TRIGONOMETRY

CLASS XI (2025-26)

1 Find the value of $\tan\frac{13\pi}{12}$

ANS: $\tan\frac{13\pi}{12} = \tan\left(\pi+\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{12}\right) = \tan(15°) = \tan(60° - 45°)$

$= \frac{tan60° - tan45°}{1+tan60°.tan45°} = \frac{\sqrt{3}-1}{1+\sqrt{3}}$

$$\frac{\sqrt{3}-1}{1+\sqrt{3}} = \frac{(\sqrt{3}-1)}{(1+\sqrt{3})} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3+1-2\sqrt{3}}{3-1} = \frac{2(2-\sqrt{3})}{2} = 2-\sqrt{3}$$

2 Show that : $\frac{\tan 69° + \tan 66°}{1 - \tan 69°.\ \tan 66°} = -1$

$\frac{\tan 69° + \tan 66°}{1 - \tan 69°.\ \tan 66°} = \tan(69° + 66°) = \tan(135°) = \tan(180 - 45°) = -1$

3 Convert $40°\ 20'$ into radian measure.

ANS: $40°\ 20' = 40°\left(\frac{20}{60}\right)^{\circ} = \frac{121°}{3} = \frac{121}{3} \times \frac{\pi}{180} = \frac{121\pi^{C}}{540}$.

4 Express the following angle in radian : $5°\ 37'\ 30''$

ANS: $5°\ 37'\ 30''$

$5°37\ \frac{30'}{60} = 5°37\ \frac{1'}{2} = 5°\ \frac{75°}{2\times 60} = \frac{45°}{8}$

$\frac{45°}{8} = \frac{\pi}{180} \times \frac{45^{C}}{8} = \frac{\pi^{C}}{32}$

5 Express the following angle in radian : 450°

$450° = \frac{\pi}{180} \times 450 = \frac{5\pi^{C}}{2}$

6 Find the value of $sin\ 75°\ cos\ 15° + cos\ 75°\ sin\ 15°$

ANS: $sin\ (75° + 15°) = sin\ 90° = 1.$

7 Find the value of $sin\ 32°\ cos\ 28° + cos\ 32°\ sin\ 28°$

ANS: $sin\ (32° + 28°) = sin\ 60° = \frac{\sqrt{3}}{2}$

8 Find the value of $cos\ 47°\ sin\ 17° - sin\ 47°\ cos\ 17°$

ANS: $sin\ (17° - 47°) = - sin\ 30° = -\frac{1}{2}$

9 Express the following as sum or difference : $cos\ 5\theta\ cos\ 3\theta$

ANS: $\frac{1}{2}[2\ cos\ 5\theta\ cos\ 3\theta]$

$= \frac{1}{2}[cos\ (5\theta + 3\theta) + cos\ (5\theta - 3\theta)]$ $= \frac{1}{2}\ [cos\ (8\theta + cos\ 2\theta)$

10 Express the following as sum or difference : $2\ sin\ 5\theta\ sin\ 3\theta$

ANS: $2\ sin\ 5\theta\ sin\ 3\theta$

$= cos\ (5\theta - 3\theta) - cos\ (5\theta + 3\theta) \quad = cos\ 2\theta - cos\ 8\theta$

11 Express each of the following as a product : $sin\ 32° + sin\ 54°$

ANS: $sin\ 54° + sin\ 32°$

$= 2\ sin\left(\frac{54°+32°}{2}\right)\left(\frac{54°- 32°}{2}\right) \quad = 2\ sin\ 43°\ cos\ 11°$

12 Express each of the following as a product : $cos\ 6\theta + cos\ 4\theta$

ANS: $cos\ 6\theta + cos\ 4\theta = 2\cos\left(\frac{6\theta+4\theta}{2}\right)\cos\left(\frac{6\theta-4\theta}{2}\right)$

$= 2\ cos\ 5\theta\ cos\ \theta.$

13 Evaluate, $sin\ 105° + cos\ 105°$.

ANS: Consider, $sin\ 105° + cos\ 105°$

$= sin\ 105° + cos\ (90° + 15°) = sin\ 105° - sin\ 15°$

$= 2\ cos\ 60°\ sin\ 45° = 2 \times \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

14 Show that : $\frac{1}{2}\left(\sqrt{3}\,cos35° - sin35°\right) = \cos 65°$

ANS: $\left(\frac{\sqrt{3}}{2}cos35° - \frac{1}{2}sin35°\right) =$

$cos30°\ cos35° - sin30° sin35° = \cos(30 + 35) = \cos 65°.$

15 Show that: $\sin(150° + x) + \sin(150° - x) = cosx$.

ANS: $\sin(150° + x) + \sin(150° - x) = 2\ sin150°\ cosx = cosx$

16 Prove that : $\frac{cos\ 29°+sin29°}{cos29°-sin29°} = tan74°$

ANS: $\frac{cos29°+sin29°}{cos29°-sin29°} = \frac{1+\tan 29°}{1-\tan 29°} = \tan(45 + 29) = tan74°$

17 Prove that : $tan36° = \frac{cos\ 9°- sin\ 9°}{cos\ 9°+ sin\ 9°}$

ANS: $tan36° = tan(45° - 9°)$

$tan(45° - 9°) = \frac{tan45°-tan9°}{1+tan45°.tan9°} = \frac{1-tan9°}{1+tan9°}\ \frac{1-\frac{sin9°}{\cos 9°}}{1+\frac{sin9°}{coc9°}} = ° = \frac{cos\ 9°- sin\ 9°}{cos\ 9°+ sin\ 9°}$

18 If $tanA = \frac{1}{2}$, $tanB = \frac{1}{3}$, find the value of $\tan(2A + B)$.

ANS: $tan2A = \frac{2tanA}{1-tan^2A} = \frac{4}{3}$

$\tan(2A + B) = \frac{tan2A+tanB}{1- tan2A\ tanB} = 3$

19 If $tanx = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$, find the value of $\sin\frac{x}{2}$.

$tanx = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$ $\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$ so $\sin\frac{x}{2}$. is positive

$sec^2x = 1 + tan^2x = \frac{25}{16}$, $cosx = \pm\sqrt{\frac{16}{25}} = cosx = -\frac{4}{5}$

$1 - 2sin^2\left(\frac{x}{2}\right) = -\frac{4}{5}$, $\sin\frac{x}{2} = \frac{3}{\sqrt{10}}$

20 Find the value of $\sqrt{3}\ cosec\ 20^\circ - sec20^\circ$.

ANS: $\left(\frac{\sqrt{3}}{2}cos35° - \frac{1}{2}sin35°\right) =$

$cos30°\ cos35° - sin30° sin35° = \cos(30 + 35) = \cos 65°$

21 Prove that : $tan\ 50° = tan\ 40° + 2\ tan\ 10°$.

ANS : Consider, tan 50° = tan (40° + 10°) = $\frac{\tan 40 + \tan 10}{1 - \tan 40\ \tan 10}$

$\Rightarrow tan\ 50° - tan\ 50°\ tan\ 40°\ tan\ 10° = tan\ 40° + tan\ 10°$ [By cross multiplication]

$\Rightarrow tan\ 50° = tan\ 40° + tan\ 10° + tan\ (90° - 40°)\ tan\ 40°\ tan\ 10°$

$= tan\ 40° + tan\ 10° + cot\ 40°\ tan\ 40°\ tan\ 10°$

$= tan\ 40° + tan\ 10° + tan\ 10°$ $[\because cot\ \theta\ tan\ \theta = 1]$

$\Rightarrow tan\ 50° = tan\ 40° + 2tan\ 10°$.

22 Prove that : $\cos 2\theta\ \cos\ \frac{\theta}{2} - \cos\ 3\theta \cos\ \frac{9\theta}{2} = \sin 5\theta \sin\frac{5\theta}{2}$.

ANS:

$\cos 2\theta\ \cos\ \frac{\theta}{2} - \cos\ 3\theta \cos\ \frac{9\theta}{2} = \frac{1}{2}\left[2cos2\theta \cos\ \frac{\theta}{2} - 2\cos\ 3\theta \cos\ \frac{9\theta}{2}\right]$

$= \frac{1}{2}\left[\cos\left(2\theta + \frac{\theta}{2}\right) + cos\left(2\theta - \frac{\theta}{2}\right) - \cos\left(\frac{9\theta}{2} + 3\theta\right) - \cos\left(\frac{9\theta}{2} - 3\theta\right)\right]$

Simplify

$= \frac{1}{2}\left[\cos\left(\frac{5\theta}{2}\right) - \cos\left(\frac{15\theta}{2}\right)\right]$ Apply $(cosC - cosD)$ Simplify

$= \sin 5\theta \sin\frac{5\theta}{2}$

23 Find the value of the expression $cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$.

$\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

$= \left(cos^2\frac{\pi}{8}\right)^2 + \left(cos^2\frac{3\pi}{8}\right)^2 + \left(\cos^2\frac{5\pi}{8}\right)^2 + \left(\cos^2\frac{7\pi}{8}\right)^2$

$= \left(\frac{1+cos\frac{\pi}{4}}{2}\right)^2 + \left(\frac{1+cos\frac{3\pi}{4}}{2}\right)^2 + \left(\frac{1+cos\frac{5\pi}{4}}{2}\right)^2 + \left(\frac{1+cos\frac{7\pi}{4}}{2}\right)^2$

$= \frac{1}{4}\left[\left(1 + \frac{1}{\sqrt{2}}\right)^2 + \left(1 - \frac{1}{\sqrt{2}}\right)^2 + \left(1 - \frac{1}{\sqrt{2}}\right)^2 + \left(1 + \frac{1}{\sqrt{2}}\right)^2\right]$ simplify

$= \frac{3}{2}$

24 Prove that: $\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x-\frac{\pi}{3}\right) = \frac{3}{2}$

$$\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x-\frac{\pi}{3}\right) = \frac{1}{2}\left[1 + cos2x + 1 + cos2\left(x+\frac{\pi}{3}\right) + 1 + cos2\left(x-\frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3 + cos2x + cos\left(2x+\frac{2\pi}{3}\right) + cos\left(2x-\frac{2\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3 + cos2x + 2\,cos2x\,.cos\frac{2\pi}{3}\right]$$

$$= \frac{1}{2}\left[3 + cos2x + 2\,cos2x\,\frac{-1}{2}\right]$$

$$= \frac{1}{2}[3 + cos2x - cos2x] = \frac{3}{2}$$

25 Show that $tan3A - tan2A - tanA = tan3A\,tan2A\,tanA$.

$$tan3A = \tan(2A + A) = \frac{tan2A + tanA}{1 - tan2A\,tanA}$$

$$tan3A\,(1 - tan2A\,tanA) = tan2A + tanA$$

$$tan3A - tan2A - tanA = tan3A\,tan2A\,tanA$$

26 Show that : $\frac{cos3A+sin3A}{cosA-sinA} = 1 + 2\,sin2A$

LHS= $\frac{cos3A+sin3A}{cosA-sinA} = \frac{\mathbf{4cos^3A-3cosA+3sinA-4sin^3A}}{cosA-sinA} = \frac{4(\mathbf{cos^3A-sin^3A})\mathbf{-3}(cosA-sinA)}{cosA-sinA}$

$$= \frac{(cosA - sinA)(4(\mathbf{cos^2A + sin^2A + cosA\,sinA}) - 3)}{cosA - sinA}$$

$=4(1 + cosA\,sinA) - 3 = 1 + 2\,sin2A$

27 Find the value of $2\,sin^2\left(\frac{3\pi}{4}\right) + 2cos^2\left(\frac{3\pi}{4}\right) - 2tan^2\left(\frac{3\pi}{4}\right)$.

ANS: $2\,sin^2\left(\frac{3\pi}{4}\right) + 2cos^2\left(\frac{3\pi}{4}\right) - 2tan^2\left(\frac{3\pi}{4}\right)$

$\sin\frac{3\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\cos\frac{3\pi}{4} = \cos\left(\pi - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$\tan\frac{3\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = -1$

$2\,sin^2\left(\frac{3\pi}{4}\right) + 2cos^2\left(\frac{3\pi}{4}\right) - 2tan^2\left(\frac{3\pi}{4}\right) = 2 \times \frac{1}{2} + 2 \times \frac{1}{2} - 2 \times 1 = 0$

28 What is the value of $cos\left(\frac{\pi}{4} - x\right)\ cos\left(\frac{\pi}{4} - y\right) - sin\left(\frac{\pi}{4} - x\right)\ sin\left(\frac{\pi}{4} - y\right)$

ANS: $cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\} = cos\left\{\frac{\pi}{2}-(x+y)\right\} = sin(x+y)$

29 Prove that: $\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x-\frac{\pi}{3}\right) = \frac{3}{2}$

$$\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x-\frac{\pi}{3}\right) = \frac{1}{2}\left[1+cos2x+1+cos2\left(x+\frac{\pi}{3}\right)+1 + cos2\left(x-\frac{\pi}{3}\right)\right]$$

$$=\frac{1}{2}\left[3+cos2x+cos\left(2x+\frac{2\pi}{3}\right)+cos\left(2x-\frac{2\pi}{3}\right)\right]$$

$=\frac{1}{2}\left[3+cos2x+2\,cos2x\,.\,cos\frac{2\pi}{3}\right]$ $\quad =\frac{1}{2}\left[3+cos2x+2\,cos2x\,\frac{-1}{2}\right]$

$=\frac{1}{2}[3+cos2x- cos2x\ ]=\frac{3}{2}$

30 Find the value of sin 15°, cos 15°, tan 15°, cot 15°

ANS: $sin\,15° = sin\,(45° - 30°)$

$= sin\,45°\,cos\,30° - cos\,45°\,sin\,30°$

$=\frac{1}{\sqrt{2}}\,.\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$ii)$ $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$ $\quad iii)$ $\tan 15° = 2-\sqrt{3}$ $\quad iv)$ $\cot 15° = 2+\sqrt{3}$

31 Find the value of the following : $tan\,(-\,1125°)$

ANS: $tan\,(-\,1125°) = -\,tan\,1125°$

$= -\,tan\,[12 \times 90° + 45°] = -\,tan\,45° = -\,1.$

32 Find the value of the following : $sin\,(-\,330°)$

ANS: $sin\,(-330°) = -\,[sin\,(360° - 30°)]$

$= -\,[-\,sin\,30°] = \frac{1}{2}$

33 Prove: $tan\,720° - cos\,270° - sin\,150°\,cos\,120° = \frac{1}{4}$

ANS: $0-0-\frac{1}{2}\times\left(-\frac{1}{2}=\frac{1}{4}\right)$

34 Prove: $cos\,570°\,sin\,510° + sin\,(-330°)\,cos\,(-390°) = 0$

ANS: $cos\,(360° + 210°)\,sin\,(360° + 150°) - sin\,(360° - 30°)\,cos\,(360° + 30°)$

$= cos\,(180° + 30°)\,sin\,(180° - 30°) + sin\,30°\,.\,cos\,30°$

$= -\,cos\,30°\,.\,sin\,30° + sin\,30°\,cos\,30° = 0$

35 Prove: $24° + cos\,55° + cos\,125° + cos\,204° + cos\,300° = \frac{1}{2}.$

ANS: $cos\,24° + cos\,55° + cos\,(180° - 55°) + cos\,(180° + 24°) + cos\,(360° - 60°)$

$cos\ 24° + cos\ 55° - cos\ 55° - cos\ 24° + cos\ 60° = \frac{1}{2}$.

36 Find the value of $cos\ 42°\ cos\ 12° + sin\ 42°\ sin\ 12°$

ANS: $cos\ (42° - 12°) = cos\ 30° = \frac{\sqrt{3}}{2}$.

37 Find the value of $cos\ 85°\ cos\ 40° + sin\ 40°\ sin\ 85°$.

ANS: $cos\ (85° - 40°) = cos\ 45° = \frac{1}{\sqrt{2}}$.

38 Find the value of $\frac{\tan 69° + \tan 66°}{1 - \tan 69°.\ \tan 66°} = -1$.

ANS: $tan\ (69° + 66°) = tan\ 135° = tan\ (180° - 45°) = -\ tan\ 45° = -1$.

39 Prove the following : $cos\ 20° + cos\ 100° + cos\ 140° = 0$.

ANS: $(cos\ 100° + cos\ 20°) + cos\ 140°$

$$= 2\ cos\ 60°\ cos\ 40° + cos\ 140°$$

$$= cos\ 40° + cos\ 140°$$

$$= 2\ cos\ 90°\ cos\ (-50°) = 0$$

40 Prove the following : $sin\ 10° + sin\ 20° + sin\ 40° + sin\ 50° = sin\ 70° + sin\ 80°$.

ANS: $(sin\ 50° + sin\ 10°) + (sin\ 40° + sin\ 20°)$

$$= 2\ sin\ 30°\ cos\ 20° + 2\ sin\ 30°\ cos\ 10°.$$

$$= cos\ 20° + cos\ 10°$$

$$= cos\ (90° - 70°) + cos\ (90° - 80°)$$

$$= sin\ 70° + sin\ 80°.$$

41 Prove the following : $\frac{sinA + \sin 3A}{\cos A + \ cos3A} = \tan 2A$

ANS: $\frac{sinA + \sin 3A}{\cos A + \ cos3A} = \frac{2sin2A\ cosA}{2cos2AcosA} = tan2A$

42 Prove the following : $\frac{\sin 7\alpha - \sin \alpha}{\sin 8\alpha - \sin 2\alpha} = \cos 4\alpha . sec5\alpha$.

ANS: $\frac{\sin 7\alpha - \sin \alpha}{\sin 8\alpha - \sin 2\alpha} = \frac{2cos4\alpha\ .sin3\alpha}{2cos5\alpha\ .sin3\alpha} = \cos 4\alpha . sec5\alpha$

43 Prove that: $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2}sinx$

LHS =

$$-2 \sin \frac{\frac{3\pi}{4} + x + \frac{3\pi}{4} - x}{2} \sin \frac{\frac{3\pi}{4} + x - \frac{3\pi}{4} + x}{2}$$

$$-2\ .\ \sin \frac{3\pi}{4} . \sin x = -\sqrt{2} \sin x$$

44 Prove that : $\frac{sinx-sin3x}{sin^2x-cos^2x}=2sinx$

ANS: $\frac{2\,cos2x.\;\sin(-x)}{-cos2x}$ = $2sinx$

45 If $\sin A=\frac{1}{2}, \cos B=\frac{\sqrt{3}}{2}$ $,where\ \frac{\pi}{2}<A<\pi,\ 0<B<\frac{\pi}{2}, find\ \tan(A+B)\ and\ \tan(A-B).$

46 If $sinx=\frac{3}{5}$ and $cosy=\frac{-12}{13}$ and x, y both lie in the second quadrant , find the value of:i. $\sin(x+y)$ ii. $\cos(x+y)$. (TRY YOURSELF)

ANS: Given, $\sin x=\frac{3}{5}$, $\cos y=-\frac{12}{13}$ and x, y both lie in the second quadrant.

We know that $\cos^2 x = 1-\sin^2 x = 1-\left(\frac{3}{5}\right)^2=\frac{16}{25} \Rightarrow \cos x=\pm\frac{4}{5}$

Since, x lies in 2nd quadrant, $\cos x$ is (–ve).

$\therefore \cos x=-\frac{4}{5}$

Also, $\sin^2 y = 1-\cos^2 y = 1-\left(\frac{-12}{13}\right)^2=\frac{25}{169} \quad \Rightarrow \sin y=\pm\frac{5}{13}$

Since, y lies in 2nd quadrant, $\sin y$ is (+ve)

$\therefore \sin y = \frac{5}{13}$

$\sin(x+y)=\sin x\cdot\cos y+\cos x\cdot\sin y$

$=\frac{3}{5}\left(-\frac{12}{13}\right)+\left(-\frac{4}{5}\right)\frac{5}{13}=-\frac{56}{65}$

47 If $\sin x=\frac{3}{5}$, $\cos y=-\frac{12}{13}$ and x, y both lie in the second quadrant, find the values of $\tan(x+y)$

ANS: $-\frac{56}{33}$

48 Show that $sin\alpha+\sin\left(\alpha+\frac{2\pi}{3}\right)+\ \sin\left(\alpha+\frac{4\pi}{3}\right)=0$

ANS $sin\alpha+\sin\left(\alpha+\frac{2\pi}{3}\right)\ +\ \sin\left(\alpha+\frac{4\pi}{3}\right)=\ sin\alpha+2\sin\frac{\left(\alpha+\frac{2\pi}{3}\right)+\left(\alpha+\frac{4\pi}{3}\right)}{2}\ cos\frac{\left(\alpha+\frac{2\pi}{3}\right)-\left(\alpha+\frac{4\pi}{3}\right)}{2}$

$=\sin\alpha+2\sin(\alpha+\pi)\,.\cos\left(-\frac{\pi}{3}\right)=\sin\alpha+2(-\sin\alpha)\times\frac{1}{2}=\sin\alpha-\sin\alpha=0=$ RHS

49 Prove that : $\frac{sin3x+sin5x+\sin7x+sin9x}{\cos3x+\cos5x+\cos7x+cos9x}=\ tan6x$

LHS =

$$\frac{\sin3x+\sin5x+\sin7x+\sin9x}{\cos3x+\cos5x+\cos7x+\cos9x}=\frac{(\sin9x+\sin3x)+(\sin7x+\sin5x)}{(\cos9x+\cos3x)+(\cos7x+\cos5x)}$$

$$\frac{2\sin6x.\cos3x+2\sin6x.\cos x}{2\cos6x.\cos3x+2\cos6x.\cos x}=\frac{2\sin6x(\cos3x+\cos x)}{2\cos6x(\cos3x+\cos x)}=\frac{\sin6x}{\cos6x}$$

$=\ tan6x$

50 Show that : $\sqrt{2+\sqrt{2+2cos4x}}$ = $2\cos x$.

LHS =

$$\sqrt{2+\sqrt{2+2\cos 4x}} = \sqrt{2+\sqrt{2+2\left(2\cos^2 2x-1\right)}}$$

$$\sqrt{2+\sqrt{2+4\cos^2 2x-2}} = \sqrt{2+\sqrt{4\cos^2 2x}} = \sqrt{2+2\cos 2x}$$

$$\sqrt{2\left(1+\cos 2x\right)} = \sqrt{2\times 2\cos^2 x} = \sqrt{4\cos^2 x}$$

= 2cos x.

51 Prove that $(sin\ 3x + sin\ x)\ sin\ x + (cos\ 3x - cos\ x)\ cos\ x = 0$

ANS: $LHS = (sin\ 3x + sin\ x)\ sin\ x + (cos\ 3x - cos\ x)\ cos\ x$

$= sin\ 3x\ sin\ x + sin2\ x + cos\ 3x\ cos\ x - cos2x$

$= (cos\ 3x\ cos\ x + sin\ 3x\ sin\ x) - (cos2\ x - sin2\ x)$

$= cos\ (3x - x) - cos\ 2x$

$[Using\ cos\ (A - B) = cos\ A\ cos\ B + sin\ A\ sin\ B\ and\ cos2\ \theta - sin2\ \theta = cos\ 2\theta]$

$= cos\ 2x - cos\ 2x = 0 = RHS$

52 Prove the following : $tan\ 13A - tan\ 7A - tan\ 6A = tan\ 13A\ tan\ 7A\ tan\ 6A$

ANS: $tan\ 13\ A = tan\ (7A + 6A) = \frac{tan7A+tan6A}{1-\ tan7A.\ tan6A}$

$\Rightarrow tan\ 13A\ (1 - tan\ 7A\ tan\ 6A) = tan\ 7A + tan\ 6A.$

$\Rightarrow tan\ 13A - tan\ 7A - tan\ 6A = tan\ 13A\ tan\ 7A\ tan\ 6A$

53 Prove that : $tan\ 80° = tan\ 10° + 2\ tan\ 70°$

ANS: $\tan 80° = \tan(70° + 10°) = \frac{tan70 + tan10}{1-\ tan70.\ tan10}$

$\Rightarrow \tan 80° - \tan 80° \tan 70° \tan 10° = \tan 70° + \tan 10°$

$\Rightarrow \tan 80° = \tan 70° + \tan 10° + \tan(90° - 10°) \tan 70° \tan 10°,$

$\Rightarrow \tan 80° = \tan 70° + \tan 10° + \cot 10° \tan 70° \tan 10°,$

$\Rightarrow$ tan 80° = tan 70° + tan 10° + $\frac{1}{\tan 10°}$. tan 70° .tan 10° = 2 tan 70° + tan 10°

54 Prove the following : $4\ sin\ \alpha\ .\ sin\ (60 - \alpha)\ .\ sin\ (60 + \alpha) = sin\ 3\alpha.$

ANS: $4\ sin\ \alpha\ [sin^2\ 60° - sin^2\ \alpha]$ = 4 sin α [$\frac{3}{4}$ – sin^2 α] = 3 sin α – 4 sin^3 α = sin 3α

55 Prove $cos\ \alpha\ .\ cos\ (60 - \alpha)\ cos\ (60 + \alpha) = \frac{1}{4}\ cos\ 3\alpha.$

ANS: LHS = $cos\ \alpha[cos^2\ \alpha - sin^2\ 60°]$

= cos α[cos^2 α – $\frac{3}{4}$] = $\frac{1}{4}$ [4 cos^3 α – 3 cos α] = $\frac{1}{4}$ cos^3 α = RHS

56 Show that : $cos^2 A + cos^2\ B - 2\ cos\ A\ cos\ B\ cos\ (A + B) = sin^2\ (A + B).$

ANS: LHS = $cos^2 A + cos^2 B - [cos(A+B) + cos(A-B)]\, cos(A+B)$

$= cos^2 A + cos^2 B - \cos^2(A+B) - \cos(A+B)\cos(A-B)$

$= cos^2 A + cos^2 B - cos^2(A+B) - cos^2 A + sin^2 B$

$= (cos^2 B + sin^2 B) - cos^2(A+B) = 1 - cos^2(A+B) = sin^2(A+B)$ = RHS

57 Show that : $cos\, A + cos\,(120° - A) + cos\,(120° + A) = 0.$

ANS: $cos\, A + 2\, cos\, \frac{240°}{2}\ \ cos\,(-A)$

$= cos\, A + 2\, cos\, 120°\, cos\, A$

$= cos\, A + 2 \times -\frac{1}{2} \times cos\, A = 0$

58 Find the value of $\sqrt{3}\ cosec\ 20^{\circ} - sec 20^{\circ}$.

ANS: $\sqrt{3}\ cosec\ 20^{\circ} - sec 20^{\circ} = \sqrt{3}\frac{1}{\sin 20} - \frac{1}{\cos 20} = \frac{\sqrt{3}\cos 20 - sin 20}{\cos 20\ .\ sin 20}$

$= \frac{2}{2} \times \left(\frac{\sqrt{3}\cos 20 - sin20}{\cos 20\ sin 20}\right) = \frac{2}{\cos 20\ .\ sin20}\ \left(\frac{\sqrt{3}}{2} cos20 - \frac{1}{2}\ sin20\right)$

$= \frac{2}{\cos 20\ .\ sin20}(\ cos30\ cos20 - sin30\ sin20)$

$= \frac{2}{\cos 20\ .\ sin20}\ cos50 = \frac{4}{2\cos 20\ .\ sin20}\ cos50 = \frac{4}{sin40} cos50$

$= \frac{4}{sin40}\cos(90-40) = \frac{4}{sin40}\ sin40 = 4$

59 Show that: $\sin(150° + x) + \sin(150° - x) = cosx$.

ANS: $\sin(150° + x) + \sin(150° - x) = 2\ sin150°\ cosx = cosx$

60 Prove that $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$

ANS: $\cos(2 \times 2x) = 1 - 2\sin^2 2x = 1 - 2(2\sin x \cos x)^2 = 1 - 8\sin^2 x \cos^2 x$

61 Prove each of the following : $\frac{\sec 8A - 1}{sec4A - 1} = \frac{tan8A}{tan2A}$

ANS: $\frac{\sec 8A - 1}{sec4A - 1} = \frac{1 - cos8A}{cos8A} \times \frac{cos4A}{1 - cos4A} = \frac{2sin^2 4A}{cos8A} \times \frac{cos4A}{2sin^2 2A}$

$$\frac{sin4A(2\ sin4A\ \ cos4A)}{cos8A.2sin^2 2A} = \frac{2sin2A\ cos2A(sin8A)}{cos8A.2sin^2 2A} = \frac{tan8A}{tan2A}$$

62 $If\ \ \tan(x+y) = \frac{3}{4},\ \tan(x-y) = \frac{8}{15}$ Find i) $tan2x$ ii) $tan2y$

63 Prove that : $tan62° = \frac{cos\ 17° + sin\ 17°}{cos\ 17° - \ sin\ 17°}$

64 Prove that : $tan74° = \frac{cos\ 29° + sin\ 29°}{cos\ 29° - \ sin\ 29°}$

65 Prove that : $tan34° = \frac{cos\ 10° - sin\ 10°}{cos\ 10° + \ sin\ 10°}$

66 Show that : $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2cos8\theta.}}}$ = 2cos θ.

Ans: LHS = $\sqrt{2+\sqrt{2+\sqrt{2(1+cos8\theta)}}}$. = $\sqrt{2+\sqrt{2+\sqrt{2(2\,cos^2 4\theta)}}}$.

= $\sqrt{2+\sqrt{2+2\,cos4\theta}}$ = $\sqrt{2+\sqrt{2(1+\ cos4\theta)}}$ = $\sqrt{2+\sqrt{2(2\,cos^2 2\theta)}}$

= $\sqrt{2+2cos2\theta}$ = $\sqrt{2(1+cos2\theta)}$ = $\sqrt{2(2\,cos^2\theta)}$ = 2cos θ.

67 Prove that: $sinA.\sin(60-A)\sin(\,60+A) = \frac{1}{4}\sin 3A$

ANS:

$sinA.\sin(60-A)\sin(\,60+A) = \frac{1}{2}\,sinA(2\sin(\,60+A)\sin(\,60-A))$

$= \frac{1}{2}\,sinA(\cos(\,60+A-\ (\,60-A)\,)-\ \cos(\,60+A+\ (\,60-A)$

$= \frac{1}{2}\,sinA\ (cos2A-\cos 120\,)$

$= \frac{1}{4}(\,2\,cos2A\sin A-2\cos 120\sin A)$, simplify LHS = $\frac{1}{4}\sin 3A$

68 Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$, if $\tan x = -\frac{4}{3}$, where x lies in 2nd quadrant

ANS: $\tan x = -\frac{4}{3}$, x lies in 2nd quadrant

$\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

$\frac{x}{2}$ *lies in the* 1*st*quadrant

$\tan x = -\frac{4}{3}$ $cosx = -\frac{3}{5}$ (2nd quadrant)

Formula: $sin^2\frac{A}{2} = \frac{1-cosA}{2}$

$\Rightarrow sin\left(\frac{x}{2}\right) = \sqrt{\frac{1-cosx}{2}}$

$sin\left(\frac{x}{2}\right) = \sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}} = \sqrt{\frac{8}{10}} = \frac{2}{\sqrt{5}}$

Formula: $cos^2\frac{A}{2} = \frac{1+cosA}{2}$

$\Rightarrow cos\left(\frac{x}{2}\right) = \sqrt{\frac{1+cosx}{2}} = \sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}} = \sqrt{\frac{2}{10}} = \frac{1}{\sqrt{5}}$

$\tan\frac{x}{2} = \sqrt{\frac{1-cosx}{1+cosx}}$ $= \frac{2}{\sqrt{5}} \div \frac{1}{\sqrt{5}} = 2$

69 Prove that $\cos 55° + \cos 65° + \cos 175° = 0$

ANS: $cos55° + \cos 65° + \cos 175° = 2cos\frac{55°+65°}{2}\cos\frac{55°-65°}{2} + \cos 175°$

$= 2\cos 60\,.\cos 5 + \cos 175° = cos5\cos(180-5) = 0$

70 If $sin\ (\,A-B) = \frac{1}{\sqrt{10}}$ *and* $\cos(A+B) = \frac{2}{\sqrt{29}}$ *where* A, B *lie between* 0 *and* $\frac{\pi}{4}$, find $tan2A$.

ANS : $\sin\ (A-B)=\frac{1}{\sqrt{10}}\ \ \tan(A-B)=\frac{1}{3}\ \ ,\ \cos(A+B)=\ \frac{2}{\sqrt{29}}=\ \tan(A+B)=\frac{5}{2}$

$tan2A .= \tan[(A+B)+(\ A-B)]=\frac{\tan[(A+B)+(\ A-B)]}{1-\tan[(A+B)+(\ A-B)]}=17.$

71 If $A\ +\ B\ =\ 45°$ show that: $(1+\ tan\ A)\ (1+tan\ B)\ =\ 2$

72 Prove that : $\sqrt{\frac{1+sinA}{1-sinA}}=tan\left(\frac{\pi}{4}+\frac{A}{2}\right)$

73 Show that : $\frac{sin3A}{sinA}-\frac{cos3A}{cosA}=2.$

74 Prove that: $\frac{sin\ 38°-cos68°}{cos68°+\sin\ \ 38°}=\ \sqrt{3}\tan 8\,°$

ANS: : $\frac{sin\ (90°-52°)\ -cos68°}{cos68°+sin\ (90°-52°)}=\frac{cos52°\ -cos68°}{cos68°+cos52°}\ =\ \frac{sin60\ sin8°}{cos60\ cos8°}=\ \sqrt{3}\tan 8\,°$

75 Prove that : $cos20°cos40°cos80°\ =\ \frac{1}{8}$

76 Show that: $\sin(y+z-x)+\sin(z+x-y)+\sin(x+y-z)-\sin(x+y+z)=4sinx\ siny\ sinz.$

77 Show that: $(\cos\alpha+cos\beta)^2\ +(\sin\alpha+sin\beta)^2\ =4\ cos^2\left(\frac{\alpha-\beta}{2}\right)$.

78 Prove that : $\frac{\sin(x+y)}{\sin(x-y)}=\frac{tanx+tany}{tanx-tany}$

79 Prove that : $\sin(40°+A)\cos(10°+A)-\cos(40°+A)\sin(10°+A)=\frac{1}{2}$

80 If $tanA\ =\ \ \frac{m}{m+1}\ and\ tanB\ =\ \ \frac{1}{2m+1}$, then show that A + B = $\frac{\pi}{4}$

81 Prove that : $\frac{sin3x-sinx}{\cos 2x}=2\ sinx$

ANS: $\frac{sin3x-sinx}{\cos 2x}=\frac{2\ cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}{\cos 2x}=\frac{2cos2x.sinx}{cos2x}=2\ sinx$