

DEFINITE INTEGRALS – CLASS XII

SUJITHKUMAR KP

1. $\int_a^b f(x)dx = \int_a^b f(t)dt$	2. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx,$ $a < b < c$	4. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
5. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$	6. $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
7. $\int_0^{2a} f(x)dx = 2 \int_0^a f(x)dx, f(2a-x) = f(x)$	8. $\int_0^{2a} f(x)dx = 0, f(2a-x) = -f(x)$
9. $\int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx, \text{ if } f(-x) = f(x)$	10. $\int_{-a}^a f(x)dx = 0, \text{ if } f(-x) = -f(x)$

- 1 The value of $\int_0^{\frac{\pi^2}{4}} \frac{\cos \sqrt{x}}{\sqrt{x}} dx = \underline{\hspace{2cm}}$
(A) -2 (B) 1 (C) 2 (D) π
- 2 Evaluate : $\int_1^3 \frac{\cos(\log x)}{x} dx$.
(A) $\cos(\log 3)$ (B) $\sin(\log 3)$ (C) $\cos 3$ (D) $-\sin(\log 3)$
- 3 Evaluate : $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x} = \underline{\hspace{2cm}}$
(A) -2 (B) 1 (C) 2 (D) 0
- 4 Let $l_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}$ and $l_2 = \int_1^2 \frac{dx}{x}$, then
(A) $l_1 = l_2$ (B) $l_1 > l_2$ (C) $l_1 < l_2$ (D) $l_1 > 2l_2$
- 5 Evaluate : $\int_8^{13} \frac{\sqrt{21-x}}{\sqrt{x} + \sqrt{21-x}} dx = \underline{\hspace{2cm}}$
(A) -5 (B) 5 (C) $\frac{5}{2}$ (D) 10
- 6 The value of $\int_0^2 x[x]dx = \underline{\hspace{2cm}}$
(A) 3 (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{3}{4}$
- 7 Evaluate the following integral $\int_1^e \log x = \underline{\hspace{2cm}}$
(A) 1 (B) $\frac{1}{2}$ (C) $\frac{5}{2}$ (D) -1
- 8 Evaluate : $\int_0^1 x(1-x)^{89} dx$
(A) $\frac{1}{8190}$ (B) $\frac{1}{8290}$ (C) $\frac{1}{8100}$ (D) 8190
- 9 Evaluate: $\int_1^2 e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$
- 10 Assertions (A) : if $\int_0^a f(x)dx = 5$ then $\int_0^a f(a-x)dx = 5$
Reason (R) : $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
- 11 Evaluate: $\int_1^e \frac{1}{x\sqrt{1-(\log x)^2}} dx$
(A) 0 (B) π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

- 12 Evaluate : $\int_0^{\frac{\pi}{4}} (\tan^2 x + \tan^4 x) dx$
- 13 Evaluate : $\int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1 + \cos 2x} dx$
- 14 Show that $\int_0^{\frac{\pi}{4}} \log(1 + \tan \theta) d\theta = \frac{\pi}{8} \log 2$
- 15 Using properties of integrals, evaluate : $\int_0^{\pi} \frac{4x \sin x}{1 + \cos^2 x} dx$
- 16 Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x} dx$
- 17 Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx$
- 18 Using properties of integrals, evaluate : $\int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1} + \sqrt[3]{11-x}} dx$
- 19 Evaluate : $\int_2^5 \{ |x - 2| + |x - 3| + |x - 5| \} dx$
- 20 (i) Evaluate : $\int_0^4 \{ |x| + |x - 2| + |x - 4| \} dx$ ANS: 20
(ii) Evaluate : $\int_1^3 \{ |x - 1| + |x - 2| + |x - 3| \} dx$ ANS: 5
- 21 Evaluate: $\int_1^5 \frac{\sqrt{x}}{\sqrt{6-x} + \sqrt{x}} dx$
- 22 Evaluate : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\tan x}} dx$
- 23 Evaluate: $\int_0^e \frac{dx}{x\sqrt{1 - (\log x)^2}}$
- 24 Evaluate: $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-x^2}} dx$
25. Evaluate : $\int_0^{\pi} |\cos x|$

Type 1: Evaluate the following definite integrals.

1. $\int_0^{\frac{\pi}{4}} \sqrt{1 + \sin 2x} dx$
2. $\int_0^{\frac{\pi}{2}} \cos 3x \cos 2x dx$
3. $\int_0^{\frac{\pi}{2}} \cos^2 x dx$
4. $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$
5. $\int_2^3 \frac{1}{x^2+1} dx$
6. $\int_0^1 \frac{2x+3}{5x^2+1} dx$
7. $\int_0^{\frac{\pi}{4}} \sec x \sqrt{\frac{1-\sin x}{1+\sin x}} dx$
8. $\int_1^2 \frac{5x^2}{x^2+4x+3} dx$
9. $\int_0^2 \frac{5x+1}{x^2+4} dx$
10. $\int_{\frac{\pi}{2}}^{\pi} \frac{1-\sin x}{1-\cos x} dx$
11. $\int_0^1 x e^x dx$
12. $\int_1^3 \frac{1}{x^2(x+1)} dx$
13. $\int_0^{\frac{\pi}{4}} 2 \tan^3 x dx$
14. $\int_0^1 \sin^{-1} x dx$
15. $\int_0^1 \frac{1}{2x-3} dx$
16. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos \theta \operatorname{cosec}^2 \theta d\theta$
17. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 \theta}{(1 + \cos \theta)^2} d\theta$
18. $\int_0^{\frac{\pi}{2}} \cos^2 \frac{\theta}{2} d\theta$
19. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{1 - \sin 2x} dx$
20. $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-x^2}} dx$
21. $\int_0^1 \frac{1}{\sqrt{2x+3}} dx$
22. $\int_0^1 \frac{2x}{1+x^2} dx$
23. $\int_0^1 \frac{e^x}{1+e^{2x}} dx$

ANSWERS

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|---|---|-----------------------------------|---|
| 1. 1 | 2. $\frac{3}{5}$ | 3. $\frac{\pi}{4}$ | 4. $\frac{\pi}{2}$ |
| 5. $\tan^{-1} 3 - \tan^{-1} 2$ | 6. $\frac{1}{5} \log 6 + \frac{3}{\sqrt{5}} \tan^{-1} \sqrt{5}$ | 7. $2 - \sqrt{2}$ | 8. $5 + 10 \log \frac{8}{15} + \frac{25}{2} \log \frac{6}{5}$ |
| 9. $\frac{5}{2} \log 2 + \frac{\pi}{8}$ | 10. $1 - \log 2$ | 11. 1 | 12. $\frac{2}{3} + \log \frac{2}{3}$ |
| 13. $1 - \log 2$ | 14. $\frac{\pi}{2} - 1$ | 15. $-\frac{1}{2} \log 3$ | 16. $\sqrt{2} - 1$ |
| 17. $-\frac{\pi}{2} + 2$ | 18. $\frac{\pi+2}{4}$ | 19. $\sqrt{2} - 1$ | 20. $\frac{\pi}{4}$ |
| 21. $\sqrt{5} - \sqrt{3}$ | 22. $\log 2$ | 23. $\tan^{-1} e - \frac{\pi}{4}$ | |

HOME WORK

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|---|---|---|
| 1. $\int_{\frac{\pi}{2}}^{\pi} \frac{1+\cos x}{1-\cos x} dx = -\frac{\pi}{2} + 2$ | 2. $\int_0^{\frac{\pi}{2}} \sin x \sin 2x dx = \frac{2}{3}$ | 3. $\int_0^{\frac{\pi}{4}} \sec x dx = \log(\sqrt{2} + 1)$ |
| 4. $\int_3^5 \frac{x^2}{x^2-4} dx = 2 + \log \frac{15}{7}$ | 5. $\int_1^2 \frac{1}{(x+1)(x+2)} dx = \log \frac{9}{8}$ | 6. $\int_1^3 \frac{1}{7-2x} dx = \frac{1}{2} \log 5$ |
| 7. $\int_0^{\frac{\pi}{4}} \frac{1}{1+\cos 2x} dx = \frac{1}{2}$ | 8. $\int_0^a 3x^2 dx = 8$, find a | 9. $\int_0^{\frac{\pi}{2}} \sqrt{1-\cos 2x} dx = \sqrt{2}$ |
| 10*. $\int_0^a \frac{dx}{4+x^2} = \frac{\pi}{8}$ find a | 11*. $\int_0^{\frac{\pi}{2}} e^x (\sin x - \cos x) dx$ | 12. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx = \frac{1}{2} \log 2$ |
| 13. $\int_3^4 \frac{1}{x^2-4} dx = \frac{1}{4} \log \frac{5}{3}$ | 14. $\int_0^{\pi} \frac{1}{1+\sin x} dx = 2$ | 15. $\int_0^{\frac{\pi}{2}} \cos^2 x dx = \frac{\pi}{4}$ |

Type 2: By substitution

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|--|---|---|---|
| 19. $\int_1^3 \frac{\cos(\log x)}{x} dx$ | 20. $\int_0^{\frac{\pi}{4}} \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ | 21. $\int_1^e \frac{\sin(\pi \log x)}{x} dx$ | 22. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x} dx$ |
| 23. $\int_0^1 \frac{e^{2x}}{1+e^{4x}} dx$ | 24. $\int_0^1 \frac{x}{1+\sqrt{x}} dx$ | 25. $\int_0^1 \frac{x^5}{1+x^6} dx$ | 26. $\int_0^1 \sin^{-1} \frac{2x}{1+x^2} dx$ |
| 27. $\int_0^1 (\cos^{-1} x)^2 dx$ | 28. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log \sin x dx$ | 29. $\int_0^{\frac{\pi}{4}} \frac{1}{4+5\cos^2 x} dx$ | 30. $\int_0^{\frac{\pi}{2}} \frac{1}{4\sin^2 x + 5\cos^2 x} dx$ |
| 31. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1+\cos x}} dx$ | 32. $\int_0^{\frac{\pi}{2}} 2 \tan^3 x dx$ | | |

ANSWERS

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|-------------------|-------|---------------------|---------------------|
| 19. $\sin \log 3$ | 20. 2 | 21. $\frac{2}{\pi}$ | 22. $\frac{\pi}{4}$ |
|-------------------|-------|---------------------|---------------------|

23. $\frac{1}{2} (\tan^{-1} e^2 - \frac{\pi}{4})$

24. $2 (\frac{5}{6} - \log 2)$

25. $\frac{1}{6} \log 2$

26. $\frac{\pi}{2} - \log 2$

27. $\pi - 2$

28. $\frac{1}{4} \log 2 - \frac{\pi}{8} + \frac{1}{4}$

29. $\frac{1}{6} \tan^{-1} \frac{2}{3}$

30. $\frac{\pi}{4\sqrt{5}}$

31. $2(\sqrt{2} - 1)$

32. $1 - \log 2$

HOME WORK

1. $\int_0^{\frac{\pi}{2}} \sqrt{\cos \theta} \sin^3 \theta d\theta = \frac{8}{21}$

2. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}} dx = \frac{3}{2}$

3. $\int_0^{\frac{\pi}{2}} \frac{1}{4+9\cos^2 x} dx = \frac{\pi}{4\sqrt{13}}$

4. $\int_0^{\frac{\pi}{2}} \frac{1}{5+4\sin x} dx = \frac{2}{3} \tan^{-1} \frac{1}{3}$

5. $\int_0^{\frac{\pi}{4}} 2 \tan^3 x dx = 1 - \log 2$

6. $\int_1^2 \frac{e^x(1+x \log x)}{x} dx = e^2 \log 2$

7. $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx = \frac{\pi^2}{32}$

8. $\int_1^2 \frac{\cos(\log x)}{x} dx = \sin \log 2$

9. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{4-\sin^2 x} dx = \frac{\log 3}{4}$

10*. $\int_e^{e^2} \frac{dx}{x \log x}$

11*. $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1+3\sin^2 x} dx$ Hint: $\int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{(1+\tan^2 x)(1+4\tan^2 x)} dx$

12*. $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x} dx$ Hint: $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{(\sin x - \cos x)^2 - 4} dx$

13. $\int_0^{\frac{\pi}{4}} \frac{1}{\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x} dx = \frac{\pi}{2}$

Type-3 Using Properties, evaluate:

31. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx = \frac{\pi}{4}$

32. $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\cos^5 x + \sin^5 x} dx = \frac{\pi}{4}$

33. $\int_0^{\frac{\pi}{2}} \log \tan x dx = 0$

34. $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$

35. $\int_0^1 x(1-x)^{99} dx = \frac{1}{10100}$

36. $\int_0^2 x\sqrt{2-x} dx = \frac{16\sqrt{2}}{15}$

37. $\int_0^{\pi} \frac{x}{1+\sin x} dx = \pi$

38. $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x} dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$

39. $\int_0^{\pi} x \sin^3 x dx = \frac{2\pi}{3}$

40. $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx = \frac{\pi^2}{4}$

41. $\int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx = \frac{\pi^2}{4}$

42. $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx = \pi(\frac{\pi}{2} - 1)$

43. $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx = 0$

44. $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt[4]{\tan x}} dx = \frac{\pi}{4}$

45. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 2$

46. $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\tan x}} dx = \frac{\pi}{4}$

47. $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx = \frac{\pi^2}{4}$

48. $\int_0^3 x^2(3-x)^{\frac{1}{2}} dx = \frac{144\sqrt{3}}{35}$

49. $\int_2^5 |x-5| dx = \frac{9}{2}$

50. $\int_0^3 |3-2x| dx = \frac{9}{2}$

51. $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx = \frac{\pi}{2}$

52. $\int_2^3 \frac{\sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}} dx = \frac{1}{2}$

53. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin |x| dx = 2$

54. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x| dx = 2$

55. $\int_0^1 \log \left(\frac{1}{x} - 1 \right) dx = 0$

56. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^5 x dx = 0$

57. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\cot x}} dx = \frac{\pi}{12}$

58. $\int_1^4 \{|x-1| + |x-2| + |x-4|\} dx = \frac{23}{2}$

59*. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\cot x}} dx = \frac{\pi}{12}$

60. $\int_0^{\frac{\pi}{2}} \sin 2x \log \tan x dx = 0$

61. $\int_0^1 \cot^{-1}(1-x+x^2) dx = \frac{\pi}{2} - \log 2$

HOME WORK: 3

1. $\int_0^{\frac{\pi}{2}} \frac{1}{1+\tan x} dx = \frac{\pi}{4}$
2. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} dx = \frac{\pi}{4}$
3. $\int_0^1 x(1-x)^n dx = \frac{1}{(n+1)(n+2)}$
4. $\int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx = -\frac{\pi}{2} \log 2$
5. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$
6. $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx = \frac{a}{2}$
7. $\int_{-2}^2 |x+1| dx = 5$
8. $\int_{-6}^6 |x+2| dx = 40$
9. $\int_0^{2\pi} |\cos x| dx = 4$
10. $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx = \frac{\pi^2}{2ab}$
11. $\int_0^4 x(4-x)^{3/2} dx = \frac{512}{35}$
12. $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx = \frac{\pi^2}{4}$
13. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\cot x}} dx = \frac{\pi}{12}$
14. $\int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$
15. $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$, hint: $x = \tan \theta$.
16. $\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx = \frac{\pi}{8}$.

HOTS

1. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$
2. $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$
3. $\int_{-a}^a \sqrt{\frac{a-x}{a+x}} dx = a\pi$
4. $\int_0^{\infty} \frac{x^3}{(1+x^2)^{9/2}} dx$
5. $\int_0^a \frac{x^4}{\sqrt{a^2-x^2}} dx$
6. $\int_0^{\frac{1}{2}} \frac{1}{(1-2x^2)\sqrt{1-x^2}} dx$
7. $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$
8. $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$
9. $\int_0^{\frac{\pi}{4}} \sqrt{\tan x} + \sqrt{\cot x} dx$

SUJITHKUMAR K