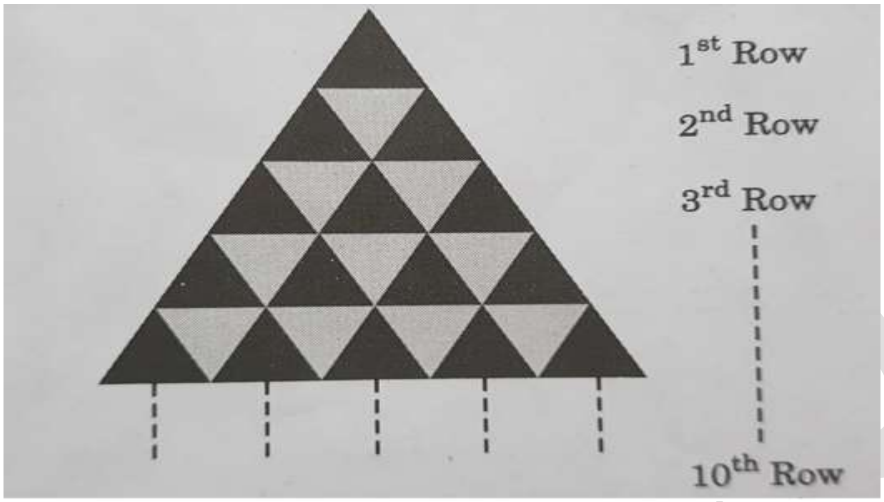
ARITHMETIC PROGRESSIONS

CLASS X (2025-26)

1 In an equilateral triangle of side 10 cm, equilateral triangles of 1 cm are formed as shown in the figure below, such that there is one triangle in the first row, three triangles in the second row, five triangles in the third row and so on. CBSE- 2025



Based on the given information, answer the following questions using AP.

(i) How many triangles will be there in the bottom most row?

(ii) How many triangles will be there in the fourth row from the bottom?

(iii) (a) Find the total number of triangles of side 1 cm each till 8^{th} row?

OR

(b) How many more number of triangles are there from 5^{th} row to 10^{th} row than in first 4 rows? Show working.

ANS: (i) 19

(ii) 13

(iii) (a) 64

(iii) (b) 68

2 **Assertion (A) :** For an AP, 3, 6, 9, ……….., 198, 10^{th} term from the end is 168. CBSE- 2025

Reason (R) : If a and l are the first term and last term of an AP with common difference d, then nth term from the end of the given AP is $l - (n-1)d$.

ANS: (D) A is false but R is true.

3 If the first term a is 6 and the common difference d is 3, then the AP is ----

ANS: 6, 9,12, 15,…….

4 Which of the following list of numbers form an AP? If they form an AP, **write the next two terms :**

(i) 4, 10, 16, 22, . . . (ii) 1, – 1, – 3, – 5, . . .

(iii) – 2, 2, – 2, 2, – 2, . . . (iv) 11, 22, 33, 44,………..

(v) 1,3, 9,27……… (vi) 10, $10+2^5$, $10+2^6$, $10+2^7$…

i) 4, 10, 16, 22, . . .

ANS : (i) $a_2 - a_1 = 10 - 4 = 6$,

$a_3 - a_2 = 16 - 10 = 6$

$a_4 - a_3 = 22 - 16 = 6$.

so it forms an AP , d = 6

next two terms are 28, 34.

(ii) 1, – 1, – 3, – 5, . . .

ANS: (ii) $a_2 - a_1 = -1 - 1 \quad = -2$,

$a_3 - a_2 = -3 - (-1) = -2,$

$a_4 - a_3 = -5 - (-3) = -2$

it forms an AP , d = -2

next two terms are $-7, -9$

ANS: (iii) $-2, 2, -2, 2, -2, \ldots$

$a_2 - a_1 = 2 - (-2) = 4,$

$a_3 - a_2 = -2 - 2 = -4$

$a_2 - a_1 \neq a_3 - a_2$ so it is not an AP.

ANS: iv) 11, 22, 33, 44,………..

$a_2 - a_1 = 11, a_3 - a_2 = 11,$

it is AP d = 11

ANS: v) 1,3, 9,27………

$a_2 - a_1 = 3 - 1 = 2, \quad a_3 - a_2 = 9 - 3 = 6$

$a_2 - a_1 \neq a_3 - a_2$ so it is not an AP.

ANS: (vi) 10, $10+2^5$, $10+2^6$, $10+2^7$………

$a_2 - a_1 = 10 + 2^5 - 10 = 32$

$a_3 - a_2 = 10+2^6 - (10 + 2^5) = 64 - 32 = 32$

$a_4 - a_3 = 10+2^7 - (10 + 2^6) = 128 - 64 = 64$

$a_4 - a_3 \neq a_3 - a_2$ it is not an AP.

5 For the following APs, write the common difference (d)

i) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots \ldots \ldots$

ANS: $a_2 - a_1 = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$, (Or $a_3 - a_2 = \frac{9}{3} - \frac{5}{3} = \frac{4}{3}$)

$d = \frac{4}{3}$

ii) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \ldots \ldots \ldots$

ANS; $a_2 - a_1 = \frac{1}{4} - 0 = \frac{1}{4}$ (Or $a_3 - a_2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$)

$d = \frac{1}{4}$

iii) $7, \ 7 + \sqrt{3}, \ 7 + 2\sqrt{3}, \ 7 + 3\sqrt{3}$…………..

ANS: $a_2 - a_1 = 7 + \sqrt{3} - 7 = \sqrt{3}$

$(a_3 - a_2 = 7 + 2\sqrt{3} - \left(7 + \sqrt{3}\right) = \sqrt{3}$)

$d = \sqrt{3}$

6 Find " d" and the next term of the A.P. $\sqrt{7}, \sqrt{28}, \sqrt{63}$,……………

ANS: $\sqrt{7}, \sqrt{28}, \sqrt{63}$,…………….

$\sqrt{7}, \ \sqrt{4 \times 7}, \ \sqrt{9 \times 7}$, ……..

ie. $\sqrt{7}, \ 2\sqrt{7}, \ 3\sqrt{7}$, ……..

$d = a_2 - a_1 = 2\sqrt{7} - \sqrt{7} = \sqrt{7}$

Next term of $\sqrt{7},\ 2\sqrt{7},\ 3\sqrt{7}, \ldots\ldots$ is $4\sqrt{7}$

7 Find d and the next term of the A.P $p,\ p + 0.12,\ p + 0.24,\ p + 0.36 \ldots \ldots \ldots$

ANS: $p,\ p + 0.12,\ p + 0.24,\ p + 0.36 \ldots \ldots \ldots$

$d = a_2 - a_1 = p + 0.12 - p = 0.1$

Next term = $p + 0.48$

8 Determine k, so that $k + 2,\ 4k - 6,\ and\ 3k - 2$ are three consecutive terms of an AP

ANS: we know that $a_2 - a_1 = a_3 - a_2$ = d

$\Rightarrow (4k - 6) - (k + 2) = (3k - 2) - (4k - 6) \Rightarrow\ (3k - 8) = (-k + 4) \Rightarrow\ 4k = 12\ \Rightarrow k = 3$

9 Find the common difference of the AP.

$x + 3y,\ 2x + 5y,\ 3x + 7y, \ldots \ldots \ldots$

ANS: $d = a_2 - a_1 = 2x + 5y - (x + 3y)$

$= x + 2y$

10 Determine k, so that $k,\ k + 4,\ and\ 3k$ are three consecutive terms of an AP.

ANS: $a_2 - a_1 = a_3 - a_2$ = d

$k + 4 - k = 3k - (k + 4)$

$4 = 2k - 4, \quad \Rightarrow 2k = 8 \quad \Rightarrow k = 4$

11 Find a, b such that the numbers a, 7 , b, 23 are in AP.

$7 - a = b - 7 = 23 - b$ (d)

$7 - a = b - 7$

$a + b$ =14 --------(1)

$b - 7 = 23 - b$

$2b = 30$, $b = 15$, $a = -1$

12 Find the value of x for which $(8x + 4)$, $(6x – 2)$ and $(2x + 7)$ are in A.P.

ANS: $x = \frac{15}{2}$

13 If $x + 1$, $3x$ and $4x + 2$ are in A.P., find the value of x.

ANS: $x = 3$

14 Find the 20^{th} term of the AP 7, 3, $-1, -5 \ldots \ldots$

ANS: Here a = 7, d = -4

$\boldsymbol{a_n = a + (n-1)d}$, $a_{20} = 7 + (20 - 1)(-4)$

$= 7 + (19)(-4) = 7 - 76 = -69$

$\therefore\ 20^{th}$ term = -69

15 Find the 18^{th} term of the AP $\sqrt{2},\ 3\sqrt{2},\ 5\sqrt{2},\ 7\sqrt{2}$, **.......**

ANS: Here a = $\sqrt{2}$, d =$3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

$\boldsymbol{a_n = a + (n-1)d}$

$a_{18} = \sqrt{2} + (18-1)(2\sqrt{2})$= $\sqrt{2} + (17)(2\sqrt{2})$ = $35\sqrt{2}$

∴ 18^{th} term of A. P. is $35\sqrt{2}$

16 Find the n^{th} term of the AP 13, 8, 3, $-$2, ...

ANS: Here a = 13 , d = 8 - 13 = -5

$\boldsymbol{a_n = a + (n-1)d}$

$\boldsymbol{a_n}$= 13 + (n $-$1) ($-$5)

= 13 $-$5n + 5

∴ n^{th} term of the A.P is a_n = 18 $-$5 n.

17 Find the 9^{th} term of the AP $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots\ldots\ldots\ldots\ldots\ldots$..

ANS: Here a =$\frac{3}{4}$, d = $\frac{5}{4}$ - $\frac{3}{4}$ = $\frac{1}{2}$

$\boldsymbol{a_n = a + (n-1)d}$

9^{th} term = a_9 = $\frac{3}{4}$ + (9 - 1) $\frac{1}{2}$

= $\frac{3}{4}$ + (8) $\frac{1}{2}$ = $\frac{3}{4} + 4 = \frac{19}{4}$

18 Which term of the AP 84, 80, 76, ... …is 0?

ANS: First term a = 84

Common difference (d) = $a_2 - a_1 = 80 - 84 = -4$

We know that, n^{th} term $\boldsymbol{a_n = a + (n-1)d}$

And, given n^{th} term is 0

0 = 84 + (n - 1) ($-$4)

0 = 84 $-$(n - 1) 4

84 = +4(n - 1)

n - 1 = $\frac{84}{4}$ = 21

n = 21 + 1 = 22

∴ 22^{nd} term in the A.P is 0.

19 The n^{th} term of an AP is $6n$ + 2. Find its common difference.

ANS: $\boldsymbol{a_n}$ = $6n$ + 2

a_1 = 6 × 1 + 2 = 8

a_2 = 6 × 2 + 2 = 14

Common difference = $a_2 - a_1$ = 14 – 8 = 6

20 Is 68 a term of the A.P. 7, 10, 13,……. ?

ANS: Given, A.P. 7, 10, 13,………

Here, a = 7 and d = $a_2 - a_1$ = 10 – 7 = 3

$\boldsymbol{a_n = a + (n-1)d}$

a + (n – 1)d = 68

7 + (n – 1)3 = 68

7 + 3n – 3 = 68

3n + 4 = 68

3n = 64 ⇒ n = $\frac{64}{3}$, which is not a whole number.

Therefore, 68 is not a term in the A.P.

21 The first term of an A.P. is 5, the common difference is 3 and the last term is 80; find the number of terms.

ANS: Given,

$a = 5$ and $d = 3$, $a_n = 80$

We know that, nth term $\boldsymbol{a_n = a + (n - 1)d}$

So, for the given A.P. $a_n = 5 + (n - 1)3 = 3n + 2$

$\Rightarrow 3n + 2 = 80$

$3n = 78 \quad \Rightarrow \quad n = \frac{78}{3} = 26$

Therefore, there are 26 terms in the A.P.

22 If 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term, show that 25^{th} term of the A.P. is zero

ANS: $\boldsymbol{a_n = a + (n - 1)d}$

$\Rightarrow 10(\boldsymbol{a_{10}}) = 15(\boldsymbol{a_{15}})$

$10(a + (10 - 1)d) = 15(a + (15 - 1)d)$

$10(a + 9d) = 15(a + 14d)$

$10a + 90d = 15a + 210d$

$5a + 120d = 0 \quad \Rightarrow 5(a + 24d) = 0$

$\Rightarrow a + 24d = 0$

$\Rightarrow a + (25 - 1)d = 0 \qquad \{a_{25} = a + (25 - 1)d\}$

$\Rightarrow a_{25} = 0$

Therefore, the 25^{th} term of the A.P. is zero.

23 Find the 12^{th} term from the end of the following arithmetic progression: 3, 5, 7, 9, …. 201

ANS: Given A.P = 3, 5, 7, 9, …. 201

Re - write the AP in the reverse order.

201, 199,197, ……… ,7,5,3.

12^{th} term from the end of AP 3, 5, 7, 9, …. 201

is same as 12^{th} term of 201, 199,197, ………9,7,5,3

Here, a = 201 and d = -2

$\boldsymbol{a_n = a + (n - 1)d} \qquad a\_12 = 201 + (12 - 1)(-2)$

$a_{12} = 201 + (11)(-2)$

$a_{12} = 179$

12^{th} term from the end = 179

Alternate method

$\boldsymbol{a_1, a_2, \ldots\ldots a_n}$ ***is the AP then*** $\boldsymbol{p^{th}}$ ***term from the end is*** $\boldsymbol{t_n - (p - 1)d}$

The given sequence is

3, 5, 7, 9, …. 201

12^{th} term from the end = $201 - (12 - 1)2$

$= 201 - (11)2 = 201 - 22 = 179$

12^{th} term from the end = 179

24 Find the number of two-digit numbers which are divisible by 6.

ANS: Two digit numbers, divisible by 6 are 12, 18, 24, ..., 96

Here a = 12 and d = 18 – 12 = 6 , a_n = 96

⇒ 12 + (n –1)6 = 96

⇒ (n – 1)6 = 96 – 12 = 84

⇒ n – 1 = $\frac{84}{6}$ = 14

⇒ n – 1 = 14

⇒ n = 14 + 1 = 15.

number of two-digit numbers divisible by 6 =15

25 Find the sum of 10 terms of AP 2, 5, 8, 11, ...

ANS: Here a = 2, d = 5 – 2 = 3 , n = 10

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

sum of 10 terms = $S_{10} = \frac{10}{2}$ [2 × 2 + 9 × 3]

$S_{10} = 5[4 + 27]$= 5 × 31 = 155

26 Find the sum: 3 + 11 + 19 + ... + 803.

ANS: Here a = 3, d = 11 – 3 = 8, $a_n = l$ = 803

$a_n = a + (n – 1)d$

⇒ 3 + (n – 1)8 = 803

(n – 1)8 = 800

n – 1 = 100, n = 101

$S_n = \frac{n}{2}(a + l)$

⇒ $\frac{101}{2}$(3 + 803) = 101 × 403 = 40703

27 Find the sum of all 2-digit odd positive numbers.

ANS: Two-digit odd positive numbers are

11, 13, 15, ... …… 99.

These numbers are in AP.

Here a = 11 and d = 2 , a_n = 99

$a_n = a + (n – 1)d = 99$

11 + (n – 1) × 2 = 99

(n – 1) × 2 = 88

n – 1 = 44 , n = 45

Since, $S_n = \frac{n}{2}(a + l)$

$S_{45} = \frac{45}{2}$ (11 + 99) = 45 × 55 = 2475

sum of all 2-digit odd positive numbers = 2475

28 The n^{th} term (a_n) of an Arithmetic Progression is given by $a_n = 4n – 5$. Find the sum of the first 25 terms of the Arithmetic Progression.

ANS: $a_n = 4n – 5$

a_1 = 4 × 1 – 5 = –1

a_2 = 4 × 2 – 5 = 3

Common difference $d = 3 - (-1) = 4$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$S_{25} = \frac{25}{2}[2 \times (-1) + (25-1)4] = \frac{25}{2} \times 94 = 1175$

sum of the first 25 terms = 1175

29 How many terms of the AP 3, 5, 7, ... must be taken so that the sum is 120?

.ANS: Here $a = 3$, $d = 2$ and $S_n = 120$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow\ 120 = \frac{n}{2}\{6 + (n-1)2\} = \frac{n}{2} \times 2\{3 + n - 1\}$

$= n\{2 + n\}$

$120 = n(n + 2)$

$n^2 + 2n - 120 = 0$ (Factorise : P= – 120, S = 2)

$\Rightarrow (n + 12)(n - 10) = 0$

$\Rightarrow$ Either $n + 12 = 0$ or $n - 10 = 0$ (*n is positive)*

$n = -12$ (rejected) $n = 10$

10 terms must be taken to make sum 120

30 Find the sum of first 25 terms of an AP whose n^{th} term is $1 - 4n$.

ANS: $a_n = 1 - 4n$

$a_1 = 1 - 4 \times 1 = -3$,

$a_2 = 1 - 4 \times 2 = -7$

$d = a_2 - a_1 = -7 - (-3) = -4$

$a_{25} = a + 24d = -3 + 24 \times (-4) = -99$

$S_n = \frac{n}{2}(a + l)$

$S_{25} = \frac{25}{2}(a_1 + a_{25}) = \frac{25}{2}(-3 - 99) = \frac{25}{2}(-102)$

$= 25 \times (-51) = -1275$

31 The sum of n terms of an AP is $3n^2 + 5n$. Find the AP. Hence, find its 16^{th} term.

ANS: Given $S_n = 3n^2 + 5n$

$S_1 = 3 \times 1^2 + 5 \times 1 = 8$

$a_1 = 8$ ....(i)

$S_2 = 3 \times 2^2 + 5 \times 2 = 22$

$a_1 + a_2 = 22$ (OR apply $S_2 - S_1 = a_2$)

$8 + a_2 = 22$ [Using (i)]

$a_2 = 14$

$\Rightarrow\ d = a_2 - a_1 = 14 - 8 = 6$

AP is 8, 14, 20, 26, ...

Now $a_{16} = a + 15d = 8 + 15 \times 6 = 98$

16^{th} term of the AP = 98.

32 In an AP, the first term is 8, n^{th} term is 33 and sum to first n terms is 123. Find n and d, the common difference.

ANS: $a = 8$, $a_n = 33$ and $S_n = 123$

Now, $S_n = \frac{n}{2}(a + a_n)$

$\Rightarrow \quad \frac{n}{2}[8 + 33] = 123 \Rightarrow n = \frac{123 \times 2}{41} \Rightarrow n = 6$

Also, $a_n = a + (n - 1)d$

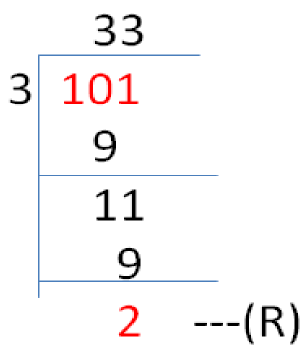
$a + (n - 1)d = 33$

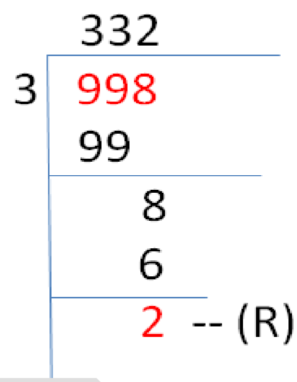
$8 + (6 - 1)d = 33$

$5d = 25 \Rightarrow d = 5$

common difference = 5

33 Find the sum of all three-digit numbers each of which leaves the remainder 2, when divided by 3





ANS: 1^{st} three digit number leaving remainder 2 when divided by 3 is 101.

Required numbers are 101, 104, 107, ...

last such no = 999 – 1 = 998

These numbers are in AP.

Here $a = 101$, $d = 104 - 101 = 3$, $a_n = 998$

$a + (n - 1)d = 998$

$101 + (n - 1) \times 3 = 998$

$(n - 1) \times 3 = 897$

$n - 1 = 299 \Rightarrow n = 300$

Now, $S_n = \frac{n}{2}(a + a_n)$

$= \frac{300}{2}(101 + 998) = 164850$

34 How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

ANS: Here, $a = 24$, $d = 21 - 24 = -3$, $S_n = 78$. We need to find n.

$S_n = \frac{n}{2}\{2a + (n - 1)d\}$, $78 = \frac{n}{2}\{2 \times 24 + (n - 1)(-3)\}$

$156 = n\{48 + (-3n + 3)\}156 = n\{51 - 3n\}$

$3n^2 - 51n + 156 = 0 \Rightarrow \quad n^2 - 17n + 52 = 0$

$\Rightarrow (n - 4)(n - 13) = 0$

or $n = 4$ or 13

Both values of n are admissible. So, the number of terms is either 4 or 13.

35 MCQs

1. If p – 1, p + 3, 3p – 1 are in AP, then p is equal to

(a) 4 (b) – 4 (c) 2 (d) – 2

2. If the third term of an AP is 12 and the seventh term is 24, then the 10^{th} term is

(a) 33 (b) 34 (c) 35 (d) 36

3. A number 15 is divided into three parts which are in AP and sum of their squares is 83. The smallest part is

(a) 2 (b) 5 (c) 3 (d) 6

4. How many terms of an AP must be taken for their sum to be equal to 120 if its third term is 9 and the difference between the seventh and second term is 20 ?

(a) 7 (b) 8 (c) 9 (d) 6

5. 9th term of an AP is 499 and 499th term is 9. The term which is equal to zero is

(a) 507th (b) 508th (c) 509th (d) 510^{th}

6. The sum of all two digit numbers which when divided by 4 yield unity as remainder is

(a) 1012 (b) 1201 (c) 1212 (d) 1210

7. An AP consist of 31 terms if its 16th term is m, then sum of all the terms of this AP is

(a) 16 m (b) 47 m (c) 31 m (d) 52 m

8. In a certain AP, 5 times the 5th term is equal to 8 times the 8th term, then its 13th term is equal to

(a) 5 (b) 1 (c) 0 (d) 13

9. The sum of 5 numbers in AP is 30 and sum of their squares is 220. Which of the following is the third term ?

(a) 5 (b) 6 (c) 7 (d) 8

10. If a, b, c, d, e and f are in AP, then e – c is equal to

(a) 2(c – a) (b) 2(f – d) (c) 2(d – c) (d) d – c

11.7th term of an AP is 40. The sum of its first 13th terms is

(a) 500 (b) 510 (c) 520 (d) 530

12. The sum of the first four terms of an AP is 28 and sum of the first eight terms of the same AP is 88 . Sum of first 16 terms of the AP is

(a) 346 (b) 340 (c) 304 (d) 268

13. Which term of the AP 4, 9, 14, 19, ….. is 109?

(a) 14th (b) 18th (c) 22nd (d) 16^{th}

14. How many terms are there in the arithmetic series

1 + 3 + 5 +…… …….. + 73 + 75?

(a) 28 (b) 30 (c) 36 (d) 38

15. The sum 51 + 52 + 53 + 54 +……. + 100 = ?

(a) 3775 (b) 4025 (c) 4275 (d) 5050

16.How many natural numbers between 1 and 1000 are divisible by 5?

(a) 197 (b) 198 (c) 199 (d) 200

17. If a, a – 2 and 3a are in AP, then the value of a is

(a) –3 (b) –2 (c) 3 (d) 2

18. How many terms are there in the AP 7, 10, 13, …. , 151?

(a) 50 (b) 55 (c) 45 (d) 49

19. The 4th term of an AP is 14 and its 12th term is 70. What is its first term?

(a) –10 (b) –7 (c) 7 (d) 10

20 . Which term of the AP 72, 63, 54,…….. is 0?

(a) 8th (b) 9th (c) 11th (d) 12th

36 If p, q, r are in AP, then $p^3 + r^3 - 8q^3$ is equal to

(a) 4pqr (b) – 6pqr c) 2pqr (d) 8pqr

ANS: (b) ∵ p, q, r are in AP.

∴ $2q = p + r \quad \Rightarrow p + r - 2q = 0$

∴ $p^3 + r^3 + (-2q)^3 = 3 \times p \times r \times -2q$

[Using if $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$]

$\Rightarrow p^3 + r^3 - 8q^3 = -6pqr$.

37 The list of numbers –10, – 6, –2, 2, ... is_________

(a) an AP with d = –16 (b) an AP with d = 4

(c) an AP with d = – 4 (d) not an AP

ANS: (b) An AP with d = 4

38 Two APs have the same common difference. The first term of one of these is –1 and that of the other is –8. Then the difference between their 4th terms is

(a) –1 (b) –8 (c) 7 (d) –9

ANS: (c) $a_4 - b_4 = (a_1 + 3d) - (b_1 + 3d) = a_1 - b_1 = -1 - (-8) = 7$

39 If p – 1, p + 3, 3p – 1 are in AP, then p is equal to ________.

ANS: ∵ p – 1, p + 3 and 3p – 1 are in AP.

∴ $2(p + 3) = p - 1 + 3p - 1$

$\Rightarrow 2p + 6 = 4p - 2$.

$\Rightarrow -2p = -8 \Rightarrow p = 4$.

40 In an AP, if d = –2, n = 5 and $a_n = 0$, the value of a is

(a) 10 (b) 5 (c) –8 (d) 8

ANS: (d) $d = -2, n = 5, a_n = 0$

$\because a_n = 0$

$\Rightarrow a + (n - 1)d = 0$

$\Rightarrow a + (5 - 1)(-2) = 0$

$\Rightarrow a = 8$

Correct option is (d).

41 If the common difference of an AP is 3, then $a_{20} - a_{15}$ is______

(a) 5 (b) 3 (c) 15 (d) 20

ANS: (c) Common difference, d = 3

$a_{20} - a_{15} = (a + 19d) - (a + 14d) = 5d = 5 \times 3 = 15$

42 The next term of the AP $\sqrt{18}, \sqrt{50}, \sqrt{98}, \ldots \ldots \ldots$ is

ANS: $\sqrt{18}, \sqrt{50}, \sqrt{98}, \ldots \ldots = 3\sqrt{2},\ 5\sqrt{2},\ 7\sqrt{2}, \ldots\ldots$

next term of the AP $9\sqrt{2} = \sqrt{162}$

43 If the nth term of an AP is (2n + 1), then the sum of its first three terms is______-

(a) 6n + 3 (b) 15 (c) 12 (d) 21

ANS: (b) $a_1 = 2 \times 1 + 1 = 3$,

$a_2 = 2 \times 2 + 1 = 5,\ a_3 = 2 \times 3 + 1 = 7$

$\therefore$ Sum = 3 + 5 + 7 = 15

44 An AP consists of 31 terms. If its 16th term is m, then sum of all the terms of this AP is_________-

(a) 16 m (b) 47 m (c) 31 m (d) 52 m

ANS: (c) $S_{31} = \frac{31}{2}$ (2a + 30d)

a_{16} = a + 15d = m

$\Rightarrow S_{31} = \frac{31}{2} \times 2(a + 15d) \Rightarrow S_{31} = 31m$

45 If the first term of an AP is 2 and common difference is 4, then sum of its first 40 terms is _______.

ANS: $S_{40} = \frac{40}{2}(2 \times 2 + 39 \times 4) = 40 \times (80) = 3200$

46 7th term of an AP is 40. The sum of its first 13th terms is _______.

ANS: a + 6d = 40

$S13 = \frac{13}{2}[2a + 12d] = 13(a + 6d) = 13 \times 40 = 520.$

47 The first term of an AP of consecutive integers is $p^2 + 1$. The sum of 2p + 1 terms of this AP is

(a) $(p + 1)^2$ (b) $(2p + 1)(p + 1)^2$

(c) $(p + 1)^3$ (d) $p^3 + (p + 1)^3$

ANS: (d) $\because a = p^2 + 1$ and d = 1.

$S_{2p+1} = \frac{(2p+1)}{2}(2p^2 + 2 + (2p)1)$

$= \frac{(2p+1)}{2}(2)(p^2 + p + 1)$

$= (2p + 1)(p^2 + p + 1) = p^3 + (p + 1)^3$

48 If the sum of first n terms of an AP is $An + Bn^2$ where A and B are constants, the common difference of AP will be

(a) A + B (b) A – B (c) 2A (d) 2B

ANS: (d) $S_n = A_n + Bn^2$

$S_1 = A \times 1 + B \times 1^2 = A + B$

$\because S_1 = a_1$

$\therefore a_1 = A + B$ ... (i)

and $S_1 = A \times 2 + B \times 2^2$

$\Rightarrow a_1 + a_2 = 2A + 4B$

$\Rightarrow (A + B) + a_2 = 2A + 4B$ [Using (i)]

$\Rightarrow a_2 = A + 3B$

$\therefore d = a_2 - a_1 = 2B$

49 Find the 10th term of the AP 2, 7, 12, ...

ANS: Here $a = 2$; $d = 7 - 2 = 5$. So, $a_{10} = a + 9d = 2 + 9 \times 5 = 47$

50 The *n*th term of an AP is $7 - 4n$. Find its common difference.

ANS: $a_n = 7 - 4n \Rightarrow a_1 = 7 - 4 \times 1 = 3$

$a_2 = 7 - 4 \times 2 = -1$ and $a_3 = 7 - 4 \times 3 = -5$

Common difference = $a_2 - a_1 = -1 - 3 = -4$.

51 Which term of the AP 21, 18, 15, ..., is zero?

ANS: Here, $a = 21$, $d = 18 - 21 = -3$

Let $a_n = 0$

$a + (n - 1)\, d = 0$, $\Rightarrow$ $21 + (n - 1)(-3) = 0$

$\Rightarrow$ $(n - 1)(-3) = -21 \Rightarrow$ $n - 1 = 7 \Rightarrow$ $n = 8$

8th term is zero.

52 For what value of p, are $2p + 1$, 13, $5p - 3$ three consecutive terms of an AP?

ANS: If terms are in AP, then $13 - (2p + 1) = (5p - 3) - 13$

$\Rightarrow$ $13 - 2p - 1 = 5p - 3 - 13 \Rightarrow$ $28 = 7p \Rightarrow$ $p = 4$

53 Find the sum of first 22 terms of the AP 8, 3, – 2, ...

ANS: Here $a = 8$, $d = 3 - 8 = -5$. So, $S_{22} = \frac{22}{2}\,[2a + (22 - 1)d]$

$\Rightarrow$ $S_{22} = 11(16 - 105) = -979$

54 If the sum of first m terms of an AP is $2m^2 + 3m$, then what is its second term?

ANS: $S_m = 2m^2 + 3m$

$S_1 = 2 \times 1^2 + 3 \times 1 = 5 = a_1$ and $S_2 = 2 \times 2^2 + 3 \times 2 = 14$ ...(i)

$\Rightarrow$ $a_1 + a_2 = 14$ $\Rightarrow$ $5 + a_2 = 14 \Rightarrow$ $a_2 = 9$

55 If the sum of first p terms of an AP is $ap^2 + bp$, find its common difference.

ANS: $S_p = ap^2 + bp$

$\Rightarrow$ $S_1 = a \times 1^2 + b \times 1 = a + b = a_1$ and $S_2 = a \times 2^2 + b \times 2 = 4a + 2b$ ...(i)

$\Rightarrow$ $a_1 + a_2 = 4a + 2b \Rightarrow$ $a + b + a_2 = 4a + 2b \Rightarrow$ $a_2 = 3a + b$ [Using eq. (i)]

Now, $d = a_2 - a_1 = (3a + b) - (a + b) = 2a$.

56 In an AP, if $a = 3$, $n = 8$, $S_n = 192$, find d.

ANS: $S_n = \frac{n}{2}\{2a + (n - 1)d\} \Rightarrow$ $192 = \frac{8}{2}\,[2 \times 3 + (8 - 1) \times d]$

$\Rightarrow \frac{192}{4} = 6 + 7d \Rightarrow 7d = 48 - 6 = 42 \Rightarrow$ $d = 6$

57 The nth term of an AP is $6n + 2$. Find its common difference.

ANS: $a_n = 6n + 2$

$\Rightarrow a_1 = 6 \times 1 + 2 = 8$

$a_2 = 6 \times 2 + 2 = 14$

Common difference $= a_2 - a_1 = 14 - 8 = 6$

58 Find the 11th term of the AP -3, $\frac{-1}{2}$, 2, ...

ANS: Here $a = -3$, $d = \frac{-1}{2} - (-3) = \frac{5}{2}$

$a_{11} = a + 10d = -3 + 10 \times \frac{5}{2} = 22$

59 The first term of an AP is p and its common difference is q. Find its 10th term.

ANS: $a = p$ and $d = q$

$C = a + (n - 1)\, d$

$a_{10} = p + 9q$

60 Find the 12th term of the AP with first term 9 and common difference 10.

ANS: $a = 9$, $d = 10$

$a_{12} = a + 11d = 9 + 11 \times 10 = 119$

61 Find the sum of the first 1000 positive integers.

ANS: $S_n = \frac{n}{2}[a + l]$

$\Rightarrow S_{1000} = \frac{1000}{2}[1 + 1000] = 500500$

62 If sum of first n terms of an AP is $2n^2 + 5n$. Then find S_{20}.

ANS: $S_n = 2n^2 + 5n$

$S_{20} = 2(20)^2 + 5 \times 20 = 2 \times 400 + 100 = 900$

63 The 6th term of an Arithmetic Progression (AP) is –10 and its 10th term is –26. Determine the 15th term of the AP.

ANS: Let Ist term of AP = a and common difference = d.

Now, $a_6 = -10 \Rightarrow a + 5d = -10$ ...(i)

Also, $a_{10} = -26 \Rightarrow a + 9d = -26$ ...(ii)

Subtract (i) from (ii), $4d = -16$ — $d = -4$

Substituting in (i), we get

$a + 5 \times (-4) = -10 \Rightarrow a = 10$

Now, $a_{15} = a + 14d = 10 + 14 \times -4 = -46$

64 Is –150 a term of the AP 17, 12, 7, 2...?

ANS: 17, 12, 7, 2, ...

Here, $a = 17$, $d = 12 - 17 = -5$

Let $a_n = -150$

$a + (n - 1)d = -150 \Rightarrow 17 + (n - 1)(-5) = -150$

$\Rightarrow (n - 1)(-5) = -150 - 17 \Rightarrow (n - 1)(-5) = -167$

$\Rightarrow n - 1 = \frac{167}{5}$ $\qquad \Rightarrow n = \frac{167}{5} + 1$

$\Rightarrow n = \frac{172}{5}$

n is not a whole number.

– 150 is not a term of the A.P.

65 Which term of the AP 21, 42, 63, 84, ... is 420?

ANS: Here $a = 21$, common difference $d = 42 - 21 = 21$.

Let $a_n = 420$

Now, $a_n = a + (n - 1)d \Rightarrow 420 = 21 + (n - 1)21$

$n = 20$.

66 Determine the 25th term of an AP whose 9th term is – 6 and common difference is $\frac{5}{4}$.

ANS: Let Ist term = a

Common difference, $d = \frac{5}{4}$ (Given)

Also, $a_9 = -6$

$\Rightarrow a + 8d = -6 \Rightarrow a + 8 \times \frac{5}{4} = -6 \Rightarrow a = -16$

Now $a_{25} = a + 24d = -16 + 24 \times \frac{5}{4} = 14$

67 Determine k so that $4k + 8$, $2k^2 + 3k + 6$ and $3k^2 + 4k + 4$ are three consecutive terms of an AP.

ANS: For consecutive terms of AP,

$2(2k^2 + 3k + 6) = (3k^2 + 4k + 4) + (4k + 8)$

$\Rightarrow 4k^2 + 6k + 12 = 3k^2 + 8k + 12 \Rightarrow k^2 - 2k = 0$

$\Rightarrow k(k - 2) = 0 \Rightarrow k = 0$ or $k = 2$

68 If 5 times the 5th term of an AP is equal to 10 times the 10th term, show that its 15th term is zero.

ANS: Let 1st term = a and common difference = d.

$a_5 = a + 4d$, $a_{10} = a + 9d$

According to the question, $5 \times a_5 = 10 \times a_{10}$

$\Rightarrow 5(a + 4d) = 10(a + 9d) \Rightarrow 5a + 20d = 10a + 90d \Rightarrow a = -14d$

Now $a_{15} = a + 14d \Rightarrow a_{15} = -14d + 14d = 0$.

69 In an AP, the 24th term is twice the 10th term. Prove that the 36th term is twice the 16th term.

ANS: Let 1st term = a, common difference = d.

$a_{10} = a + 9d$, $a_{24} = a + 23d$

According to the question, $a_{24} = 2 \times a_{10}$

$\Rightarrow a + 23d = 2(a + 9d) \Rightarrow a + 23d = 2a + 18d \Rightarrow a = 5d$

Now, $a_{16} = a + 15d = 5d + 15d = 20d$ ...(i)

$a_{36} = a + 35d = 5d + 35d = 40d$ ...(ii)

From (i) and (ii), we get

$a_{36} = 2 \times a_{16}$ Hence proved.

70 Find the number of terms in the AP $17, 14\frac{1}{2}, 12, \ldots\ldots, -38$.

ANS: Here $a = 17$, $d = 14\frac{1}{2} - 17 = \frac{29}{2} - 17 = -\frac{5}{2}$

Let number of terms in AP = n

$a_n = -38$

$a + (n - 1)d = -38 \Rightarrow 17 + (n - 1) \times \left(-\frac{5}{2}\right) = -38$

$\Rightarrow (n - 1)\left(-\frac{5}{2}\right) = -55 \Rightarrow (n - 1) = -55 \times \left(-\frac{5}{2}\right) = 22$

$n = 23$

71 Find 10th term from end of the AP 4, 9, 14, ... , 254.

ANS: 10th term from end of AP 4, 9, 14, ..., 254 is 10th term of the AP 254, 249, 244, ... 14, 9, 4

Here $a = 254$

$d = 249 - 254 = -5$

$a_{10} = a + 9d \Rightarrow a_{10} = 254 + 9 \times (-5) = 209$

72 How many terms are there in an AP whose first term and 6th term are –12 and 8 respectively, and sum of all its terms is 120?

ANS: Ist term $a = -12$

Let common difference be d.

Now $a_6 = 8 \Rightarrow a + 5d = 8 \Rightarrow -12 + 5d = 8 \Rightarrow d = 4$.

$S_n = \frac{n}{2}\{2a + (n - 1)d\} = 120$

$\Rightarrow n[2 \times (-12) + (n - 1)4] = 240 \Rightarrow n(-28 + 4n) = 240$

$\Rightarrow 4n^2 - 28n - 240 = 0 \Rightarrow n^2 - 7n - 60 = 0$

$\Rightarrow (n - 12)(n + 5) = 0 \Rightarrow$ Either $n = 12$ or $n = -5$

Number of terms = 12.

73 The first term, common difference and last term of an AP are 12, 6 and 252 respectively. Find the sum of all terms of this AP.

ANS: Given; $a = 12$, $d = 6$, $a_n = 252$

$\Rightarrow$ $a + (n-1)d = 252 \Rightarrow 12 + (n-1)6 = 252 \Rightarrow (n-1)6 = 240$

$\Rightarrow n - 1 = \frac{240}{6} = 40 \Rightarrow n = 41$

$S_{41} = \frac{41}{2}(12 + 252) = 5412$

74 Find the common difference of an AP whose first term is 4, the last term is 49 and the sum of all its terms is 265.

ANS: Given; $a = 4$, $l = 49$ and $S_n = 265$

$265 = \frac{n}{2}(4 + 49) \Rightarrow 530 = 53n \Rightarrow n = 10$

$l = a_{10} = a + 9d$

$\Rightarrow 49 = 4 + 9d \Rightarrow 9d = 45 \Rightarrow d = 5$

Common difference = 5

75 Find the sum of the:

(i) first 11 terms of the AP: 2, 6, 10, ...

(ii) first 51 terms of the AP whose second term is 2 and fourth term is 8.

ANS: (i) $a = 2$, common difference $d = 6 - 2 = 4$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{11} = \frac{11}{2}[2 \times 2 + (11-1)4] = \frac{11}{2}[4 + 40] = 242.$

(ii) Let Ist term = a, common difference = d

$a_n = a + (n-1)d$

$a_2 = a + (2-1)d$ $\quad$ $a + d = 2$ ...(i)

and $a_4 = a + 3d$ $\quad$ $a + 3d = 8$ ...(ii)

Solving (i) and (ii), we get

$d = 3$, $a = -1$

Now $\quad S_n = \frac{n}{2}\{2a + (n-1)d\}$

$S_{51} = \frac{51}{2}[2 \times (-1) + (51-1) \times 3]$

$S_{51} = \frac{51}{2}[2 \times (-1) + 50 \times 3] = 3774.$

76 Find the sum: 2 + 4 + 6 + ... + 200

ANS: Here, $a = 2$, $d = 4 - 2 = 2$

$a_n = a + (n-1)d \quad \Rightarrow 200 = 2 + (n-1)2 \Rightarrow n = 100$

There are 100 terms in the given AP.

Now $S_n = \frac{n}{2}[a + l]$

$\Rightarrow S_{100} = 50[2 + 200] = 10100.$

77 Find the sum: $-5 + (-8) + (-11) + ... + (-230)$.

ANS: Here $a = -5$, $d = (-8) - (-5) = -3$

$a_n = l = -230$

Now $a_n = a + (n-1)d$

$a + (n-1)d = -230 - 5 + (n-1)(-3) = -230$

$(n-1)(-3) = -225$

$(n-1) = \frac{-225}{-3} = 75$

$\Rightarrow n = 76$

$S_n = \frac{n}{2}\ (a + l) = \frac{76}{2}\ (-5 - 230) = 38(-235) = -8930$

78 Find the sum of the first 25 terms of an AP whose nth term is given by $t_n = 7 - 3n$.

ANS: $t_n = 7 - 3n$

$t_1 = 7 - 3 \times 1 = 4$

$t_2 = 7 - 3 \times 2 = 1$

Common difference = 1 – 4 = – 3

$S_n = \frac{n}{2}\ [2a + (n-1)d] = \frac{25}{2}\ [2 \times 4 + (25 - 1) \times (-3)] = \frac{25}{2}\ [8 - 72] = -800$

79 If the sum of first n terms of an AP is given by $S_n = 3n^2 + 2n$, find the nth term of the AP.

ANS: Given: $S_n = 3n^2 + 2n$. Let t_n be the nth term.

$t_n = S_n - S_{n-1} = (3n^2 + 2n) - \{3(n-1)^2 + 2(n-1)\} = 3n^2 + 2n - 3n^2 + 6n - 3 - 2n + 2$

Hence, $t_n = 6n - 1$.

80 Find the sum of all the natural numbers less than 100 which are divisible by 6.

ANS: Natural numbers less than 100 and divisible by 6 are 6, 12, 18, ..., 96.

$a = 6$, $d = 6$, $n = 16$, $a_n = 96$

Sum = 8 [6 + 96] = 816

81 Using AP, find the sum of all 3-digit natural numbers which are the multiples of 7.

ANS: Ist three-digit number which is a multiple of 7 = 105

IInd three-digit number which is a multiple of 7 = 112

IIIrd three-digit number which is a multiple of 7 = 119

Last three-digit number which is a multiple of 7 = 994

So, numbers are 105, 112, 119, ..., 994

These numbers are in AP.

Here, $a = 105$, $d = 112 - 105 = 7$

Let $a_n = 994$

$a + (n-1)d = 994 \Rightarrow 105 + (n-1)7 = 994$

$\Rightarrow (n-1)7 = 889 \Rightarrow (n-1) = \frac{889}{7} = 127$

$\Rightarrow n = 128$

Now, $S_n = \frac{n}{2}\ (a + l) = \frac{128}{2}\ (105 + 994) = 70336$

82 The sum of three numbers of an AP is 27 and their product is 405. Find the numbers.

ANS: Let three numbers in AP are $a - d$, a and $a + d$

$(a - d) + a + (a + d) = 27$

$\Rightarrow 3a = 27 \Rightarrow a = 9$

Also $(a - d)\ (a)\ (a + d) = 405 \Rightarrow (9 - d)\ (9)\ (9 + d) = 405$

$\Rightarrow (9 - d)\ (9 + d) = 45 \Rightarrow 81 - d^2 = 45$

$\Rightarrow d^2 = 36 \Rightarrow d = 6, -6$

When $d = 6$, numbers are 3, 9, 15

When $d = -6$, numbers are 15, 9, 3

83 Which term of the AP 14, 11, 8, ...is –1?

ANS: Here, $a = 14$, $d = 11 - 14 = -3$

Let $a_n = -1 \Rightarrow a + (n-1)d = -1$

$\Rightarrow 14 + (n-1)(-3) = -1$

$\Rightarrow (n-1)(-3) = -1 - 14$

$\Rightarrow n - 1 = \frac{-15}{-3} = 5 \Rightarrow n = 6$

6th term of the AP is –1.

84 Write the next two terms of the AP: 1, –1, –3, –5, ...

ANS: $a_1 = 1$

$a_1 = a_2 - a_1 = -1 - 1 = -2$

$a_5 = a_1 + 4d = 1 + 4(-2) = 1 - 8 = -7$

$a_6 = a_5 + d = -7 - 2 = -9$

Next two terms are – 7 and – 9.

85 Find the 6th term from the end of the AP 17, 14, 11, ..., – 40.

ANS: 6th term from end of 17, 14, 11, ... –40 is 6th term of –40, –37, –34, ..., 11, 14, 17.

Here, $a = -40$

$d = -37 - (-40) = 3$

$a_6 = -40 + (6-1)3 = -40 + 15 = -25$

86 Which term of AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

ANS: $a = 3$, $d = 15 - 3 = 12$.

$a_n = a + (n-1)d$

$\Rightarrow a_{21} = 3 + (21-1) \times 12 = 243$

Let $a_m = a_{21} + 120$

$\Rightarrow a_m = 243 + 120 = 363$

$363 = a + (m-1)\, d$

$\Rightarrow 363 = 3 + (m-1) \times 12$

$\Rightarrow 360 = (m-1) \times 12$

$\Rightarrow m - 1 = 30$

$\Rightarrow m = 31$

31st term is 120 more than 21st term.

87 Which term of the AP 5, 2, – 1, ... is – 22?

ANS: Here $a = 5$, $d = 2 - 5 = -3$

Let $a_n = -22$

$\Rightarrow a + (n-1)d = -22$

$\Rightarrow 5 + (n-1)(-3) = -22$

$\Rightarrow 5 - 3n + 3 = -22$

$\Rightarrow -3n = -30 \Rightarrow n = 10$

10th term of the AP is –22.

88 Find the sum of n terms of AP where $a_n = 5 - 2n$.

ANS: $a_2 = 5 - 2n$,

$\Rightarrow a_1 = 5 - 2 \times 1 = 3$

$a_2 = 5 - 2 \times 2 = 1$

$\Rightarrow d = 1 - 3 = -2$

$S_n = \frac{n}{2}\{2a + (n-1)d\} = \frac{n}{2}[2 \times 3 + (n-1) \times -2] = \frac{n}{2}[6 - 2n + 2] = \frac{n}{2}(8 - 2n) = 4n - n^2$

89 Find the sum of 10 terms of AP 2, 5, 8, 11, ...

ANS: Here $a = 2$, $d = 5 - 2 = 3$ $n = 10$

Now,

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S_{10} = 5[2 \times 2 + 9 \times 3] = 155$

90 Find the sum of first 25 terms of an AP whose nth term is $1 - 4n$.

ANS: $a_n = 1 - 4n$

$\Rightarrow a_1 = 1 - 4 \times 1 = -3$

$a_2 = 1 - 4 \times 2 = -7$

$d = a_2 - a_1 = -7 - (-3) = -4$

$a_{25} = a + 24d = -3 + 24 \times (-4) = -99$

Now, $S_{25} = \frac{25}{2}(a_1 + a_{25}) = \frac{25}{2}(-3 - 99) = 25 \times (-51) = -1275$

91 In an AP, the first term is – 4, the last term is 29 and the sum of all its terms is 150. Find its common difference.

ANS: $a = -4$, $l = 29$, $S_n = 150$. $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$150 = \frac{n}{2}(-4 + 29)$

$\Rightarrow 300 = 25n \Rightarrow n = 12$

$l = a_{12} \Rightarrow 29 = -4 + 11d$

$\Rightarrow 11d = 33 \Rightarrow d = 3$

Common difference = 3.

92 Find the sum: 3 + 11 + 19 + ... + 803.

ANS: Here $a = 3$, $d = 11 - 3 = 8$,

$a_n = l = 803$

$a_n = a + (n-1)d$

$\Rightarrow 3 + (n-1)8 = 803$

$\Rightarrow (n-1)8 = 800$

$\Rightarrow n - 1 = 100 \Rightarrow n = 101$

Now $S_n = \frac{n}{2}(a + l) = \frac{101}{2}(3 + 803) = 101 \times 403 = 40703$

93 The nth term (t_n) of an Arithmetic Progression is given by $t_n = 4n - 5$. Find the sum of the first 25 terms of the Arithmetic Progression.

ANS: $t_n = 4n - 5$

$t_1 = 4 \times 1 - 5 = -1$

$t_2 = 4 \times 2 - 5 = 3$

Common difference $d = 3 - (-1) = 4$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S_{25} = \frac{25}{2}[2 \times (-1) + (25 - 1)4] = \frac{25}{2} \times 94 = 1175$

94 Find the sum of all 2-digit odd positive numbers.

ANS: Two-digit odd positive numbers are 11, 13, 15, ... 99.

These numbers are in AP.

Here $a = 11$ and $d = 2$

$a_n = 99$

$\Rightarrow a + (n - 1)d = 99$

$\Rightarrow 11 + (n - 1) \times 2 = 99$

$\Rightarrow (n - 1) \times 2 = 88$

$\Rightarrow n - 1 = 44 \quad \Rightarrow \quad n = 45$

Since, $S_n = \frac{n}{2}(a + l)$

$\Rightarrow S_{45} = \frac{45}{2}(11 + 99) = 2475$

95 Find the sum of all 2-digit positive numbers divisible by 3.

ANS: Two-digit positive numbers di visible by 3 are 12, 15, 18, ..., 99

These numbers are in A.P. with $a = 12$, $d = 15 - 12 = 3$ and $a_n = 99$

$a + (n - 1)d = 99$

$\Rightarrow 12 + (n - 1)3 = 99$

$\Rightarrow (n - 1)3 = 87$

$\Rightarrow n - 1 = 29 \Rightarrow n = 30$

Now $S_n = \frac{n}{2}(a + l)$

$\Rightarrow S_{30} = 15 (12 + 99) = 1665$

96 The sum of n terms of an AP is $3n^2 + 5n$. Find the AP. Hence, find its 16th term.

ANS: Given; $S_n = 3n^2 + 5n$

$S_1 = 3 \times 1^2 + 5 \times 1 = 8$

$\Rightarrow a_1 = 8$ ...(i)

$S_2 = 3 \times 2^2 + 5 \times 2 = 22$

$\Rightarrow a_1 + a_2 = 22$

$\Rightarrow 8 + a_2 = 22$ [Using (i)]

$\Rightarrow a_2 = 14$

$d = a_2 - a_1 = 14 - 8 = 6$

AP is 8, 14, 20, 26, ...

Now $a_{16} = a + 15d = 8 + 15 \times 6 = 98$

97 If m times the mth term of an AP is equal to n times its nth term, find the $(m + n)$th term of the AP.

ANS: Let Ist term = a, common difference = d.

$a_m = a + (m - 1)d$ and $a_n = a + (n - 1)d$

A.T.Q.,

$m \,.\, a_m = n \,.\, a_n \Rightarrow m\{a + (m - 1)d\} = n\{a + (n - 1)d\}$

$\Rightarrow ma - na = n(n - 1)d - m(m - 1)d \Rightarrow (m - n)a = (n^2 - n - m^2 + m)d$

$\Rightarrow (m - n)a = (n - m)(m + n - 1)d \Rightarrow a = -(m + n - 1)d$

Now $a_{m+n} = a + (m + n - 1)d = -(m + n - 1)d + (m + n - 1)d = 0$

98 If 9th term of an AP is zero, prove that its 29th term is double of its 19th term.

ANS: Let Ist term of AP be a and common difference be d.

Now, $a_9 = 0$

$\Rightarrow a + 8d = 0 \Rightarrow a = – 8d$ ...(i)

Now, $a_{29} = a + 28d = – 8d + 28d$ [Using eq. (i)]

$\Rightarrow a_{29} = 20d$ ...(ii)

Also, $a_{19} = a + 18d = – 8d + 18d = 10d$

$\Rightarrow 2 \times a_{19} = 2 \times 10d = 20d$ ...(iii)

From (ii) and (iii), we have

$a_{29} = 2 \times a_{19}$

99 Find the value of the middle term of the following AP: –6, –2, 2, ..., ………..58.

ANS: Here, $a = – 6$, $d = – 2 + 6 = 4$ and $a_n = 58$

$a_n = 58$

$\Rightarrow a + (n – 1)d = 58 \Rightarrow – 6 + (n – 1)4 = 58$

$\Rightarrow (n – 1)4 = 64 \Rightarrow n – 1 = 16 \Rightarrow n = 17$ (odd)

Middle term = $\frac{17+1}{2} = \frac{18}{2}$ = 9th term

9th term is the middle term.

Now, $a_9 = a + 8d = –6 + 8 \times 4 = –6 + 32 = 26$

100 Determine the AP whose fourth term is 18 and the difference of the ninth term from the fifteenth term is 30.

ANS: Given; $a_4 = 18$

$\Rightarrow a + 3d = 18$ ...(i)

and $a_{15} – a_9 = 30$

$\Rightarrow (15 – 9)d = 30 \Rightarrow 6d = 30 \Rightarrow d = 5$

Putting the value of d in (i), we have

$a + 3d = 18 \Rightarrow a + 3 \times 5 = 18$

$\Rightarrow a + 15 = 18 \Rightarrow a = 3$

Required AP is 3, 8, 13, ...

101 How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?

ANS: Numbers should be of the form $4m + 3$, i.e. 11, 15, 19, ..., 299

$a = 11$, $d = 4$, $a_n = 299$

$a_n = a + (n – 1)d$

$299 = 11 + (n – 1)4 \Rightarrow 299 – 11 = (n – 1)4$

$\Rightarrow \frac{288}{4} = n – 1 \Rightarrow 72 = n – 1 \Rightarrow 73 = n \Rightarrow n = 73$

102 If the pth, qth, rth terms of an AP be x, y, z respectively, show that $x(q – r) + y(r – p) + z(p – q) = 0$.

ANS: Let a be the first term and d be the common difference of the AP.

$t_p = x \Rightarrow a + (p – 1) d = x$ ...(i)

$t_q = y \Rightarrow a + (q – 1) d = y$ ...(ii)

$t_r = z \quad a + (r – 1) d = z$ ...(iii)

Substituting the values of x, y and z from (i), (ii) and (iii), we get

$x(q – r) + y(r – p) + z(p – q)$

$= [a + (p – 1)d] (q – r) + [a + (q – 1)d] (r – p) + [a + (r – 1)d] (p – q)$

$= a[(q – r) + (r – p) + (p – q)] + d [(p – 1) (q – r) + (q – 1)(r – p) + (r – 1) (p – q)]$

$= a(0) + d [p(q – r) + q (r – p) + r (p – q) – (q – r + r – p + p – q)] = d(0 – 0) = 0.$

103 The sum of n terms of an AP is $5n^2 - 3n$. Find the AP and also its 10th term.

ANS: Sum of n terms of given AP, $S_n = 5n^2 - 3n$

Sum of $(n - 1)$ terms of given AP,

$S_{n-1} = 5(n - 1)^2 - 3(n - 1)$

$\Rightarrow S_{n-1} = 5(n^2 - 2n + 1) - 3n + 3$

$= 5n^2 - 10n + 5 - 3n + 3 = 5n^2 - 13n + 8$

nth term of AP $= a_n = S_n - S_{n-1} = (5n^2 - 3n) - (5n^2 - 13n + 8)$

$\Rightarrow \boldsymbol{a_n} = 10n - 8$

1st term of AP $= 10 \times 1 - 8 = 2$

2nd term of AP $= 10 \times 2 - 8 = 12$ and 3rd term of AP $= 10 \times 3 - 8 = 22$

Required AP is 2, 12, 22, ...

$a_{10} = 10 \times 10 - 8 = 92$

104 Find the number of two-digit numbers which are divisible by 6.

ANS: Two digit numbers, divisible by 6 are 12, 18, 24, ..., 96

Here $a = 12$ and $d = 18 - 12 = 6$

$a_n = 96$

From formula, $a + (n - 1)d = a_n$, we get

$12 + (n - 1)6 = 96$

$(n - 1)6 = 96 - 12 = 84$

$\Rightarrow n - 1 = \frac{84}{6}$

$\Rightarrow n - 1 = 14$

$\Rightarrow n = 14 + 1 = 15$

105 In an AP, the first term is 12 and the common difference is 6. If the last term of the AP is 252, find its middle term.

ANS: Here $a = 12$, $d = 6$.

Let no. of terms $= n$

$a_n = 252$

$\Rightarrow a + (n - 1)d = 252$

$\Rightarrow 12 + (n - 1)6 = 252 \Rightarrow (n - 1)6 = 240$

$\Rightarrow n - 1 = 40 \Rightarrow n = 41$

Now middle term $= \frac{(41+1)}{2}$ = 21st term

$a_{21} = a + 20d = 12 + 20 \times 6 = 132$

106 Find the number of three-digit natural numbers which are divisible by 11.

ANS: Three-digit natural numbers divisible by 11 are 110, 121, 132, ..., 990

These form an AP with $a = 110$ and $d = 121 - 110 = 11$.

Last term = 990

$\Rightarrow a_n = 990$

$\Rightarrow a + (n - 1)d = 990$

$\Rightarrow 110 + (n - 1)11 = 990$

$\Rightarrow 11(n - 1) = 880$

$\Rightarrow n - 1 = 80$

$\Rightarrow n = 81$

There are 81 three-digit natural numbers which are divisible by 11.

107 Find $a_{30} - a_{20}$ for the AP

(i) – 9, – 14, – 19, – 24, ...

(ii) $a, a + d, a + 2d, a + 3d, ...$

ANS: (i) $a = -9$, $d = a_2 - a_1 = -14 - (-9) = -5$

$a_{30} - a_{20} = (a + 29d) - (a + 19d)$

$= 29d - 19d = 10d$

$= 10 \times (-5) = -50$

(ii) $a_1 = a$, $d = (a + d) - a = d$

$a_{30} - a_{20} = (a + 29d) - (a + 19\text{d}) = 10d$

108 The fifth term of an AP is 1, whereas its 31st term is –77. Which term of the AP is –17?

ANS: Let Ist term of the AP = a and common difference = d

Now $a_5 = 1 \Rightarrow\ a + 4d = 1$

$\Rightarrow\ a = 1 - 4d$ ...(i)

and $a_{31} = -77$

$\Rightarrow a + 30d = -77$

$\Rightarrow\ (1 - 4d) + 30d = -77$ [Using equation (i)]

$\Rightarrow 26d = -78 \Rightarrow d = -3$

When $d = -3$, equation (i) becomes,

$a = 1 - 4 \times (-3) = 13$

Let $a_n = -17$

$\Rightarrow a + (n - 1)d = -17$

$\Rightarrow 13 + (n - 1)(-3) = -17$

$\Rightarrow 13 - 3n + 3 = -17$

$\Rightarrow -3n = -33\ \Rightarrow n = 11$

11th term will be –17.

109 The 8th term of an Arithmetic Progression (AP) is 37 and its 12th term is 57. Find the AP.

ANS: Let Ist term of AP = a and common difference = d.

$t_n = a + (n - 1)\ d$

Now $t_8 = a + 7d$

$\Rightarrow\ 37 = a + 7d$ ...(i)

and $t_{12} = a + 11d$

$\Rightarrow 57 = a + 11d$ ...(ii)

Solving equations (i) and (ii), we get

$d = 5$ and $a = 2$

AP is 2, 7, 12, 17, ...

110 The 8th term of an arithmetic progression is zero Prove that its 38th term is triple of its 18th term.

ANS: Let Ist term = a, common difference = d.

$a_8 = 0 \Rightarrow a + 7d = 0$

$\Rightarrow a = -7d$

Now, $a_{18} = a + 17d = -7d + 17d = 10d$

$\Rightarrow 3 \times a_{18} = 30d$ ...(i)

Also, $a_{38} = a + 37d = -7d + 37d = 30d$ ...(ii)

From (i) and (ii), we get

$a_{38} = 3 \times a_{18}$ Hence proved.

111 The 19th term of an AP is equal to three times its sixth term. If its 9th term is 19, find the AP.

ANS: Let Ist term of the AP = a and common difference = d.

A.T.Q., $a_{19} = 3\,a_6$

$\Rightarrow a + 18d = 3(a + 5d)$

$\Rightarrow a = \frac{3}{2}d$ ....(i)

Also, $a_9 = 19 \Rightarrow a + 8d = 19$

$\Rightarrow \frac{3}{2}d + 8d = 19$ [Using eq. (i)]

$\Rightarrow 19d = 38 \Rightarrow d = 2$

When $d = 2$, equation (i) becomes

$a = \frac{3}{2} \times 2 = 3$

AP is 3, 5, 7, 9, ...

112 How many terms of the AP 3, 5, 7, ... must be taken so that the sum is 120?

ANS: Let the sum of n terms be 120.

Given AP is 3, 5, 7, ...

Here $a = 3$, $d = 2$ and $S_n = 120$.

Using

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

we get

$120 = \frac{n}{2}\{6 + (n-1)\,2\}$

$\Rightarrow 120 = n(n + 2)$

$\Rightarrow n^2 + 2n - 120 = 0$

$\Rightarrow (n + 12)(n - 10) = 0$

$\Rightarrow$Either $n + 12 = 0$ or $n - 10 = 0$

$\Rightarrow n = -12$ (rejected)

$n = 10$

10 terms must be taken to make sum 120.

113 Find the number of terms of the AP 54, 51, 48, ... so that their sum is 513.

ANS: $a = 54$, $d = 51 - 54 = -3$.

Let number of terms = n

Since, $S_n = 513$

$\Rightarrow S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow 513 = \frac{n}{2}[2 \times 54 + (n-1)(-3)]$

$\Rightarrow 513 \times 2 = n[108 - 3n + 3]$

$\Rightarrow 1026 = 111n - 3n^2$

$\Rightarrow 3n^2 - 111n + 1026 = 0$

$\Rightarrow n^2 - 37n + 342 = 0$

$\Rightarrow (n - 18)(n - 19) = 0$

$n = 18, 19$

$a_{19} = 54 + 18 \times (-3) = 0$

Sum of 18 or 19 terms = 513.

114 The sum of first six terms of an AP is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth terms of the AP.

ANS: Let the Ist term of the AP = a and common difference = d

Then $S_6 = \frac{6}{2}\{2a + (6 - 1)d\}$

$\Rightarrow 42 = 3(2a + 5d)$

$\Rightarrow 2a + 5d = 14$ ...(i)

Also, $\frac{a_{10}}{a_{30}} = \frac{1}{3}$

$\Rightarrow \frac{a+9d}{a+29d} = \frac{1}{3}$

$\Rightarrow 3a + 27d = a + 29d$

$\Rightarrow 2a = 2d$

$\Rightarrow a = d$

Substituting the value of d in equation (i), we get

$2a + 5a = 14 \Rightarrow a = 2$

$d = 2$

Now $a_{13} = a + 12d$

$\Rightarrow a_{13} = 2 + 12 \times 2 = 26$

Ist term = 2

Thirteenth term = 26a

115 In an AP, the sum of first ten terms is –150 and the sum of its next ten terms is –550. Find the AP.

ANS: Let a be the first term and d be the common difference.

Given $S_{10} = -150$

$-150 = \frac{10}{2}[2a + 9d]$

$\Rightarrow 2a + 9d = -30$ ...(i)

and $S_{20} - S_{10} = -550$

$\Rightarrow S_{20} = -550 - 150 = -700$

and $-700 = \frac{20}{2}[2a + 19d]$

$\Rightarrow 2a + 19d = -70$ ...(ii)

Solving (i) and (ii) for a and d, we get

$d = -4$ and $a = 3$

AP is 3, – 1, – 5, ...

116 How many multiples of 4 lie between 10 and 250? Also find their sum.

ANS: The multiples of 4 between 10 and 250 be 12, 16, 20, ..., 248.

Here, $a = 12$, $d = 16 - 12 = 4$ and $a_n = 248$. From formula, $a_n = a + (n - 1)d$, we get $12 + (n - 1)4 = 248$

$\Rightarrow 4(n - 1) = 248 - 12 \Rightarrow 4(n - 1) = 236 \Rightarrow n - 1 = \frac{236}{4} = 59 \Rightarrow n = 59 + 1 = 60$

$S_n = \frac{n}{2}(a + a_n) \Rightarrow S_{60} = 30(12 + 248) = 30 \times 260 = 7800$

117 If the sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

ANS: $S_6 = 36$ and $S_{16} = 256$.

$S_6 = 3[2a + 5d]$ $[\ S_n = \frac{n}{2}\{2a + (n-1)d\}]$

$S_6 = 3(2a + 5d)$

$\frac{36}{3} = 2a + 5d$

$12 = 2a + 5d$ ...(i)

and $S_{16} = \frac{16}{2}[2a + 15d]$

$\frac{256}{8} = 2a + 15d$

$32 = 2a + 15d$ ...(ii)

Subtracting (i) and (ii), $2a + 5d = 12$

$2a + 15d = 32$

$-\quad -\quad\quad -$

$-10d = -20 \Rightarrow d = 2$

From (i), $12 = 2a + 5(2)$

$12 - 10 = 2a \Rightarrow 2a = 2 \Rightarrow a = 1$

Hence, $S_{10} = \frac{10}{2}[2a + 9d] = 5(2 \times 1 + 9 \times 2) = 5(2 + 18)$

$S_{10} = 5 \times 20 = 100$.

118 150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped the second day, four more worker dropped the third day and so on. It takes 8 more days to finish the work now. Find the number of days in which the work was completed.

ANS: Let the number of days in which work was finished be n.

Number of workers on Ist day = 150

Number of workers on IInd day = 146

Number of workers on IIIrd day = 142 and so on

One day equivalent of all the worker

= 150 + 146 + 142 + ... up to n terms

$= \frac{n}{2}[2 \times 150 + (n-1) \times (-4)]$

$= \frac{n}{2}(304 - 4n) = 152n - 2n^2$ ...(i)

If 150 workers would have worked every day then number of days required to finish the work = $(n - 8)$

One day equivalent of workers = $150(n - 8) = 150n - 1200$ ...(ii)

From (i) and (ii), we have

$152n - 2n^2 = 150n - 1200$

$2n^2 - 2n - 1200 = 0$

$n^2 - n - 600 = 0$

$(n - 25)(n + 24) = 0$

$n = -24, n = 25$

Work was completed in 25 days.

119 Interior angles of a polygon are in AP. If the smallest angle is 120° and common difference is 5°, find the number of sides of the polygon.

ANS: Let number of sides of polygon be n.

The smallest angle = 120°.

angle are in AP

a = 120°

and common difference d = 5°

Angles are 120°, 125°, 130°, ... up to n terms

Now, sum of all the interior angles $\frac{n}{2}\{2a + (n-1)d\}$

$= \frac{n}{2}$ [2 × 120 + (n – 1)5] = $\frac{n}{2}$ (235 + 5n) ...(i)

Also, sum of interior angles of a polygon = (n – 2)(180) ...(ii)

From (i) and (ii), we get

$\frac{n}{2}$ (235 + 5n) = (n – 2)(180)

$235n + 5n^2 = 360n - 720$

$5n^2 - 125n + 720 = 0$

$5(n^2 - 25n + 144) = 0$

$n^2 - 25n + 144 = 0$

$(n - 9)(n - 16) = 0$

n = 9 or n = 16

When n = 16,

the sixteenth angle = $a + 15d$ = 120 + 15 × 5 = 195°

which cannot be an interior angle of a polygon.

n = 9.

120 Which term of the AP 121, 117. 113, ... is its first negative term?

ANS: Given A.P is 121, 117, 113, ...

a = 121

d = 117 – 121 = -4

$a_n = a + (n - 1)d$,

And, for some nth term is negative i.e., $a_n < 0$

121 + (n - 1) (-4) < 0

121 + 4 - 4n < 0

125 - 4n < 0

4n > 125

n > 125/4

n > 31.25

The integer which comes after 31.25 is 32.

∴ $32nd$ term in the A.P will be the first negative term.

121 With all round development in infrastructure METRO plays an important role in producing transport as well as beautifying the city. If we look around different designs of pillars have been erected between two stations. As class XI student I can consider two stations as two numbers and pillars erected at equal distances as means in sequence terms. Let as take the positions of two stations A and B as 1 km stone and 3 km stone and 14 pillars have been erected between two stations.

) If station A is considered first term then station B will be which term with respect to 14 pillars?

ii) What is the position of 14th means (14th pillars)?

iii) Sum of positions of 14 means (14 pillars) is

OR

iii) What is difference between two consecutive means (pillars)?

(i) Station B is 14 + 2 = 16th term

(ii) Position of 14th pillar $P_{14} = a_{15}$

$a_{16} - d = 3 - \frac{2}{15} = \frac{43}{15} km$

(iii) Sum of 14 means $\frac{17}{15} + \frac{19}{15} + \frac{21}{15} \dots \dots \frac{43}{15} = \frac{1}{15}\ (17 + 19 + \cdots .43) = \frac{1}{15} \times \frac{14}{2}\ (17 + 43) = 28$

(iii) $a_{16} = 3$, $a_1 = 1$ let difference = d

$3 = 1 + 15\,d$

$d = \frac{2}{15}$

122 Write the first 3 terms of each of the following sequences whose n^{th} term are:

(i) $a_n = 2n + 1$ (ii) $a_n = \frac{n-2}{3}$ (iii) $a_n = 5n$

(iv) $a_n = \frac{3n-2}{2}$ (v) $a_n = (-1)^n . 3n$ vi) $a_n = (2)^n$

ANS: (i) $a_n = 2n + 1$

$a_1 = 2 \times 1 + 1 = 3,$

$a_2 = 2 \times 2 + 1 = 5\,,$

$a_3 = 2 \times 3 + 1 = 7$

∴ the required first 3 terms of the sequence : 3, 5, 7

(ii) $a_n = \frac{n-2}{3}$

$a_1 = \frac{1-2}{3} = -\frac{1}{3},$

$a_2 = \frac{2-2}{3} = 0,$

$a_3 = \frac{3-2}{3} = \frac{1}{3}$

∴ the required first 3 terms of the sequence: $-\frac{1}{3}, 0\,, \frac{1}{3}$

(iii) $a_n = 5n$

$a_1 = 5 \times 1 = 5,$ $a_2 = 5 \times 2 = 10\,,$ $a_3 = 5 \times 3 = 15$

∴ the required first 3 terms of the sequence : 5, 10, 15

(iv) $a_n = \frac{3n-2}{2}$

$a_1 = \frac{3\times1-2}{2} = \frac{1}{2}$, $a_2 = \frac{3\times2-2}{2} = 2$, $a_3 = \frac{3\times3-2}{2} = \frac{7}{2}$

∴ the required first 3 terms of the sequence: $\frac{1}{2}$, 2, $\frac{7}{2}$

(v) $a_n = (-1)^n \,.\, 3n$

$a_1 = (-1)^1 \,.\, 3\times1 = -3,$ $a_2 = (-1)^2 \,.\, 3\times2 = 6$

$a_3 = (-1)^3 \,.\, 3\times3 = -9$

∴ the required first 3 terms of the sequence: $-3,\ 6, -9.$

vi) $a_n = (2)^n$ ANS: 2, 4, 8.

123 If the first term *a* is 10 and the common difference *d* is 4, then the AP is -----

ANS: 10, 14, 18, 22, ……..