student choose 10 questions?

    (The possibilities are: 4 from Part A and 6 from Part B or 5 from Part A and 5 from Part B or 6 from Part A and 4 from Part B. Therefore, the required number of ways is 105 + 126 + 35 = 266.)

38. Suppose m men and n women are to be seated in a row so that no two women sit together. If m > n, show that the number of ways in which they can be seated is $\frac{m!(m+1)!}{(m-n+1)!}$.

    (Let the men take their seats first. They can be seated in m! ways as shown in the following figure

    ◯ M ◯ M ◯ M ◯ ……. ... M ◯

    From the above figure, we observe, that there are (m + 1) places for n women. It is given that m > n and no two women can sit together. Therefore, n women can take their seats (m + 1)Pn ways and hence the total number of ways so that no two women sit together is $m! \,.\, (m+1)P_n = \frac{m!(m+1)!}{(m-n+1)!}$)

39. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics Part II, unless Mathematics Part I is also borrowed. In how many ways can he choose the three books to be borrowed?

    (Case (i) Boy borrows Mathematics Part II, then he borrows Mathematics Part I also. So the number of possible choices is 6 C_1 = 6. Case (ii) Boy does not borrow Mathematics Part II, then the number of possible choices is $7C_3$ = 35. Hence, the total number of possible choices is 35 + 6 = 41.)

40. There are four bus routes between A and B; and three bus routes between B and C. A man can travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make round trip?

    (there are 4 bus routes from A to B and 3 routes from B to C. Therefore, there are 4 × 3 = 12 ways to go from A to C. It is round trip so the man will travel back from C to A via B. It is restricted that man cannot use same bus routes from C to B and B to A more than once. Thus, there are 2 × 3 = 6 routes for return journey. Therefore, the required number of ways = 12 × 6 = 72.)

41. In how many ways a committee consisting of 3 men and 2 women, can be chosen from 7 men and 5 women?

    (Out of 7 men, 3 men can be chosen in $7C_3$ ways and out of 5 women, 2 women can be chosen in $5C_2$ ways. Hence, the committee can be chosen in $7C_3 \times 5C_2$ = 350 ways.)

42. All the letters of the word 'EAMCOT' are arranged in different possible ways. The number of such arrangements in which no two vowels are adjacent to each other is

    (We note that there are 3 consonants and 3 vowels E, A and O. Since no two vowels have to be together, the possible choice for vowels are the places marked as 'X'. X M X C X T X, these vowels can be arranged in 4 P_3 ways 3 consonants can be arranged in 3 ways. Hence, the required number of ways = 3! × 4 P_3 = 144.

43. Ten different letters of alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is

    (Number of 5 letters words (with the condition that a letter can be repeated) = 105 . Again number of words using 5 different letters is 10 P_5 . Therefore, required number of letters = Total number of words – Total number of words in which no letter is repeated = 105 – 10 P_5 = 69760).

44. In an examination there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answer correct is

    (There are three multiple choice question, each has four possible answers. Therefore, the total number of possible answers will be 4 × 4 × 4 = 64. Out of these, possible answer only one will be correct and hence the number of ways in which a student can fail to get correct ans is 64 – 1 = 63.)

45. Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements.
    (2 women occupy the chair, from 1 to 4 in $4P_2$ ways and 3 men occupy the remaining chairs in 6 P3 ways.)
46. Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of lines that can be formed joining the point.
    (Number of straight lines = 18C2 – $5C_2$+ 1)
47. How many committee of five persons with a chairperson can be selected from 12 persons
    (Chairman can be selected in 12 ways and remaining in $11C_4$.)
48. How many automobile license plates can be made if each plate contains two different letters followed by three different digits?
49. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected from the lot.
50. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.
51. There are 10 persons named P1, P2, P3,…… P10. Out of 10 persons, 5 persons are to be arranged in a line such that in each arrangement P1 must occur whereas P4 and P5 do not occur. Find the number of such possible arrangements.
    (Required number of arrangement = $7C_4$ × 5!]
52. The number of different words that can be formed from the letters of the word INTERMEDIATE such that two vowels never come together is _____.
    (Number of ways of arranging 6 consonants of which two are alike is 6!/ 2! and number of ways of arranging vowels $is \quad 7C_6.\frac{1}{3!}.\frac{1}{2!}$.
53. In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together.
    (First we take books of a particular subject as one unit. Thus there are 4 units which can be arranged in 4! = 24 ways. Now in each of arrangements, mathematics books can be arranged in 3! ways, history books in 4! ways, chemistry books in 3! ways and biology books in 2! ways. Thus the total number of ways = 4! × 3! × 4! × 3! × 2! = 41472.)