

INTEGRATION CLASS XII

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- 1 Find $\int \frac{2\cos x}{3\sin^2 x} dx$
 A) $-\frac{2}{3} \operatorname{cosec} x + C$ B) $\frac{2}{3} \operatorname{cosec} x + C$ C) $\frac{3}{2} \operatorname{cosec} x + C$ D) $\frac{2}{3} \sec x + C$ ANS: (A)
- 2 Find $\int (ax + b)^3 dx$
 A) $(ax + b)^4 + C$ B) $\frac{(ax+b)^4}{4} + C$ C) $\frac{(ax+b)^4}{a} + C$ D) $\frac{(ax+b)^4}{4a} + C$ ANS: (D)
- 3 Find $\int \sec x (\sec x + \tan x) dx$
 A) $\tan x - \sec x + C$ B) $\tan x + \sec x + C$ C) $\tan x + \operatorname{cosec} x + C$ D) $\sec x - \cot x + C$ ANS: (B)
- 4 Find $\int \frac{1}{(3+x)^2 + 1} dx$
 A) $\tan^{-1}(3 + x) + C$ B) $\log(3 + x) + C$ C) $(3 + x)^3 + C$ D) $\frac{(3+x)^3}{3} + C$ ANS: (A)
- 5 Find $\int \frac{1}{4+9x^2} dx$
 A) $\frac{1}{3} \tan^{-1} \frac{3x}{2} + C$ B) $\frac{1}{6} \log \frac{3x}{2} + C$ C) $\frac{1}{6} \tan^{-1} \frac{3x}{2} + C$ D) $\frac{1}{2} \tan^{-1} \frac{x}{2} + C$ ANS: (C)
- 6 If $f(x) = \int_0^x t \sin t dt$, then write the value of $f'(x)$.
 A) $\sin x$ B) $x \sin x$ C) $x \cos x$ D) $x \cos x + \sin x$ ANS: (B)
- 7 Find $\int \frac{1}{(2-x)^2 + 1} dx$
 A) $-\log(2 - x) + C$ B) $\tan^{-1}(x) + C$ C) $-\tan^{-1}(2 - x) + C$ D) $\tan^{-1}(2 - x) + C$ ANS: (C)
- 8 Find $\int \tan^2(x) dx$
 A) $\tan x - x + C$ B) $\tan x + x + C$ C) $\tan + C$ D) $x - \tan x + C$ ANS: (A)
- 9 Find $\int \tan^{-1}(\cot x) dx$
 A) $\frac{\pi}{2} x + \frac{x^2}{2} + C$ B) $\frac{\pi}{2} - \frac{x^2}{2} + C$ C) $\frac{\pi}{2} x - \frac{x^2}{2} + C$ D) $\frac{x^2}{2} - \frac{\pi}{2} x + C$ ANS: (C)
- 10 Find $\int \cos^2 x dx$
 A) $\frac{1}{2} \left(x + \frac{\sin 2x}{2} \right) + C$ B) $\left(x + \frac{\sin 2x}{2} \right) + C$ C) $\sin 2x + C$ D) $-\sin 2x + C$ ANS: (A)
- 11 Find $\int \frac{1}{3 - x^2} dx$
 A) $\frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + x}{\sqrt{3} - x} \right| + C$ B) $\frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{3} + x}{\sqrt{3} - x} \right| + C$ C) $\frac{1}{2\sqrt{3}} \tan^{-1} x + C$ D) $\frac{1}{\sqrt{3}} \tan^{-1}(3 - x) + C$ ANS: (A)
- 12 Find $\int \sec^2(7 - x) dx$
 A) $\tan(7 - x)$ B) $-\tan(7 - x)$ C) $\cot(7 - x)$ D) none ANS: (B)
- 13 Find $\int \frac{x^2}{1 + x^3} dx$

A) $\frac{1}{3} \log|1 + x^3|$ B) $\frac{1}{2} \log|1 + x^3|$ C) $\frac{1}{3} \tan^{-1}(1 + x^3)$ D) none ANS: (A)

14. Evaluate: $\int \sqrt{1 + \sin\left(\frac{x}{4}\right)} dx$

(A) $8\left(\cos\frac{x}{8} + \sin\frac{x}{8}\right) + C$

(B) $8\left(\sin\frac{x}{8} - \cos\frac{x}{8}\right) + C$

(C) $\left(\sin\frac{x}{8} - \cos\frac{x}{8}\right) + C$

(D) $4\left(\sin\frac{x}{4} - \cos\frac{x}{4}\right) + C$

ANS: (B)

15. Evaluate: $\int \frac{x+2}{\sqrt{x+5}} dx$

(A) $\frac{2}{3}(x+5)^{\frac{3}{2}} - 3\sqrt{x+5}$

(B) $\frac{2}{3}(x+5)^{\frac{3}{2}} - 6\sqrt{x+5}$

(C) $\frac{2}{3}(x+5)^{\frac{3}{2}} + 6\sqrt{x+5}$

(D) $\frac{2}{3}(x+5)^{\frac{3}{2}} - 2\sqrt{x+5}$

ANS: (B)

16. Find $\int \frac{1}{9x^2+4} dx$

(A) $\frac{1}{9} \tan^{-1} \frac{3x}{2} + C$

(B) $\frac{1}{2} \tan^{-1} \frac{3x}{2} + C$

(C) $\frac{1}{6} \tan^{-1} \frac{3x}{2} + C$

(D) $\frac{1}{4} \log \frac{3x+2}{3x-2} + C$

ANS: (C)

17. Evaluate: $\int \frac{x + \cos 6x}{3x^2 + \sin 6x} dx$

(A) $\log|3x^2 + \sin 6x| + C$

(B) $\frac{1}{3} \log|3x^2 + \sin 6x| + C$

(C) $\frac{1}{6} \log|3x^2 + \sin 6x| + C$

(D) $\frac{1}{6} \log|x + \cos 6x| + C$

ANS: (C)

18. Evaluate: $\int \frac{e^x}{1+e^{2x}} dx$

(A) $\tan^{-1} x + C$

(B) $\tan^{-1} e^x + C$

(C) $\log x + C$

(D) $\log e^x + C$

ANS: (B)

19. Evaluate: $\int_1^3 \frac{\cos(\log x)}{x} dx$

(A) $\cos(\log 3)$

(B) $\sin(\log 3)$

(C) $\cos 3$

(D) $-\sin(\log 3)$

ANS: (B)

20. The value of $\int_0^{\frac{\pi^2}{4}} \frac{\cos \sqrt{x}}{\sqrt{x}} dx =$ _____

(A) -2

(B) 1

(C) 2

(D) π

ANS: (C)

REVISION

1. Evaluate $\int \tan^3 x dx$

ANS: $\frac{\tan^2 x}{2} - \log \sec x + C$

2. Evaluate $\int \frac{2 dx}{1 + \cos 2x}$

ANS: $\tan x + C$

3. Evaluate $\int \frac{\sec^2 x dx}{\operatorname{cosec}^2 x}$

4. If $\int \frac{x^3}{\sqrt{1+x^2}} dx = a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$, then find the values of a and b . ANS: $a = \frac{1}{3}, b = -1$

5. Evaluate $\int \sqrt{1 + \cos x} dx$

6. Evaluate $\int \frac{x^4+1}{x^2+1} dx$

7. Evaluate $\int \frac{x+2}{(x+1)^2} dx$	8. Evaluate $\int (3x-2)^3 dx$
9. Evaluate $\int e^{4t+3} dt =$	10. Evaluate $\int \sec^2 8x dx =$
11. Evaluate $\int \operatorname{cosec}(2-7x) \cot(2-7x) dx$	12. Find $\int \tan^{-1}(\cot x) dx$
13. Find $\int (1-x)\sqrt{x} dx$	14. Find $\int \sin^3 x dx$
15. Find $\int \frac{x+4}{(x+3)^2} dx$	16. Find $\int \sin^2 bx dx$
17. Find $\int \frac{1}{1+\cos x} dx$	18. Find $\int \frac{1}{1+\sin x} dx$
19. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos^2 x dx$	20. Evaluate: $\int_1^e \frac{1}{x\sqrt{1-(\log x)^2}} dx$

1. Evaluate : $\int e^x \cos x dx$

ANS: $\int e^x \cos x dx = e^x \sin x - \int e^x \sin x dx$
 $I = e^x \sin x - [e^x(-\cos x) - \int e^x(-\cos x) dx]$
 $I = e^x \sin x + e^x(\cos x) - I$
 $2I = e^x(\sin x + \cos x),$
 $I = \frac{1}{2} e^x(\sin x + \cos x) + C$

2. Find $\int \sec x (\sec x + \tan x) dx$

ANS: $\int (\sec^2 x + \sec x \tan x) dx = \tan x + \sec x + C$

3 Find $\int \frac{2-3 \sin x}{\cos^2 x} dx$

ANS: $\int 2 \sec^2 x - 3 \sec x \tan x dx =$
 $2 \tan x - 3 \sec x + C$

4 Find $\int \tan^2 x dx$

ANS: $= \int (\sec^2 x - 1) dx = \tan x - x + C$

5 Find $\int \tan^{-1}(\cot x) dx$

ANS: $\int \tan^{-1}\left(\tan\left(\frac{\pi}{2} - x\right)\right) dx =$
 $\int \left(\frac{\pi}{2} - x\right) dx = \frac{\pi}{2}x - \frac{x^2}{2} + C$

6 Find $\int \cos^2 x dx$

ANS: $\int \frac{1+\cos 2x}{2} dx = \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + C$

7 Evaluate: $\int \sin^2 bx dx$

ANS: $I = \int \frac{1-\cos 2bx}{2} dx$
 $= \frac{1}{2} \int (1 - \cos 2bx) dx = \frac{1}{2} \left(x - \frac{\sin 2bx}{2b}\right) + C$

8 Evaluate: $\int \cos^2 x dx$

ANS: $\int \frac{1+\cos 2x}{2} dx$
 $= \frac{1}{2} \int (1 + \cos 2x) dx = \frac{1}{2} \left(x + \frac{\sin 2x}{2}\right) + C$

9 Evaluate: $\int \sin^3 x dx$

ANS: $\sin 3A = 3 \sin A - 4 \sin^3 A$
 $\sin^3 A = \frac{3}{4} \sin A - \frac{1}{4} \sin 3A$

$$\int \sin^3 x \, dx = \int \left(\frac{3}{4} \sin x - \frac{1}{4} \sin 3x \right) dx$$

$$= -\frac{3}{4} \cos x + \frac{\cos 3x}{12} + C$$

10 Evaluate: $\int \cos^3 x \, dx$

ANS: $\cos 3A = 4\cos^3 A - 3\cos A$

$$\int \cos^3 x \, dx = \int \left(\frac{1}{4} \cos 3x + \frac{3}{4} \cos x \right) dx$$

$$\frac{1}{12} \sin 3x + \frac{3}{4} \sin x + C$$

11 Evaluate: $\int \frac{1}{1+\sin x} \, dx$

ANS: $\int \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx = \int \frac{1-\sin x}{\cos^2 x} \, dx$

$$= \int \left[\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \right] dx$$

$$= \int \sec^2 x \, dx - \int \sec x \cdot \tan x \, dx$$

$$= \tan x - \sec x + C$$

12 Evaluate: $\int \frac{1}{1+\cos x} \, dx$

ANS: $\int \frac{1}{2\cos^2(\frac{x}{2})} \, dx$

$$= \int \frac{1}{2} \sec^2 x/2 \, dx = \frac{1}{2} \tan\left(\frac{x}{2}\right) \times 2 = \tan\left(\frac{x}{2}\right) + C$$

13 Evaluate: $\int \tan x \, dx$

ANS: $\int \frac{\tan x \cdot \sec x}{\sec x} \, dx = \int \frac{dt}{t} = \log t + C =$

$$\log|\sec x| + C = -\log|\cos x| + C$$

put $\sec x = t$, $\sec x \tan x \, dx = dt$

14 Evaluate: $\int \cot x \, dx$

ANS: $\int \frac{\cos x}{\sin x} \, dx = \log|\sin x| + C$

put $\sin x = t$, $\cos x \, dx = dt$

15 Evaluate: $\int \operatorname{cosec} x \, dx =$

$$\int \frac{\operatorname{cosec} x \cdot (\operatorname{cosec} x - \cot x)}{(\operatorname{cosec} x - \cot x)} \, dx = \int \frac{dt}{t} = \log t + C$$

$$= \log|(\operatorname{cosec} x - \cot x)| + C$$

$$(\operatorname{cosec} x - \cot x) = t,$$

$$(-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x) dx = dt, \operatorname{cosec} x \cdot (\operatorname{cosec} x - \cot x) = dx = dt$$

16 Evaluate: $\int \sec x \, dx$

ANS: $\int \frac{\sec x (\sec x + \tan x)}{(\sec x + \tan x)} \, dx = \int \frac{dt}{t} \, dx$

$$= \log t + C = \log|(\sec x + \tan x)| + C$$

$$(\sec x + \tan x) = t$$

$$(\sec x \cdot \tan x + \sec^2 x) dx = dt$$

17 Evaluate: $\int \tan^4 x \cdot \sec^2 x \, dx$,

ANS: $\tan x = t$, $\sec^2 x \, dx = dt$

$$\int \tan^4 x \cdot \sec^2 x \, dx = \int t^4 \cdot dt = \frac{t^5}{5} + C$$

$$= \frac{\tan^5 x}{5} + C$$

18 Evaluate: $\int \sec^4 x \cdot \tan x \, dx$

$$\text{ANS: } \int \sec^2 x \cdot \tan x \cdot \sec^2 x \cdot dx$$

$$\tan x = t, \sec^2 x \, dx = dt \int (1 + \tan^2 x) \cdot \tan x \cdot \sec^2 x \cdot dx$$

$$\int (1 + t^2) \cdot t \cdot dt = \int (t + t^3) \cdot dt$$

$$\frac{t^2}{2} + \frac{t^4}{4} + C = \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + C$$

19 Evaluate: $\int \frac{1}{1 - \tan x} \, dx$

$$\text{ANS: } I = \int \frac{1}{1 - \tan x} \, dx = \int \frac{\cos x}{\cos x - \sin x} \, dx =$$

$$\frac{1}{2} \int \frac{2 \cos x}{\cos x - \sin x} \, dx = \frac{1}{2} \int \frac{\cos x - \sin x + \cos x + \sin x}{\cos x - \sin x} \, dx$$

$$\frac{1}{2} \int \left[1 + \frac{\cos x + \sin x}{\cos x - \sin x} \right] \, dx,$$

$$\cos x - \sin x = t$$

$$(-\sin x - \cos x) \, dx = dt$$

$$I = \frac{1}{2} x + \frac{1}{2} \int \frac{-dt}{t} = \frac{1}{2} x - \frac{1}{2} \log(\cos x - \sin x) + C$$

20 Evaluate: $\int \frac{1}{1 + \cot x} \, dx$

$$\text{ANS: } \int \frac{1}{1 + \cot x} \, dx = \frac{1}{2} \int \frac{2 \sin x}{\sin x + \cos x} \, dx =$$

$$\frac{-1}{2} \int \frac{\cos x - \sin x - \cos x - \sin x}{\sin x + \cos x} \, dx \quad \text{Try...}$$

21 Evaluate: $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} \, dx$

$$\text{ANS: } \int \frac{\sin x + \cos x}{\sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x}} \, dx =$$

$$\int \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}} \, dx = \int 1 \, dx = x + C$$

22 Evaluate: $\int \frac{\sqrt{\tan x}}{\sin 2x} \, dx$

$$\text{ANS: } \int \frac{\sqrt{\tan x}}{\sin 2x} \, dx = \int \frac{\sqrt{\tan x}}{2 \sin x \cos x} \, dx = \int \frac{\sqrt{\tan x}}{2 \tan x (\cos x)^2} \, dx$$

$$I = \int \frac{\sqrt{\tan x}}{2 \tan x} \sec^2 x \, dx$$

$$\text{Put } \tan x = t \Rightarrow \sec^2 x \, dx = dt$$

$$I = \int \frac{\sqrt{\tan x}}{2 \tan x} \sec^2 x \, dx = I = \int \frac{\sqrt{t}}{2t} \, dt = \frac{1}{2} \int \frac{1}{\sqrt{t}} \, dt$$

$$I = \sqrt{t} + C \Rightarrow \sqrt{\tan x} + C$$

23 Evaluate: $\int \sqrt{1 + 2 \tan x (\tan x + \sec x)} \, dx$

$$\text{ANS: } \int \sqrt{1 + 2 \tan x (\tan x + \sec x)} \, dx$$

$$= \int \sqrt{\sec^2 x - \tan^2 x + 2 \tan^2 x + 2(\tan x \cdot \sec x)} \, dx$$

$$= \int \sqrt{\sec^2 x + \tan^2 x + 2(\tan x \cdot \sec x)} \, dx = \int \sqrt{(\sec x + \tan x)^2} \, dx = \int \sec x + \tan x \, dx$$

$$= \log|\sec x + \tan x| + \log|\sec x| + c$$

24 Evaluate: $\int_0^{\frac{\pi}{4}} (\tan^2 x + \tan^4 x) \, dx$

$$\text{ANS: } \int_0^{\frac{\pi}{4}} (\tan^2 x + \tan^4 x) dx$$

$$= \int_0^{\frac{\pi}{4}} \tan^2 x (1 + \tan^2 x) dx = \int_0^{\frac{\pi}{4}} \tan^2 x \sec^2 x dx$$

$$\tan x = t, \quad \sec^2 x dx = dt$$

$$I = \int_0^1 t^2 dt = \frac{1}{3}$$

25 Evaluate : $\int \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$

$$\text{ANS: } \int \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx :$$

$$I = \int \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx = \int \frac{\sin x + \cos x}{25 - 16 + 16 \sin 2x} dx$$

$$\int \frac{\sin x + \cos x}{25 - 16 (1 - \sin 2x)} dx = \int \frac{\sin x + \cos x}{25 - 16 (\cos x - \sin x)^2} dx$$

$$\cos x - \sin x = t, \quad (\sin x + \cos x) dx = -dt$$

$$I = \int \frac{1}{25 - 16 t^2} dt = \int \frac{1}{5^2 - (4t)^2} dt$$

$$\frac{1}{40} \log \left| \frac{5 + 4(\cos x - \sin x)}{5 - 4(\cos x - \sin x)} \right| + C$$

26 **Prove that** $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

$$\frac{1}{a^2 - x^2} = \frac{1}{(a+x)(a-x)} = \frac{A}{a+x} + \frac{B}{a-x}$$

$$1 = A(a-x) + B(a+x)$$

$$\text{sub: } x = a, \quad B = \frac{1}{2a}$$

$$\text{sub: } x = -a, \quad A = \frac{1}{2a}$$

$$\frac{1}{(a+x)(a-x)} = \frac{1/2a}{a+x} + \frac{1/2a}{a-x} = \frac{1}{2a} \left(\frac{1}{a+x} + \frac{1}{a-x} \right)$$

$$\int \frac{1}{a^2 - x^2} dx = \int \frac{1}{2a} \left(\frac{1}{a+x} + \frac{1}{a-x} \right) dx =$$

$$= \frac{1}{2a} \int \frac{1}{a+x} + \frac{1}{2a} \int \frac{1}{a-x}$$

$$= \frac{1}{2a} \log(a+x) + \frac{1}{2a} \log(a-x)(-1) + C$$

$$= \frac{1}{2a} \log \frac{a+x}{a-x} + C$$

FORMULAS

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \log |x + \sqrt{a^2 + x^2}| + C$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log|x + \sqrt{x^2 - a^2}| + C$$

27 Evaluate: $\int \frac{1}{9x^2 - 1} dx$

ANS: $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$

$$= \frac{1}{9} \int \frac{1}{x^2 - \frac{1}{9}} dx = \frac{1}{9} \int \frac{1}{x^2 - \left(\frac{1}{3}\right)^2} dx$$

$$= \frac{1}{9} \times \frac{1}{2 \times \frac{1}{3}} \log \frac{x - 1/3}{x + 1/3} + C$$

$$= \frac{1}{6} \log \frac{3x-1}{3x+1} + C$$

28 Evaluate: $\int \frac{1}{x^2 - 121} dx$

ANS: $\int \frac{1}{x^2 - 11^2} dx$

$$= \frac{1}{22} \log \left| \frac{x-11}{x+11} \right| + C$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

29 ANS: $\int \frac{1}{\sqrt{4+x^2}} dx$

ANS: $\int \frac{1}{\sqrt{4+x^2}} dx = \int \frac{1}{\sqrt{2^2+x^2}} dx = \log|x + \sqrt{4+x^2}| + C$

$$\int \frac{1}{\sqrt{a^2+x^2}} dx = \log|x + \sqrt{a^2+x^2}| + C$$

30 Evaluate: $\int \frac{1}{\sqrt{9-25x^2}} dx$

ANS: $\int \frac{1}{\sqrt{3^2-(5x)^2}} dx = \frac{1}{5} \sin^{-1} \frac{5x}{3} + C$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{x}{a} + C$$

31 Evaluate: $\int \frac{1}{\sqrt{4x^2-1}} dx$

ANS: $\int \frac{1}{\sqrt{4x^2-1}} dx = \int \frac{1}{\sqrt{(2x)^2-1}} dx$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \frac{1}{2} \log|2x + \sqrt{4x^2-1}| + C$$

32 Evaluate: $\int \frac{x+2}{\sqrt{x^2-1}} dx$

ANS: $\int \frac{x+2}{\sqrt{x^2-1}} dx = \int \frac{x}{\sqrt{x^2-1}} dx + \int \frac{2}{\sqrt{x^2-1}} dx$

$$I_1 = \int \frac{x}{\sqrt{x^2-1}} dx = \int \frac{2x}{2\sqrt{x^2-1}} dx$$

Sub. $x^2 - 1 = t$, $2x dx = dt$

$$I_1 = \int \frac{dt}{2\sqrt{t}} = \frac{1}{2} 2\sqrt{t} + C_1 = \sqrt{x^2-1} + C_1 \text{----(1)}$$

$$I_1 = \int \frac{2}{\sqrt{x^2-1}} dx = 2 \log|x + \sqrt{x^2-1}| + C_2 \text{----(2)}$$

$$I = \sqrt{x^2-1} + 2 \log|x + \sqrt{x^2-1}| + C$$

33 Evaluate: $\int \frac{\sin x}{3+4 \cos^2 x} dx$

$$\text{ANS: } I = \int \frac{\sin x}{3+4 \cos^2 x} dx$$

$$\cos x = t, \quad -\sin x \, dx = dt$$

$$I = \int \frac{-dt}{3+4t^2} = \int \frac{-dt}{(\sqrt{3})^2 + (2t)^2}$$

$$I = -\frac{1}{2\sqrt{3}} \tan^{-1} \frac{2t}{\sqrt{3}} + C$$

$$I = \frac{-1}{2\sqrt{3}} \tan^{-1} \frac{2\cos x}{\sqrt{3}} + C$$

34 Evaluate: $\int \frac{x^2-1}{x^2+4} dx$

$$\text{ANS: } I = \int \frac{x^2-1}{x^2+4} dx$$

$$I = \int \frac{x^2-1}{x^2+4} dx = \int \frac{x^2+4-5}{x^2+4} dx$$

$$= \int dx - \int \frac{5}{x^2+4} dx$$

$$= x - \int \frac{5}{x^2+2^2} dx$$

$$= x - 5 \cdot \frac{1}{2} \tan^{-1} \frac{x}{2} + C \quad I = x - \frac{5}{2} \tan^{-1} \frac{x}{2} + C$$

35 Evaluate: $\int \frac{x^2}{x^6+1} dx$

$$\text{ANS: } I = \int \frac{x^2}{x^6+1} dx = \int \frac{x^2}{(x^3)^2+1} dx$$

$$x^3 = t, \quad 3x^2 dx = dt, \quad x^2 dx = \frac{dt}{3}$$

$$I = \frac{1}{3} \int \frac{dt}{t^2+1} = \frac{1}{3} \tan^{-1} t + C$$

$$\frac{1}{3} \tan^{-1} x^3 + C$$

36 Evaluate: $\int \sqrt{1 + \sin x} \, dx$

$$\text{ANS: } \int \sqrt{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2\sin \frac{x}{2} \cos \frac{x}{2}} \, dx =$$

$$= \int \sqrt{\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2} \, dx = \int \left[\sin \frac{x}{2} + \cos \frac{x}{2}\right] dx$$

$$= -2 \cos \frac{x}{2} + 2 \sin \frac{x}{2} + C$$

37 Evaluate: $\int \frac{1}{x^2+4x+7} dx$

$$\text{ANS: } \int \frac{1}{x^2+4x+7} dx$$

$$\int \frac{1}{x^2+4x+7} dx = \int \frac{1}{(x+2)^2-4+7} dx$$

$$\int \frac{1}{(x+2)^2+3} dx = \int \frac{1}{(x+2)^2+(\sqrt{3})^2} dx$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x+2}{\sqrt{3}} \right) + C$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

38 Evaluate: $\int \frac{1}{2x^2+7x+13} dx$

$$I = \int \frac{1}{2x^2+7x+13} dx$$

$$-\frac{49}{16} + \frac{13}{2} = \frac{-49 + 104}{16} = \frac{55}{16}$$

$$\begin{aligned}
&= \frac{1}{2} \int \frac{1}{x^2 + \frac{7}{2}x + \frac{13}{2}} dx = \\
&\frac{1}{2} \int \frac{1}{\left(x + \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{13}{2}} dx = \frac{1}{2} \int \frac{1}{\left(x + \frac{7}{4}\right)^2 + \frac{55}{16}} dx = \\
&\frac{1}{2} \int \frac{1}{\left(x + \frac{7}{4}\right)^2 + \left(\frac{\sqrt{55}}{4}\right)^2} dx = \\
&\frac{1}{2} \times \frac{4}{\sqrt{55}} \tan^{-1} \frac{x + \frac{7}{4}}{\frac{\sqrt{55}}{4}} = \frac{2}{\sqrt{55}} \tan^{-1} \frac{4x+7}{\sqrt{55}} + C
\end{aligned}$$

39 Evaluate: $\int \frac{1}{1-6x-9x^2} dx$

$$\int \frac{1}{1-6x-9x^2} dx = \frac{1}{9} \int \frac{1}{\frac{1}{9}-\frac{6}{9}x-x^2} dx$$

$$\frac{1}{9} \int \frac{1}{\left(\frac{\sqrt{2}}{3}\right)^2 - \left(x + \frac{1}{3}\right)^2} dx =$$

$$\frac{1}{9} \cdot \frac{1}{2 \cdot \frac{\sqrt{2}}{3}} \log \frac{\frac{\sqrt{2}}{3} + \left(x + \frac{1}{3}\right)}{\frac{\sqrt{2}}{3} - \left(x + \frac{1}{3}\right)}$$

$$\frac{1}{6\sqrt{2}} \log \frac{\sqrt{2}+3x+1}{\sqrt{2}-3x-1} + C$$

40 Evaluate: $\int \frac{1}{\sqrt{5-4x-2x^2}} dx$

ANS: $\int \frac{1}{\sqrt{5-4x-2x^2}} dx$

$$\frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{5}{2}-2x-x^2}} dx = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{7}{2}-(x+1)^2}} dx$$

$$\frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\frac{7}{2}\right)^2 - (x+1)^2}} dx = \frac{1}{\sqrt{2}} \sin^{-1} \frac{\sqrt{2}(x+1)}{\sqrt{7}}$$

41 Evaluate: $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$

$$e^x = t, \quad e^x dx = dt$$

$$\begin{aligned}
\text{ANS: } \int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx &= \int \frac{dt}{\sqrt{5-4t-t^2}} \\
&= \int \frac{dt}{\sqrt{5-4t-t^2}} = \int \frac{dt}{\sqrt{-(t^2+4t-5)}} \\
&= \int \frac{dt}{\sqrt{-(t+2)^2-4-5}} = \int \frac{dt}{\sqrt{9-(t+2)^2}}
\end{aligned}$$

$$I = \int \frac{dt}{\sqrt{5-4t-t^2}} = \int \frac{dt}{\sqrt{3^2-(t+2)^2}} = \sin^{-1} \frac{t+2}{3} + C$$

$$\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx = \sin^{-1} \frac{e^x+2}{3} + C$$

42 Evaluate: $\int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}} dx$

ANS: $\int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}} dx$

$$\begin{aligned}
&\frac{1}{9} - \frac{6}{9}x - x^2 \\
&= -\left(x^2 + \frac{2}{3}x - \frac{1}{9}\right) \\
&= -\left[\left(x + \frac{1}{3}\right)^2 - \frac{1}{9} - \frac{1}{9}\right] \\
&= -\left[\left(x + \frac{1}{3}\right)^2 - \frac{2}{9}\right] \\
&= \frac{2}{9} - \left(x + \frac{1}{3}\right)^2 \\
&= \left(\frac{\sqrt{2}}{3}\right)^2 - \left(x + \frac{1}{3}\right)^2
\end{aligned}$$

$$\sin x = t, \quad \cos x dx = dt$$

$$\int \frac{\cos x}{\sqrt{\sin^2 x - 2 \sin x - 3}} dx = \int \frac{dt}{\sqrt{t^2 - 2t - 3}}$$

$$I = \int \frac{dt}{\sqrt{(t-1)^2 - 1 - 3}} = \int \frac{dt}{\sqrt{(t-1)^2 - 2^2}} =$$

$$I = \log[(\sin x - 1 + \sqrt{\sin^2 x - 2 \sin x - 3}) + C$$

43 Evaluate: $\int \frac{1}{x [6(\log x)^2 + 7 \log x + 2]} dx$

43 ANS: $\int \frac{1}{x [6(\log x)^2 + 7 \log x + 2]} dx$

$$\log x = t, \frac{1}{x} dx = dt$$

$$I = \int \frac{1}{x [6(\log x)^2 + 7 \log x + 2]} dx = \int \frac{1}{[6(t)^2 + 7t + 2]} dt$$

$$\frac{1}{6} \int \frac{dt}{t^2 + \frac{7}{6}t + \frac{1}{3}} = \frac{1}{6} \int \frac{dt}{\left(t + \frac{7}{12}\right)^2 - \frac{49}{144} + \frac{1}{3}} = \frac{1}{6} \int \frac{dt}{\left(t + \frac{7}{12}\right)^2 - \left(\frac{1}{12}\right)^2}$$

$$\frac{1}{6} \cdot \frac{1}{2 \cdot \frac{1}{12}} \log \left[\frac{t + \frac{7}{12} - \frac{1}{12}}{t + \frac{7}{12} + \frac{1}{12}} \right] + C$$

$$\log \frac{2t+1}{3t+2} + C = \log \frac{2 \log x + 1}{3 \log x + 2} + C$$

$$1. \int \frac{1}{\sqrt{7-6x-x^2}} dx$$

$$2. \int \frac{1}{\sqrt{(x-1)(x-2)}} dx$$

$$3. \int \frac{1}{\sqrt{8+3x-x^2}} dx$$

$$4. \int \frac{5x-2}{1+2x+3x^2} dx$$

$$5. \int \frac{x+2}{\sqrt{x^2-1}} dx$$

44 Evaluate : $\int e^x \frac{1+\sin x}{1+\cos x} dx$

$$\text{ANS: } \int e^x \frac{1+\sin x}{1+\cos x} dx = \int e^x \left(\frac{1+2\sin(x/2) \cos(x/2)}{2 \cos^2(x/2)} \right) dx$$

$$\int e^x \left(\frac{1}{2 \cos^2(x/2)} + \tan(x/2) \right) dx$$

$$= \int e^x \left(\frac{1}{2} \sec^2(x/2) + \tan(x/2) \right) dx = e^x \tan \frac{x}{2} + C$$

45 Evaluate: $\int \frac{1}{x(x^5-2)} dx$

$$\text{ANS: } \int \frac{1}{x(x^5-2)} dx = \int \frac{x^4}{x^5(x^5-2)} dx$$

$$x^5 = t \Rightarrow x^4 dx = \frac{dt}{5}$$

$$I = \frac{1}{5} \int \frac{dt}{t(t-2)}$$

$$\frac{1}{t(t-2)} = \frac{A}{t} + \frac{B}{t-2}$$

$$1 = A(t-2) + Bt$$

$$A = -\frac{1}{2}, B = \frac{1}{2}$$

$$I = \frac{1}{10} \left[\int -\frac{1}{t} + \frac{1}{t-2} \right] dt = \frac{1}{10} [-\log t + \log(t-2)]$$

$$= \frac{1}{10} \left(\log \frac{(t-2)}{t} \right) + C = \frac{1}{10} \left(\log \frac{(x^5-2)}{x^5} \right) + C$$

46 Evaluate using properties : $\int_0^3 x^2(3-x)^{\frac{1}{2}} dx$

ANS: $I = \int_0^1 x(1-x)^{\frac{3}{2}} dx$.

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$I = \int_0^1 (1-x)(x)^{\frac{3}{2}} dx$$

$$I = \int_0^1 \left[x^{\frac{3}{2}} - x^{\frac{5}{2}} \right] dx$$

$$I = \left[\frac{2}{5} x^{\frac{5}{2}} - \frac{2}{7} x^{\frac{7}{2}} \right]_0^1 = \frac{2}{5} - \frac{2}{7} = \frac{4}{35}$$

47 Evaluate : $\int e^x \frac{2-x}{(1-x)^2} dx$

ANS: $\int e^x \frac{2-x}{(1-x)^2} dx$

$$\int e^x \frac{2-x}{(1-x)^2} dx = \int e^x \frac{(1-x)+1}{(1-x)^2} dx$$

$$I = \int e^x \left[\frac{1}{1-x} + \frac{1}{(1-x)^2} \right] dx$$

Apply $\int e^x (f(x) + f'(x)) dx = e^x f(x) + C$, $I = \frac{e^x}{1-x} + C$

48. $\int \sec^2(10-x) dx$

$$\int \sec^2(10-x) dx = -\int \sec^2 t dt = -\tan t + C = -\tan(10-x) + C$$

1. $\int \frac{2x+3}{x^2+3x+7} dx$

10. $\int 5x\sqrt{1-2x^2} dx$

2. $\int \frac{1-\sin x}{x+\cos x} dx$

11. $\int e^{\cos^2 x} \sin 2x dx$

3. $\int x^{n-1} \sin x^n dx$

12. $\int \frac{\sin(2+3\log x)}{x} dx$

4. $\int x^3 \cos x^4 dx$

13. $\int \sqrt{1 + \sin\left(\frac{x}{4}\right)} dx$

5. $\int x^9 \sec^2 x^{10} dx$

6. $\int \frac{x^5}{\sqrt{1-x^{12}}} dx$

14. $\int \frac{\cot x}{\log \sin x} dx$

7. $\int x e^{x^2+2} dx$

15. $\int \frac{(x+1)e^x}{\sin^2(xe^x)} dx$

8. $\int \frac{2x^4}{1+x^{10}} dx$

16. $\int \frac{\log(\sin e^x) e^x}{\tan(e^x)} dx$

9. $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} dx$

INTEGRATION CLASS XII- 2025

SUJITHKUMAR K P

TYPE I: Formula Based.

Evaluate:

1. $\int \left(\frac{1}{x} + e^x + 5 \right) dx$
2. $\int \frac{2+3 \sin x}{\cos^2 x} dx$
3. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$
4. $\int \cos^{-1}(\sin x) dx$
5. $\int (\tan x + \cot x)^2 dx$
6. $\int \sqrt{1 + \sin 2x} dx$
7. $\int \sec^2 x \operatorname{cosec}^2 x dx$
8. $\int \frac{1}{\cos 2x + \sin^2 x} dx$
9. $\int (e^{a \log x} + e^{x \log a}) dx$
10. $\int \frac{x-1}{\sqrt{x+4}} dx$
11. $\int \frac{1}{x^{3/2}} + 5\sqrt{x} - x^a dx$
12. $\int \tan^2 x dx$
13. $\int \frac{1}{1+\cos x} dx$
14. $\int \frac{1}{1+\sin x} dx$
15. $\int \sqrt{1 + \sin x} dx$
16. $\int \sec^2(10-x) dx$

HOME WORK-1

1. $\int \frac{1}{x^2} dx$
2. $\int \frac{1}{\sqrt{x}} dx$
3. $\int \sqrt{x} dx$
4. $\int x^{-6} dx$
5. $\int e^{3 \log x} dx$
6. $\int (x+2)^3 dx$
7. $\int \frac{a^2-x^2}{x^2} dx$
8. $\int \frac{3x^2+2x-3}{\sqrt{x}} dx$
9. $\int 2x^3 - 3 \sin x + 5\sqrt{x} dx$
10. $\int \frac{2}{1+\cos 2x} dx$
11. $\int \frac{3 \cos x + 4}{\sin^2 x} dx$
12. $\int (1-x)\sqrt{x} dx$
13. $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$
14. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$
15. $\int \left(\frac{1}{x+1} + e^{-x} - \frac{1}{x^2} \right) dx$
16. $\int \frac{x}{\sqrt{x+2}} dx$
17. $\int (1-\sqrt{x})(1+\sqrt{x}) dx$
18. $\int \frac{\sin x + \operatorname{cosec} x}{\tan x} dx$
19. $\int \frac{x}{x+1} dx$
20. $\int \frac{x^2}{x+1} dx$
21. $\int \sqrt{1-\sin x} dx$

TYPE II: $\int f(ax+b)dx$ **Method:** $ax+b=u$, $a dx = du$ $dx = \frac{1}{a} du$.

$$\text{Then } \int f(ax+b)dx = \int f(u) \frac{1}{a} du = \frac{1}{a} \int f(u) du.$$

If the numerator in integrand is exact differentiation of denominator, then its integral is logarithm of denominator. $\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$

Evaluate:

16. $\int \frac{1}{1+5x} dx$
17. $\int \frac{1}{12-9x} dx$
18. $\int e^{11-6x} dx$
19. $\int \sin(7x-13) dx$
20. $\int \cos(3-10x) dx$
21. $\int \sec 5x \tan 5x dx$
22. $\int \sin^2 x dx$
23. $\int \cos^2 x dx$
24. $\int \frac{2x}{1+x^2} dx$
25. $\int \tan^2 \left(\frac{x}{2} \right) dx$
26. $\int \sin^3 x dx$
27. $\int \sin 3x \sin x dx$
28. $\int \sin 3x \cos 2x dx$
29. $\int \cos 5x \cos 2x dx$
30. $\int \cos 4x \sin 2x dx$
31. $\int (3x+5)^{10} dx$

32. $\int \sqrt{2x+3} dx$

33. $\int \sqrt{3-2x} dx$

34. $\int \frac{1}{\sqrt{ax+b}} dx$

35. $\int \frac{1}{3x-7} dx$

HOME WORK: 2

1. $\int (2-3x)^3 dx$ 2. $\int e^{1-2x} dx$ 3. $\int 10^{6x} dx$ 4. $\int \frac{1}{5-2x} dx$ 5. $\int \frac{1}{2x-8} dx$.

6. $\int \sec^2(1-7z) dz$ 7. $\int \frac{\cos x}{1+\cos x} dx$ 8. $\int x\sqrt{x+2} dx$ 9. $\int \left(\frac{10}{5x+8} - \frac{6}{x+1} + 7\right) dx$ 10. $\int \cos^2 2x dx$

11. $\int \frac{\sin x}{1+\sin x} dx$ 12. $\int \cos^3 x dx$ 13. $\int \sqrt{1+\cos x} dx$ 14. $\int \sqrt{1-\cos x} dx$ 15. $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx$

16. $\int \cos 2x \cos 4x \cos 6x dx$ 17. $\int \sin x \sin 2x \sin 3x dx$ 18. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ 19. $\int \frac{x}{x^2+1} dx$

20. $\int \frac{x^3}{(x^4+1)} dx$ 21. $\int \frac{6x-8}{3x^2-8x+5} dx$ 22. $\int a^{4x-5} dx$ 23. $\int \frac{2x+3}{x^2+3x+7} dx$ 24. $\int \frac{1-\sin x}{x+\cos x} dx$.

25. $\int \frac{\cos x - \sin x}{\cos x + \sin x} dx$ 26. $\int \frac{1}{x+x \log x} dx$ 27. $\int \frac{5}{\cos^2(4x-3)} dx$ 28. $\int \frac{1+\cos x}{1-\cos x} dx$ 29. $\int \frac{dx}{1+\cos x}$ 30. $\int \frac{(\log x)^2}{x} dx$

TYPE III: $\int f(x^n) x^{n-1} dx$ **Method:** $x^n = u$ then $nx^{n-1} dx = du$, $\int f(x^n) x^{n-1} dx = \int f(u) \frac{1}{n} du = \frac{1}{n} \int f(u) du$.

36. $\int x^5 \cos^2 x^6 dx$

37. $\int x^2 \sec(x^3) \tan(x^3) dx$

38. $\int 4x^5 e^{x^6} dx$

39. $\int x^{n-1} \sec^2 x^n dx$

40. $\int \frac{2x^4}{1+x^{10}} dx$

41. $\int \frac{x^5}{\sqrt{1-x^{12}}} dx$

42. $\int x e^{x^2+2} dx$

43. $\int \frac{x^3}{\sqrt{1-x^8}} dx$

44. $\int \frac{6x^5}{1+x^6} dx$

45. $\int \frac{\tan x}{\log \sec x} dx$

46. $\int x \sin x^2 dx$

47. $\int x^3 \cos x^4 dx$

48. $\int x^{12} \sec^2 x^{13} dx$

TYPE IV: $\int (f(x))^n f'(x) dx$ **Method:** $f(x) = u$ then $f'(x) dx = du$

$$\int (f(x))^n f'(x) dx = \int (u)^n du = \frac{(f(x))^{n+1}}{n+1} + C$$

49. $\int x^2 \sqrt{x^3+1} dx$

50. $\int \sin^4 x \cos x dx$

51. $\int \sec^9 x \tan x dx$

52. $\int \frac{1+\cos x}{(x+\sin x)^2} dx$

53. $\int \frac{\cos x}{\sqrt{\sin x}} dx$

54. $\int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx$

55. $\int \frac{(1+\log x)^4}{x} dx$

56. $\int \frac{3x+2}{(3x^2+4x+7)^3} dx$

57. $\int \frac{x}{x^2-8} dx$

58. $\int \frac{e^x}{a+be^x} dx$

59. $\int \frac{1+\tan x}{x+\log \sec x} dx$

60. $\int \frac{x^3}{(1+x^2)^5} dx$.

61. $\int \frac{x^2}{(8+x^3)^4} dx$

62. $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} dx$

63. $\int \frac{x}{\sqrt{x^2+a^2}} dx$

64. $\int 5x\sqrt{1-2x^2} dx$

65. $\int x^2 \cos(x^3) dx$

66. $\int \cos^5 x \sin x dx$

67. $\int \frac{\cos x}{\sqrt{\sin x}} dx$

68. $\int \sin^2 x \cos^3 x dx$

69. $\int \tan x \, dx$

70. $\int \cot x \, dx$

71. $\int \sec x \, dx$

72. $\int \operatorname{cosec} x \, dx$

TYPE V: $\int \phi\{f(x)\} f^1(x) dx$ **Method:** $f(x) = u$ $f^1(x) dx = du$

73. $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

74. $\int e^{\sin x} \cos x \, dx$

75. $\int e^{\tan x} \sec^2 x \, dx$

76. $\int e^{\cos^2 x} \sin 2x \, dx$

77. $\int \frac{\cos(\log x)}{x} dx$

78. $\int e^{\sin^2 x} \sin x \cos x \, dx$

79. $\int \frac{\sin(2+3 \log x)}{x} dx$

80. $\int \frac{\cos x}{1+\sin^2 x} dx$

81. $\int \frac{1}{x+x(\log x)^2} dx$

82. $\int \frac{1-\tan x}{1+\tan x} dx$

83. $\int \cos^7 x \, dx$

84. $\int \sin^5 x \, dx$

HOME WORK:3 1. $\int \frac{x^4}{1+x} dx$ 2. $\int \frac{2x^2+x}{x-1} dx$ 3. $\int \frac{x^3+3x^2+2x+1}{x-1} dx$ 4. $\int \frac{x^4+1}{x^2+1} dx$ 5. $\int \frac{x+2}{(x+1)^2} dx$

6. $\int \sin(a + \frac{x}{b}) dx$ 7. $\int (\tan^2 x - 3x^2) dx$ 8. $\int (\sec^2 x + \cos x) dx$ 9. $\int (\sqrt{x} - \cos^2 \frac{x}{2}) dx$ 10. $\int \frac{x+2}{\sqrt{x+5}} dx$

11. $\int \frac{\tan x}{\sqrt{1+\cos 2x}} dx$ 12. $\int \frac{1-\sin x}{x+\cos x} dx$ 13. $\int \frac{1+\tan x}{x+\log \sec x} dx$ 14. $\int \frac{(\tan^{-1} x)^2}{1+x^2} dx$ 15. $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$ 16. $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

17. $\int \frac{\sec^2(2 \tan^{-1} x)}{1+x^2} dx$ 18. $\int \tan^3 x \, dx$ 19. $\int \frac{\sin x}{\sin(x-a)} dx$ 20. $\int \frac{1}{x-\sqrt{x}} dx$ 21. $\int \frac{\sin^6 x}{\cos^8 x} dx$ 22. $\int \frac{x}{9-4x^2} dx$

23. $\int \frac{1}{x+\sqrt{x}} dx$ 24. $\int \frac{(x+1)e^x}{\sin^2(x e^x)} dx$ 25. $\int \frac{\cot x}{\log \sin x} dx$ 26. $\int \frac{1}{1+\cot x} dx$ 27. $\int \frac{1}{1-\tan x} dx$

28. $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}} dx$

TYPE VI: Formula Based: $\int \frac{1}{x^2-a^2} dx$, $\int \frac{1}{a^2-x^2} dx$, $\int \frac{1}{x^2+a^2} dx$, $\int \frac{1}{\sqrt{x^2-a^2}} dx$, $\int \frac{1}{\sqrt{a^2-x^2}} dx$, $\int \frac{1}{\sqrt{x^2+a^2}} dx$

85. $\int \frac{1}{9x^2-1} dx$

86. $\int \frac{1}{1-9x^2} dx$

87. $\int \frac{1}{\sqrt{16-9x^2}} dx$

88. $\int \frac{1}{9x^2+1} dx$

89. $\int \frac{1}{x^2-121} dx$

90. $\int \frac{1}{x^2+1} dx$

91. $\int \frac{1}{4+x^2} dx$

92. $\int \frac{1}{\sqrt{(2-x)^2+1}} dx$

93. $\int \frac{1}{(x+2)^2+1} dx$

94. $\int \frac{1}{\sqrt{9-25x^2}} dx$

95. $\int \frac{x^2}{\sqrt{x^6+4}} dx$

96. $\int \frac{x-1}{\sqrt{x^2-1}} dx$

97. $\int \frac{\cos x}{\sqrt{4-\sin^2 x}} dx$

98. $\int \frac{\sec^2 x}{\sqrt{16+\tan^2 x}} dx$

99. $\int \frac{\sin x}{16+\cos^2 x} dx$

100. $\int \frac{3x}{1+2x^4} dx$

HOME WORK:4

1. $\int \frac{1}{\sqrt{a^2-b^2x^2}} dx$ 2. $\int \frac{1}{a^2-b^2x^2} dx$ 3. $\int \frac{1}{50+2x^2} dx$ 4. $\int \frac{1}{32-2x^2} dx$ 5. $\int \frac{1}{\sqrt{64-25x^2}} dx$

TYPE VII: When the given fraction is of the form. $\int \frac{1}{ax^2+bx+c} dx$ Or $\int \frac{1}{\sqrt{ax^2+bx+c}} dx$

101. $\int \frac{1}{7+2x+x^2} dx$

102. $\int \frac{1}{4x^2-4x+3} dx$

103. $\int \frac{1}{x^2+4x+7} dx$

104. $\int \frac{1}{9x^2+6x+1} dx$

$$\begin{array}{llll}
 105. \int \frac{1}{1+x-x^2} dx & 106. \int \frac{1}{\sqrt{4x^2+4x+5}} dx & 107. \int \frac{1}{\sqrt{8+3x-x^2}} dx & 108. \int \frac{1}{\sqrt{2x-x^2}} dx \\
 109. \int \frac{1}{3x^2-2x+4} dx & 110. \int \frac{1}{7-4x-2x^2} dx & 111. \int \frac{1}{x^2-6x-16} dx & 112. \int \frac{1}{\sqrt{5-4x-2x^2}} dx \\
 113. \int \frac{1}{\sqrt{1+3x-x^2}} dx & 114. \int \frac{1}{\sqrt{x(1-2x)}} dx & &
 \end{array}$$

TYPE VIII: $\int \frac{px+q}{ax^2+bx+c} dx$ and $\int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$

Method: Express $px + q = A(\text{derivative of } ax^2 + bx + c) + B$

ie $A(2ax + b) + B$ then integrals reduces to known forms.

$$px + q = \frac{p}{2a}(2ax + b) + \left(q - \frac{pb}{2a}\right)$$

$$\begin{array}{llll}
 115. \int \frac{3x+1}{2x^2-2x+3} dx & 116. \int \frac{2x+1}{x^2+2x+3} dx & 117. \int \frac{5x+1}{x^2-2x-35} dx & 118. \int \frac{2x+3}{x^2+2x+5} dx \\
 119. \int \frac{5x+7}{6x^2+4x-1} dx & 120. \int \frac{2x-3}{3x^2+4x+5} dx & 121. \int \frac{4x}{(x-2)(x-1)} dx & 122. \int \frac{3x+2}{\sqrt{x^2+x+1}} dx \\
 123. \int \frac{2x+5}{\sqrt{x^2-2x+10}} dx & 124. \int \frac{x+2}{\sqrt{x^2+5x}} dx & 125. \int \frac{x+1}{\sqrt{2x^2+x-3}} dx & 126. \int \sqrt{\frac{x-1}{x-2}} dx \\
 127. \int \sqrt{\frac{1-x}{1+x}} dx & 128. \int \frac{x+2}{\sqrt{(x-2)(x-3)}} dx & 129. \int \frac{5x+3}{\sqrt{x^2+4x+10}} dx & 130. \int \frac{x}{\sqrt{x^2+x+1}} dx
 \end{array}$$

HOME WORK- 5

$$\begin{array}{llll}
 1. \int \frac{3x}{1+2x^4} dx & 2. \int \frac{x^2}{1-x^6} dx & 3. \int \frac{x-1}{\sqrt{x^2+1}} dx & 4. \int \frac{x-1}{\sqrt{x^2-1}} dx \\
 5. \int \frac{1}{x^2+2x+7} dx & 6. \int \frac{1}{4x^2-4x+3} dx & & \\
 7. \int \frac{1}{9x^2+6x+1} dx & 8. \int \frac{1}{x^2+4x+7} dx & 9. \int \frac{1}{1-x-3x^2} dx & 10. \int \frac{1}{\sqrt{8+3x-x^2}} dx \\
 11. \int \frac{6x+5}{\sqrt{6+x-2x^2}} dx & 12. \int \frac{4x+1}{\sqrt{2x^2+x-3}} dx & 13. \int \frac{x+2}{\sqrt{x^2-1}} dx & 14. \int \frac{\cos x}{\sqrt{4-\sin^2 x}} dx \\
 15. \int \frac{\sec^2 x}{\sqrt{\tan^2 x+16}} dx & & &
 \end{array}$$

TYPE IX. Partial Fraction.

$$\begin{array}{llll}
 131. \int \frac{3x+2}{(x-1)(2x+3)} dx & 132. \int \frac{2x+3}{(x-1)^2(x+2)} dx & 133. \int \frac{x^2}{(x+2)^2(x-1)} dx & 134. \int \frac{1}{(x^2+1)(x-1)} dx \\
 135. \int \frac{1}{x^3+1} dx & 136. \int \frac{x}{x^3-1} dx & 137. \int \frac{2x+3}{2x^2+5x+3} dx & 138. \int \frac{5}{(1+2x)(1-3x)} dx \\
 139. \int \frac{x^2}{x^2+7x+10} dx & 140. \int \frac{1}{x^3-3x^2+2x} dx & 141. \int \frac{x+1}{(x-1)^2(x+2)} dx & 142. \int \frac{x^2}{(x+1)^2(x-2)} dx
 \end{array}$$

$$\begin{array}{llll}
143. \int \frac{1}{x(x^5+1)} dx & 144. \int \frac{1}{x(x^4+1)} dx & 145. \int \frac{1}{x(x^n+1)} dx & 146. \int \frac{1}{x(x^5-2)} dx \\
147. \int \frac{x^2}{(x-1)(x-2)(x-3)} dx & 148. \int \frac{x^3}{(x-2)(x-1)} dx & 149. \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx & 150. \int \frac{1}{x[6(\log x)^2+7\log x+2]} dx \\
151. \int \frac{x^2-3x}{(x-2)(x-1)} dx & 152. \int \frac{\cos \theta}{(2+\sin \theta)(3+4\sin \theta)} d\theta & 153. \int \frac{x^2+5x+3}{x^2+3x+2} dx & 154. \int \frac{1}{\sin x+\sin 2x} dx \\
155. \int \frac{x^2}{(x^2+1)(x^2+4)} dx & 156. \int \frac{2x}{(x^2+1)(x^2+3)} dx & 157. \int \frac{\sin x}{\sin 4x} dx & 158. \int \frac{e^x}{1+3e^x+2e^{2x}} dx \\
159. \int \frac{x^3+x+1}{x^2-1} dx & 160. \int \frac{1-\cos x}{\cos x(1+\cos x)} dx & 161. \int \frac{1}{x(x^5+3)} dx & 162. \int \frac{3x-1}{(x-2)^2} dx \\
163. \int \frac{(3\sin \varphi-2)\cos \varphi}{5-\cos^2 \varphi-4\sin \varphi} d\varphi & 164. \int \frac{\tan \theta}{1-\sin \theta} d\theta & 165. \int \frac{\sec x}{(1+\operatorname{cosec} x)} dx & 166. \int \frac{1}{x+x^2+x^3} dx \\
167. \int \frac{5x}{(x+1)(x^2+9)} dx & 168. \int \frac{\sec^2 \theta}{\tan^3 \theta+4\tan \theta} d\theta & 169. \int \frac{1}{1+x+x^2+x^3} dx & 170. \int \frac{2\sin 2\theta-\cos \theta}{6-\cos^2 \theta-4\sin \theta} d\theta \\
171. \int \frac{\sin x}{(1-\cos x)(2-\cos x)} dx & & &
\end{array}$$

TYPE X INTEGRATION BY PARTS

$$\int uv dx = u \int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx$$

$$\begin{array}{llll}
172. \int x e^{4x} dx & 173. \int x e^{-2x} dx & 174. \int x \sec x \tan x dx & 175. \int x \cos 2x dx \\
176. \int x^2 \sin x dx & 177. \int x^2 \cos 3x dx & 178. \int x^2 e^{3x} dx & 179. \int x^2 \log x dx \\
180. \int x^2 \log(1+x) dx & 181. \int x \log 2x dx & 182. \int x^2 \log 3x dx & 183. \int (\log x)^2 dx \\
184. \int e^x \cos x dx & 185. \int e^x \sin x dx & 186. \int e^{3x} \cos 2x dx & 187. \int e^{2x} \sin 3x dx \\
188. \log x dx & 189. \int \cos^{-1} x dx & 190. \int \sin^{-1} x dx & 191. \int \tan^{-1} x dx \\
192. \int \frac{x}{1+\sin x} dx & 193. \int \operatorname{cosec}^3 x dx & 194. \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx & 195. \int x^2 \tan^{-1} x dx \\
196. \int (x \operatorname{cosec}^{-1} x) dx & 197. \int \tan^{-1} \sqrt{x} dx. & 198. \int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx &
\end{array}$$

HOME WORK: 6 1. $\int \log 2x dx$ 2. $\int x^2 \tan^{-1} \frac{x}{2} dx$ 3. $\int \sin^{-1} \frac{2x}{1+x^2} dx$ 4. $\int \tan^{-1} \frac{2x}{1-x^2} dx$

$$\begin{array}{llll}
5. \int \frac{\log x}{(x+1)^3} dx & 5. \int \cos^{-1} \frac{1-x^2}{1+x^2} dx & 6. \int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx & 7. \int (\sin^{-1} x)^2 dx \quad 8. \int \sec^3 \theta d\theta. \\
9. \int e^x \sin^2 x dx & 10. \int \operatorname{cosec}^3 x dx. & 11. \int \sec^2 \sqrt{x} dx. & 12. \int \frac{1}{1-\tan x} dx \quad 13. \int \frac{1}{1+\cot x} dx
\end{array}$$

$$14. \int \frac{1}{6x-5x^2+x^3} dx \quad 15. \int \frac{x}{(x^3-1)} dx \quad 16. \int \sin \sqrt{x} dx$$

TYPE – XI- Integral of the form: $\int \sqrt{x^2 + a^2} dx$, $\int \sqrt{x^2 - a^2} dx$, $\int \sqrt{a^2 - x^2} dx$

$$199. \int \sqrt{x^2 + 4x + 6} dx$$

$$200. \int \sqrt{x^2 - 4x + 2} dx$$

$$201. \int \sqrt{2x^2 + 3x + 4} dx$$

$$202. \int \sqrt{x^2 + 8x + 4} dx$$

$$203. \int \sqrt{8 + 2x - x^2} dx$$

$$204. \int \sqrt{3 + 8x - 3x^2} dx$$

$$205. \int \sqrt{3 - 2x - 2x^2} dx$$

$$206. \int \sqrt{1 - 4x - x^2} dx$$

$$207. \int \sqrt{x^2 + 3x} dx$$

$$208. \int \sqrt{4 - 3x - 2x^2} dx$$

$$209. \int \sqrt{3 - 2x - 2x^2} dx$$

$$210. \int \sqrt{3 - 2x - 2x^2} dx$$

TYPE XII: Integral of the form: $\int e^x (f(x) + f'(x))dx = e^x f(x) + C$

$$211. \int e^x (\cot x - \operatorname{cosec}^2 x) dx$$

$$212. \int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$$

$$213. \int e^x \sec x (1 + \tan x) dx$$

$$214. \int e^x \frac{1 + \sin x}{1 + \cos x} dx$$

$$215. \int e^x \frac{2 + \sin 2x}{1 + \cos 2x} dx$$

$$216. \int e^x \frac{x^2}{(x+2)^2} dx$$

$$217. \int e^{x/2} \frac{2 + \sin x}{1 + \cos x} dx$$

$$218. \int e^x \frac{x^2 + 1}{(x+1)^2} dx$$

$$219. \int e^x (\sec x + \log(\sec x + \tan x)) dx$$

$$220. \int e^x (\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}) dx$$

$$221. \int e^{2x} \left(\frac{2x-1}{4x^2} \right) dx$$

$$222. \int e^{2x} (-\sin x + 2 \cos x) dx.$$

HOME WORK -7

$$1. \int e^x (\sin x + \cos x) dx$$

$$2. \int e^x (\cot x + \log \sin x) dx$$

$$3. \int e^x (\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}) dx$$

$$4. \int e^x \frac{x+1}{(x+2)^2} dx$$

$$5. \int e^x \frac{2-x}{(1-x)^2} dx$$

$$6. \int e^x \frac{2 + \sin 2x}{1 + \sin 2x} dx$$

$$7. \int \frac{\log x - 1}{(\log x)^2} dx$$

$$8. \int e^x \frac{1 + \sin x \cos x}{\cos^2 x} dx$$

TYPE –XIII Integral of the form: $\int \frac{1}{x^4+1} dx$

$$223. \int \frac{x^2+1}{x^4+1} dx$$

$$224. \int \frac{x^2+4}{x^4+16} dx$$

$$225. \int \frac{x^2+1}{x^4+7x^2+1} dx$$

$$226. \int \frac{1}{x^4+1} dx$$

$$227. \int \sqrt{\tan x} dx$$

$$228. \int \sqrt{\cot x} dx$$

$$229. \int \frac{x^2}{x^4+x^2+1} dx$$

$$230. \int \frac{x^2}{x^4+1} dx$$

Miscellaneous Exercise:

$$231. \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$$

$$234. \int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)} dx$$

$$237. \int e^{2x} \frac{1 - \sin 2x}{1 - \cos 2x} dx$$

$$240. \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$$

$$243. \int \frac{1}{\sin^4 x + \cos^4 x} dx$$

$$246. \int \frac{\sqrt{x^2+1} (\log(x^2+1) - 2\log x)}{x^4} dx$$

$$249. \int \frac{x}{1 + \cos x} dx$$

$$251. \int \tan^{-1} \left(\frac{2x}{1-x^2} \right) dx$$

$$254. \int \frac{1}{x^2+4} dx$$

$$257. \int \frac{x^2}{x^2-4} dx$$

$$260. \int \frac{x}{\sqrt{x^2-4}} dx$$

$$263. \int \frac{x^2}{\sqrt{x^2-4}} dx$$

$$266. \int \frac{1}{4-x^2} dx$$

$$269. \int \frac{x}{\sqrt{4-x^2}} dx$$

$$272. \int \frac{1}{x^2-1} dx$$

$$275. \int \frac{x^2}{x^4-1} dx$$

$$278. \int \frac{\tan \theta}{1 - \sin \theta} d\theta$$

$$281. \int \tan^3 x \sec^3 x dx$$

$$284. \int \frac{1}{1 + \tan x} dx$$

$$287. \int \frac{1}{2 \sin 2x + 1} dx$$

$$232. \int \frac{e^{\tan^{-1} x}}{(1+x^2)^2} dx$$

$$235. \int \frac{1 + \cos 4x}{\cot x - \tan x} dx$$

$$238. \int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx$$

$$241. \int \frac{\sin x + \cos x}{\sqrt{9+16\sin 2x}} dx$$

$$244. \int \frac{\sin 2x}{a \cos^2 x + b \sin^2 x} dx$$

$$247. \int \frac{1}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x} dx$$

$$250. \int \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$$

$$252. \int \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) dx$$

$$255. \int \frac{x}{x^2-4} dx$$

$$258. \int \frac{1}{\sqrt{x^2-4}} dx$$

$$261. \int \frac{x-1}{\sqrt{x^2-4}} dx$$

$$264. \int \frac{x^3}{x^2-4} dx$$

$$267. \int \frac{x}{4-x^2} dx$$

$$270. \int \frac{x^2}{\sqrt{4-x^2}} dx$$

$$273. \int \frac{1}{x^4-1} dx$$

$$276. \int \frac{1}{\cos(x-a)\cos(x-b)} dx$$

$$279. \int \frac{\sin(x+a)}{\sin(x-a)} dx$$

$$282. \int \tan^4 x dx$$

$$285. \int \frac{1}{\cos x + \cos \alpha} dx .$$

$$289. \int \frac{\cos x}{\cos 3x} dx$$

$$233. \int \sin x \sin 2x \sin 3x dx$$

$$236. \int \frac{\cos x}{\cos(x-a)} dx$$

$$239. \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$$

$$242. \int (x-3)\sqrt{x^2+3x-18} dx$$

$$245. \int \frac{1}{x-4} dx$$

$$248. \int \tan^{-1} \left(\frac{x-5}{1+5x} \right) dx$$

$$253. \int \frac{1}{x^2-4} dx$$

$$256. \int \frac{x}{x^2+4} dx$$

$$259. \int \frac{1}{\sqrt{x^2+4}} dx$$

$$262. \int \frac{x-4}{\sqrt{x^2-4}} dx$$

$$265. \int \frac{1}{x(x^2-4)} dx$$

$$268. \int \frac{x^2}{4-x^2} dx$$

$$271. \int \frac{x^3}{\sqrt{4-x^2}} dx$$

$$274. \int \frac{x}{x^4-1} dx$$

$$277. \int \frac{1}{\cos(x+a)\cos(x+b)} dx$$

$$280. \int \frac{1}{\sin(x-a)\sin(x-b)} dx$$

$$283. \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$

$$286. \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$$

$$290. \int \frac{\cos x - \sin x}{3+2 \sin 2x} dx$$

$$291. \int \frac{1}{\sin^4 x + \cos^4 x} dx$$

$$292. \int \frac{1}{\cos^4 x + \sin^4 x} dx$$

$$293. \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$$

$$294. \int \frac{\cos x - \sin x}{\sqrt{1+\sin 2x}} dx$$

$$295. \int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$296. \int \frac{x^3}{x^4 + 3x^2 + 2} dx$$

$$297. \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$$

$$298. \int \frac{\sqrt{1+x^2}}{x^4} dx$$

$$299. \int \frac{1-x^2}{x-2x^2} dx$$

$$300. \int \frac{e^{\tan^{-1} x}}{(1+x^2)^2} dx$$

$$301. \int \frac{x^4}{(x-1)(x^2+1)} dx$$

$$302. \int e^{2x} \frac{\sin 4x - 2}{1 - \cos 4x} dx$$

$$303. \int e^{2x} \sin x \cos x dx$$

$$304. \int \frac{x^2+1}{x^4+1} dx$$

$$305. \int \frac{x+2}{(x+1)^2} dx$$

$$306. \int (\sqrt{\tan x} + \sqrt{\cot x}) dx$$

$$307. \int \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right) dx$$

$$308. \int x \sin^{-1} x dx$$

$$309. \int \log(2+x^2) dx$$

$$310. \int \frac{\log x}{(1+x)^2} dx$$

$$311. \int \frac{x^2}{(x^2+4)(x^2+9)} dx$$

$$312. \int \frac{1}{x[6(\log x)^2 + 7 \log x + 2]} dx$$

$$313. \int \frac{e^x}{(e^x+1)(e^x+2)} dx$$

$$314. \int \frac{x^2+x+1}{(x^2-x+1)} dx$$

$$315. \int \frac{1}{4\sin^2 x + 4\sin x \cos x + 5\cos^2 x} dx$$

$$316. \int \frac{1}{\sin^2 x + \tan^2 x} dx$$

$$317. \int \frac{\sin x}{\sin 3x} dx$$

$$318. \int \frac{\cos x}{\cos(x+a)} dx$$

$$319. \int \frac{\sin x}{\sin(x-a)} dx$$

$$320. \int \frac{\sin(x-a)}{\sin(x+a)} dx$$

$$321. \int x^2 \tan^{-1} x dx$$

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