TRIGONOMETRY - CLASS X (2025-26)

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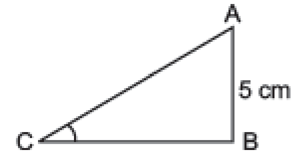
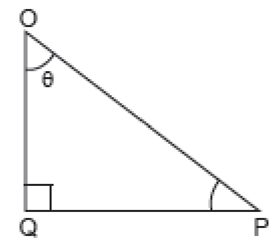
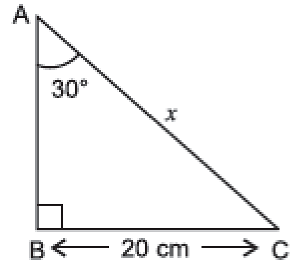
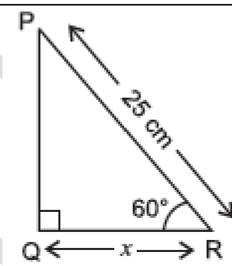
1	If $\sin\theta = \cos\theta$, then the value of θ is ______. $(0° \le \theta \le 90°)$
	ANS : 45°
2	If cosec θ – cot θ = $\frac{1}{3}$ the value of (cosec θ + cot θ) is ______. A) 1 B) 2 C) 3 D) 0
	ANS: C) 3
3	If $tanA = \frac{5}{12}$, find the value of $(sinA + \cos A) . secA$. (a) $\frac{13}{12}$ (b) $\frac{17}{12}$ (c) $\frac{12}{17}$ (d) $\frac{12}{13}$
	(b) $\frac{17}{12}$
4	If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $0° < A+B \le 90°$, $A > B$. A and B are _____ and _____.
	ANS: $A = 45°$, B =15°)
5	$4cot^2 45° - sec^2 60° + sin^2 60° + cos^2 90° =$ _________.
	ANS : $\frac{3}{4}$
6	If sec^2 θ (1 + sin θ) (1 – sin θ) = k, then find the value of k is _______.
	ANS: k = 1
7	The value of $\frac{1}{5}$ sec^{2}A – $\frac{1}{5}$ tan^{2}A = ______________. A) 5 B) 9 C) $\frac{1}{5}$ D) 0
	ANS: $\frac{1}{5}$
8	If $5\tan\theta = 4$, evaluate $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$
	ANS: $\tan\theta = 4/5$ $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{4-3}{4+2} = \frac{1}{6}$
9	Find the value of x , if $\tan 3x = \sin 45° \cdot \cos 45° + \sin 30°$.
	ANS: $\tan 3x = \sin 45° \cdot \cos 45° + \sin 30°$. $= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1 = \tan 45°$ $tan3x = 1 = tan\,45° \Rightarrow 3x = 45° \Rightarrow x = 15°$.
10	In Δ ABC, right angled at B, AB = 5 cm and $\sin C = \frac{1}{2}$. Determine the length of side AC
	ANS: sin C = $\frac{AB}{AC} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{5}{AC}$ AC = 10 cm 
11	If sec θ = $\frac{25}{7}$, find the values of tan θ and cosec θ.

	ANS : In ΔPQO, right angle at Q, sec θ = $\frac{OP}{OQ} = \frac{25}{7}$ So, $OP = 25k$ and $OQ = 7k$ $PQ^2 = OP^2 - OQ^2$ $= (25k)^2 - (7k)^2$ $= 625k^2 - 49k^2 = 576k^2$ PQ = $\sqrt{576\,k^2}$ = 24k tan θ = $\frac{PQ}{OQ} = \frac{27}{7}$ and cosec θ = $\frac{QP}{PQ} = \frac{25}{24}$	
12	In Δ ABC, right angle at B, if AB = 12 cm and BC = 5 cm, find (i) sin A and tan A, (ii) sin C and cot C.	
	(i) In ΔABC, right-angled at B, AC is the hypotenuse. Since, $AC^2 = BC^2 + AB^2$ $= 5^2 + 12^2 = 25 + 144 = 169$ AC = $\sqrt{169}$ = 13 cm Now, sin A = $\frac{BC}{AC} = \frac{5}{13}$ and tan A = $\frac{BC}{AB} = \frac{5}{12}$ (ii) sin C = $\frac{AB}{AC} = \frac{12}{13}$ and cot C = $\frac{BC}{AB} = \frac{5}{12}$	
13	Given A = 30°, verify sin 2A = 2 sin A cos A.	
	ANS: LHS = sin 2A = sin 2 × 30° = sin 60° = $\frac{\sqrt{3}}{2}$ RHS = 2 sin A cos A = 2 sin 30° cos 30° = $2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ So, LHS = RHS	
14	If tan θ = $\frac{1}{\sqrt{3}}$, $(0° \le \theta \le 90°)$ then evaluate $\frac{cosec^2\theta - sec^2\theta}{cosec^2\theta + sec^2\theta}$	
	ANS: tan θ = $\frac{1}{\sqrt{3}}$ θ = 30° $\frac{cosec^2\theta - sec^2\theta}{cosec^2\theta + sec^2\theta} = \frac{cosec^2 30 - sec^2 30}{cosec^2 30 + sec^2 30} = \frac{2^2 - \left(\frac{2}{\sqrt{3}}\right)^2}{2^2 + \left(\frac{2}{\sqrt{3}}\right)^2}$ $= \frac{4 - \frac{4}{3}}{4 + \frac{4}{3}} = \frac{1}{2}$	
15	If sin (A – B) = $\frac{1}{2}$, cos (A + B) = $\frac{1}{2}$, find A and B.	
	ANS: sin (A – B) = $\frac{1}{2}$ ⇒ A – B = 30° ...(*i*) and cos (A + B) = $\frac{1}{2}$, ⇒ A + B = 60° ...(*ii*) Solving equation (*i*) and (*ii*), we get	

	A = 45° and B = 15°
16	Simplified form of $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$ is ______. A) 1 B) $\frac{1}{2}$ C) $cot^2\theta$ D) $\tan^2 \theta$
	ANS A) 1
17	Find the value of x if $\cos 2x = \sin 60° \cdot \cos 30° - \cos 60° \cdot \sin 30°$. A) 60° B) 36° C) 27° D) 30°
	ANS: D) 30°
18	The value of $\frac{1-sin60°}{cos60°} =$ _____
	ANS: $\frac{1-sin60°}{cos60°} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$
19	If $\sec \theta - \tan \theta = \frac{1}{2}$ the value of $(\sec \theta + \tan \theta)$ is _____.
	ANS: $\sec \theta - \tan \theta = \frac{1}{2}$ Also, $\sec^2 \theta - \tan^2 \theta = 1$ $\Rightarrow (\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$ $\Rightarrow (\sec \theta + \tan \theta) \times \frac{1}{2} = 1 \Rightarrow \sec \theta + \tan \theta = 2$
20	If $\tan A = \frac{5}{12}$, find the value of $(\sin A + \cos A) . \sec A$.
	ANS: $(\sin A + \cos A). \sec A = \sin A . \sec A + \cos A . \sec A = \sin A \times \frac{1}{cosA} + 1$ $= \tan A + 1 = \frac{5}{12} + 1 = \frac{17}{12}$
21	If $cot\,\theta = \frac{7}{8}$ evaluate $\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\,\theta)}$
	ANS : $\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\,\theta)} = \frac{1-sin^2\theta}{1-cos^2\theta} = cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$
22	If $\sin \theta = \frac{1}{3}$, then find the value of $(2 \cot^2 \theta + 2)$
	ANS: $2(\cot^2 \theta + 1) = 2. \text{cosec}^2 \theta = \frac{2}{cosec^2\theta} = \frac{2}{1/9} = 18$
23	If $3x = \text{cosec}\,\theta$ and $\frac{3}{x} = \cot \theta$, find the value of $3\left(x^2 - \frac{1}{x^2}\right)$
	ANS: $3x = \text{cosec}\,\theta$ and $\frac{3}{x} = \cot \theta$ $3\left[x^2 - \frac{1}{x^2}\right] = 3\left[\left(\frac{\text{cosec}\,\theta}{3}\right)^2 - \left(\frac{\cot\theta}{3}\right)^2\right] = 3\left[\frac{\text{cosec}^2\theta - \cot^2\theta}{9}\right] = 3 \times \frac{1}{9} = \frac{1}{3}$
24	If $\sin \theta = x$ and $\sec \theta = y$, then find the value of $\cot \theta$.
	ANS: Here $\sec \theta = y \Rightarrow \cos \theta = \frac{1}{y}$ Now $\cot \theta = \frac{cos\theta}{sin\theta} = \frac{1/y}{x} = \frac{1}{xy}$
25	Find the value of x from the figure.

	ANS: Here, $\frac{BC}{AC}$ = sin 30° $\Rightarrow \frac{20}{x} = \frac{1}{2}$ $\Rightarrow$ x = 40 cm	
26	Find the value of x from the figure.	
	ANS: Here, $\frac{QR}{PR} = cos\ 60° \Rightarrow \frac{x}{25} = \frac{1}{2}$ $\Rightarrow$ $x = \frac{25}{2}$ = 12.5 cm	
27	Evaluate: 3 $\cot^2$ 60° + $\sec^2$ 45°	
	ANS: $3\cot^2 60° + \sec^2 45°$ = $3\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{2}\right)^2$ [cot 60° = $\frac{1}{\sqrt{3}}$ and sec 45° = $\sqrt{2}$] = 3 × $\frac{1}{3}$ + 2 = 3	
28	If sin A = $\frac{\sqrt{3}}{2}$ find the value of 2 $\cot^2$ A – 1.	
	ANS: sin A = $\frac{\sqrt{3}}{2}$ $\Rightarrow$ A = 60° Now 2 $\cot^2$ A – 1 = 2.$\cot^2$ 60° – 1 = $2 \times \left(\frac{1}{\sqrt{3}}\right)^2 - 1 = -\frac{1}{3}$	
29	If $cos\ (40° + x) = sin\ 30°$, find the value of x.	
	ANS: $cos\ (40° + x) = sin\ 30° = \frac{1}{2}$ So, $cos\ (40° + x) = \frac{1}{2}$ $\Rightarrow$ $40° + x = 60°$ Thus, $40° + x = 60°$ $\Rightarrow$ $x = 60° - 40° = 20°$	
30	By taking $A = 30°$, evaluate : $4cos^3A - 3cosA$	
	$cos^3A - 3cosA = 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3 \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$	
31	If $tanx = sin45° \cos 45° + sin30°$ then x = ______.	
	ANS: B) 45° $tanx = sin45° \cos 45° + sin30°$ $tanx = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} = 1 \Rightarrow x = 45°$	
32	If $A = 60°\ and\ B = 30°$, verify that $\sin(A + B) = sinA\ cosB + cosA\ sinB$	
	ANS: LHS = $\sin(A + B) = \sin(60° + 30°) = sin90° = 1$ $sinA\ cosB + cosA\ sinB = sin60° \cos 30° + cos60°\ sin30° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$	
33	If $a \cos\theta - b \sin\theta = x$ and $a \sin\theta + b \cos\theta = y$. Prove that $a^2 + b^2 = x^2 + y^2$.	
	ANS: Given. $a \cos\theta - b \sin\theta = x$	

	and $a \sin\theta + b\cos\theta = y$ To show. $a^2 + b^2 = x^2 + y^2$ Sol. RHS = $x^2 + y^2$ $= (a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2$ $= a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta + 2ab\cos\theta\sin\theta$ $= a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$ $= a^2(1) + b^2(1) = a^2 + b^2$ = LHS
34	Prove that : $\frac{sin\theta + cos\theta}{sin\theta - cos\theta} + \frac{sin\theta - cos\theta}{sin\theta + cos\theta} = \frac{2\,sec^2\theta}{tan^2\theta - 1}$
	$= \frac{sin^2\theta + cos^2\theta + 2sin\theta.cos\theta + sin^2\theta + cos^2\theta - 2sin\theta.cos\theta}{sin^2\theta - cos^2\theta}$ $= \frac{2}{sin^2\theta - cos^2\theta} = \frac{\frac{2}{cos^2\theta}}{\frac{sin^2\theta}{cos^2\theta} - 1} = \frac{2sec^2\theta}{tan^2\theta - 1}$
35	Prove that : $\frac{cosA}{1+sinA} + \frac{1+sinA}{cosA} = 2\,secA$
	LHS $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$ $= \frac{\cos^2 A + \sin^2 A + 1 + 2\sin A}{(1+\sin A)\cos A} = \frac{2(1+\sin A)}{(1+\sin A)\cos A}$ $= \frac{2}{\cos A}$ = **2 secA = RHS**
36	The value of $\frac{1}{2}\sin^2 A + \frac{1}{2}\cos^2 A$ = _____________.
	ANS: $\frac{1}{2}$
37	If $\tan(A+B) = \sqrt{3}$, $\tan(A-B) = \frac{1}{\sqrt{3}}$, $0 < A+B < 90$, $A > B$, find A and B.
	ANS: $\tan(A+B) = \sqrt{3} \Rightarrow A + B = 60°$ $\tan(A-B) = \frac{1}{\sqrt{3}}$ $\Rightarrow A - B = 30$ $2A = 90° \Rightarrow A = 45°,\ B = 15°$
38	If $\sec^2\theta(1+\sin\theta)(1-\sin\theta) = k$, then find the value of k.
	ANS: $\sec^2\theta(1+\sin\theta)(1-\sin\theta) = k$ $\Rightarrow \sec^2\theta(1-\sin^2\theta) = k$ $\Rightarrow \sec^2\theta . \cos^2\theta = k$ $\Rightarrow k = 1$
39	If $6x = \sec\theta$ and $\frac{6}{x} = \tan\theta$, find the value of $9\left(x^2 - \frac{1}{x^2}\right)$
	ANS: $6x = \sec\theta$ and $\frac{6}{x} = \tan\theta$ $\Rightarrow x = \frac{sec\theta}{6}$ and $\frac{1}{x} = \frac{tan\theta}{6}$ Consider, $9\left(x^2 - \frac{1}{x^2}\right) = 9\left(\frac{sec^2\theta}{36} - \frac{tan^2\theta}{36}\right) = \frac{9}{36}(\sec^2\theta - \tan^2\theta)$ $= \frac{1}{4} \times 1 = \frac{1}{4}$

40	If $3\tan\theta = 4$, evaluate $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$
	ANS: $3\tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{3}$ Now given expression is $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ Dividing numerator and denominator by cos θ and Putting $\tan\theta = \frac{4}{3}$ we get, we get $$\frac{5\tan\theta - 3}{5\tan\theta + 2} = \frac{5 \times \frac{4}{3} - 3}{5 \times \frac{4}{3} + 2} = \frac{11}{26}$$
41	In a triangle ABC, right angled at B, the ratio of AB to AC is $1 : \sqrt{2}$. Find the value of $\frac{2\tan A}{1+\tan^2 A}$
	$AB : AC = 1 : \sqrt{2}$. AB = x and AC = $\sqrt{2}\,x$ for some x. By Pythagoras Theorem, we have $AC^2 = AB^2 + BC^2$ $\Rightarrow (\sqrt{2}\,x\ ^2) = x^2 + BC^2$ $\Rightarrow BC^2 = 2x^2 - x^2 \quad \Rightarrow \quad BC = x$ $$\tan A = \frac{BC}{AB} = \frac{x}{x} = 1$$ $$\frac{2\tan A}{1+\tan^2 A} = \frac{2 \times 1}{1+1} = 1$$
42	If A = 60° and B = 30°, verify that sin (A – B) = sin A cos B – cos A sin B.
	ANS: A = 60°, B = 30° LHS = sin (A – B) = sin (60° – 30°) = sin 30° = $\frac{1}{2}$ ...(*i*) RHS = sin A. cos B – cos A . sin B = sin 60° . cos 30° – cos 60° . sin 30° $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$ ...(*ii*) From (*i*) and (*ii*), sin (A – B) = sin A . cos B – cos A . sin B
43	If $7\sin^2\theta + 3\cos^2\theta = 4$, then show that $\tan\theta = \frac{1}{\sqrt{3}}$
	If $7\sin^2\theta + 3\cos^2\theta = 4$, then show that $\tan\theta = \frac{1}{\sqrt{3}}$ Ans: $3 + 4\sin^2\theta = 4$ $3 = 4 - 4\sin^2\theta \quad \Rightarrow \frac{3}{4} = \cos^2\theta \quad \Rightarrow \quad \sec^2\theta = \frac{4}{3} \quad \Rightarrow 1 + \tan^2\theta = \frac{4}{3} \quad \tan\theta = \frac{1}{\sqrt{3}}$.
44	Find the value of x if $4\left(\frac{\sec^2 59° - \cot^2 31°}{3} - \frac{2}{3}\sin 90° + 3\tan^2 56° \times \tan^2 34°\right) = \frac{x}{3}$
	Ans: LHS= $4\left(\frac{\sec^2 59° - \tan^2(90-31°)}{3} - \frac{2}{3} \times 1 + 3\tan^2 56° \times \cot^2(90-34°)\right) = \frac{x}{3}$

	$$4\left(\frac{sec^2 59° - \ tan^2(59°)}{3} - \frac{2}{3} \times 1 + 3\,tan^2 56° \ \times cot^2(56°)\right) = \frac{x}{3}$$ $Simplify \quad and \quad x = 11$
45	Prove that : $(1 + cot\theta - cosec\theta\,)(1 + tan\theta + sec\theta) = 2$
	ANS: LHS= $\left(1 + \frac{cos\theta}{sin\theta} - \frac{1}{sin\theta}\right)\left(1 + \frac{sin\theta}{cos\theta} + \frac{1}{cos\theta}\right)$ Take LCM $\frac{sin^2\,\theta + cos^2\,\theta\ \ +2\,sin\theta\,cos\theta - 1}{sin\theta\,cos\theta}$ Simplify ans = 2
46	Prove that: $\frac{tan\theta + sec\theta - 1}{tan\theta - sec\theta + 1} = \frac{1 + sin\theta}{cos\theta}$
	ANS: $LHS = \frac{tan\theta + sec\theta - 1}{tan\theta - sec\theta + 1} =$ $= \frac{tan\theta + sec\theta - (sec^2\theta - tan^2\theta)}{tan\theta - sec\theta + 1}$ $= \frac{tan\theta + sec\theta - (\,tan\theta + sec\theta\,)(sec\theta - \tan\theta)}{tan\theta - sec\theta + 1}$ $= \frac{tan\theta + sec\theta(1 + \,tan\theta - sec\theta\,)}{tan\theta - sec\theta + 1}$ $= \ tan\theta + sec\theta \ = \frac{\sin\theta}{\csc\theta} + \frac{1}{cos\theta} = \frac{1 + sin\theta}{cos\theta} \ = RHS$
47	Prove that: $\frac{1}{(secx - tanx)} - \frac{1}{cosx} = \frac{1}{cosx} - \frac{1}{(secx + tanx)}$
	: using $sec^2x - tan^2x = 1$ LHS = $\frac{sec^2x - tan^2x}{(secx - tanx)} - \frac{1}{cosx}$ $= \frac{(secx - tanx)(secx + tanx)}{(secx - tanx)} - \frac{1}{cosx} = \ (secx + tanx) - \frac{1}{cosx}$ $= \frac{1}{cosx} + \frac{sinx}{cosx} - \frac{1}{cosx} = \frac{1 + \,sinx\,-1}{cosx}$ $= \ tanx$ RHS $= \frac{1}{cosx} - \frac{sec^2x - tan^2x}{(secx + tanx)}$ $= \frac{1}{cosx} - \frac{(secx + tanx)(secx - tanx)}{(secx + tanx)} = \frac{1}{cosx} - [secx + tanx] =$ $= \frac{1}{cosx} - [\frac{1}{cosx} + \frac{sinx}{cosx}]$

	$= tan x$
48	Evaluate : 4 $\cot^2 45° - \sec^2 60° + \sin^2 60° + \cos^2 90°$.
	ANS: $4 \cot^2 45° - \sec^2 60° + \sin^2 60° + \cos^2 90°$ $= 4 (\cot 45°)^2 - (\sec 60°)^2 + (\sin 60°)^2 + (\cos 90°)^2$ $= 4 \times (1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 0 = 4 - 4 + \frac{3}{4} + 0 = \frac{3}{4}$
49	Find the value of x if $\tan 3x = \sin 45°. \cos 45° + \sin 30°$.
	ANS: $\tan 3x = \sin 45° . \cos 45° + \sin 30°$ $\Rightarrow \tan 3x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$ $\Rightarrow \tan 3x = 1 = \tan 45°$ $\Rightarrow 3x = 45° \Rightarrow x = 15°$
50	Prove without using trigonometric tables: $\sin^2 5° + \sin^2 10° + ... + \sin^2 85° + \sin^2 90° = 9\frac{1}{2}$
	ANS: LHS $= \sin^2 5° + \sin^2 10° + ... + \sin^2 85° + \sin^2 90°$ $= (\sin^2 5° + \sin^2 85°) + (\sin^2 10° + \sin^2 80°) + ... + (\sin^2 40° + \sin^2 50°) + (\sin^2 45° + \sin^2 90°)$ $= \{\sin^2 5° + \cos^2 (90° - 85°)\} + \{\sin^2 10° + \cos^2 (90° - 80°\} + ... + \{\sin^2 40° + \cos^2 (90° - 50°) +$ $\left(\frac{1}{\sqrt{2}}\right)^2 + 1^2$ $= (1) + (1) + ...$ + 8 times $+ \frac{1}{2} + 1 = 8 + \frac{1}{2} + 1 = 9\frac{1}{2}$ = RHS
51	Prove without using trigonometric tables: $\tan 10° . \tan 75° . \tan 15° . \tan 80° = 1$
	ANS: LHS $= \tan 10° . \tan 75° . \tan 15° . \tan 80° = \tan 10° . \tan 75° . \cot (90° - 15°) . \cot (90° - 80°)$ $= \frac{1}{cot10} . \frac{1}{cot75} .\cot 75° . \cot 10° = 1$ = RHS
52	Without using the trigonometric tables, evaluate : $(\sin^2 25° + \sin^2 65°) + \sqrt{3}$ (tan 5° tan 15° tan 30° tan 75° tan 85°)
	ANS: $(\sin^2 25° + \sin^2 65°) + \sqrt{3}(\tan 5° \tan 15° \tan 30°. \tan 75°. \tan 85°)$ $= \sin^2 25° + \cos^2 (90° - 65°) + \sqrt{3} \tan 5°.\tan 15° \times \frac{1}{\sqrt{3}} \cot (90° - 75°).\cot (90° - 85°)$ $= \sin^2 25° + \cos^2 25° + \tan 5°.\tan 15° . \cot 15°. \cot 5° = 1 + \tan 5°.\tan 15° \times \frac{1}{tan15} \times \frac{1}{\tan 5} = 1 + 1 = 2$
53	Evaluate : $\cos^2 20° + \cos^2 70° + \sin 48° \sec 42° + \cos 40° \operatorname{cosec} 50°$.
	ANS: $\cos^2 20° + \cos^2 70° + \sin 48° . \sec 42° + \cos 40° . \operatorname{cosec} 50°$ $= \cos^2 20° + \sin^2 (90° - 70°) + \sin 48° . \operatorname{cosec} (90° - 42°) + \cos 40° . \sec (90° - 50°)$ $= \cos^2 20° + \sin^2 20° + \frac{1}{cosec48} . \operatorname{cosec} 48° + \frac{1}{sec40} \sec 40° = 1 + 1 + 1 = 3$
54	Simplify : $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$.
	ANS: $(1 + \tan^2 \theta) (1 - \sin \theta) (1 + \sin \theta) = \sec^2 \theta (1 - \sin^2 \theta)$ ($1 + \tan^2 \theta = \sec^2 \theta$) $= \sec^2 \theta \times \cos^2 \theta$ ($\cos^2 \theta + \sin^2 \theta = 1$)

$= \frac{1}{cos^2\theta} \times \cos^2\theta = 1$

55 If $\sec\theta + \tan\theta = m$ and $\sec\theta - \tan\theta = n$, find the value of $\sqrt{mn}$.

ANS: $\sec\theta + \tan\theta = m$ ...(*i*),

$\sec\theta - \tan\theta = n$ ...(*ii*)

Multiplying (*i*) and (*ii*), we get

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = m \,.\, n$

$\sec^2\theta - \tan^2\theta = mn \Rightarrow mn = 1 \Rightarrow \sqrt{mn} = \pm 1$

56 If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$, show that $q(p^2 - 1) = 2p$.

ANS: $\sin\theta + \cos\theta = p$, $\sec\theta + \text{cosec}\,\theta = q$

LHS $= q(p^2 - 1) = (\sec\theta + \text{cosec}\,\theta)\,[(\sin\theta + \cos\theta)^2 - 1]$

$= \left[\frac{1}{cos\theta} + \frac{sin\theta}{cos\theta}\right] [\sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta - 1]$

$= \left[\frac{cos\theta + sin\theta}{cos\theta}\right] [1 + 2\cos\theta\sin\theta - 1]$ ($\sin^2\theta + \cos^2\theta = 1$)

$= \left[\frac{cos\theta + sin\theta}{cos\theta}\right] \times 2\cos\theta\sin\theta$

$= 2(\sin\theta + \cos\theta) = 2p$ ($\sin\theta + \cos\theta = p$)

LHS = RHS. Hence proved.

57 Prove the following identity : $\frac{sin\theta}{1-cos\theta} + \frac{tan\theta}{1+cos\theta} = \sec\theta \,.\, \text{cosec}\,\theta + \cot\theta$

LHS= $\frac{sin\theta}{1-cos\theta} + \frac{tan\theta}{1+cos\theta}$

$= \frac{sin\theta}{1-cos\theta} + \frac{sin\theta}{cos\theta\,(1+cos\theta)}$

$= \frac{sin\theta.cos\theta\,(1+cos\theta)+sin\theta(1-cos\theta)}{(1-cos\theta)cos\theta\,(1+cos\theta)}$

$= \frac{sin\theta.cos\theta + sin\theta\,cos^2\theta + sin\theta - sin\theta(cos\theta)}{(1-cos\theta)cos\theta\,(1+cos\theta)}$

$= \frac{sin\theta\,cos^2\theta + sin\theta}{(1-cos^2\theta)cos\theta} = \frac{sin\theta\,cos^2\theta + sin\theta}{sin^2\theta\,cos\theta} = \frac{cos^2\theta + 1}{sin\theta\,.\,cos\theta} = \sec\theta \,.\, \text{cosec}\,\theta + \cot\theta$

58 Prove the following identity : $\cos^4 A - \cos^2 A = \sin^4 A - \sin^2 A$

ANS: LHS $= \cos^4 A - \cos^2 A = \cos^2 A(\cos^2 A - 1) = (1 - \sin^2 A)(-\sin^2 A)$

$= \sin^4 A - \sin^2 A =$ RHS.

59 Prove the following identity : $\sin^4 A + \cos^4 A = 1 - 2\sin^2 A\cos^2 A$

ANS: LHS $= \sin^4 A + \cos^4 A = (\sin^2 A)^2 + (\cos^2 A)^2$

$= (\sin^2 A + \cos^2 A)^2 - 2\sin^2 A \,.\, \cos^2 A$ [Using $a^2 + b^2 = (a + b)^2 - 2ab$]

$= 1 - 2\sin^2 A \,.\, \cos^2 A =$ RHS

60 Prove that: $\frac{cos\theta}{1-sin\theta} = \frac{1+sin\theta}{cos\theta}$

	ANS: $\frac{cos\theta}{1-sin\theta} = \frac{cos\theta}{1-sin\theta} \times \frac{1+sin\theta}{1+sin\theta} = \frac{cos\theta\,(1+sin\theta)}{1-sin^2\theta} = \frac{cos\theta\,(1+sin\theta)}{cos^2\theta} = \frac{1+sin\theta}{cos\theta}$
61	Prove the following identity : If cos θ – sin θ = 1, show that cos θ + sin θ = 1 or – 1.
	ANS: $(\cos\theta - \sin\theta)^2 = (1)^2$ $\Rightarrow \cos^2\theta + \sin^2\theta - 2\sin\theta \, . \cos\theta = 1$ $\Rightarrow 2\sin\theta \, . \cos\theta = 0$ ...(*i*) Now, $(\cos\theta + \sin\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta . \cos\theta$ $\Rightarrow (\cos\theta + \sin\theta)^2 = 1 + 0$ [Using (*i*)] $\Rightarrow \cos\theta + \sin\theta = \pm 1$
62	If $a\cos\theta - b\sin\theta = x$ and $a\sin\theta + b\cos\theta = y$. Prove that $a^2 + b^2 = x^2 + y^2$.
	ANS: Given. $a\cos\theta - b\sin\theta = x$ and $a\sin\theta + b\cos\theta = y$ To show. $a^2 + b^2 = x^2 + y^2$ Sol. RHS = $x^2 + y^2$ $= (a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2$ $= a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta + 2ab\cos\theta\sin\theta$ $= a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$ $= a^2(1) + b^2(1) = a^2 + b^2$ = LHS
63	If A = 60° and B = 30°, verify that cos (A – B) = cos A cos B + sin A sin B.
	ANS: A = 60°, B = 30 ° LHS = cos (A – B) = cos (60° – 30°) = cos 30° = $\frac{\sqrt{3}}{2}$ RHS = cos A cos B + sin A sin B = cos 60° cos 30° + sin 60° sin 30° $\frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$ LHS = RHS. Hence verified.
64	Determine the value of x such that $2\,\text{cosec}^2 30° + x\sin^2 60° - \frac{3}{4}\tan^2 30° = 10$
	ANS: $2\,\text{cosec}^2 30° + x\sin^2 60° - \frac{3}{4}\tan^2 30° = 10 \Rightarrow 2 \times 2^2 + x \times \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} \times \left(\frac{1}{\sqrt{3}}\right)^2 = 10$ $\Rightarrow 8 + x \times \frac{3}{4} - \frac{1}{4} = 10 \Rightarrow x = 3$
65	If sin 3θ = cos (θ – 6°) where 3θ and (θ – 6°) are acute angles, find the value of θ.
	ANS: sin 3θ = cos (θ – 6°) ⇒ cos (90° – 3θ) = cos (θ – 6°) ⇒ 90° – 3θ = θ – 6° ⇒ 90° + 6° = 4θ ⇒ θ = $\frac{96}{4}$ = 24°
66	Prove that $(cosec\theta - \cot\theta)^2 = \frac{1-\,cos\theta}{1+\,cos\theta}$
	ANS: LHS = $((cosec\theta - \cot\theta)^2$ $(cosec\theta - \cot\theta)^2 = \left(\frac{1}{sin\theta} - \frac{\cos\theta}{sin\theta}\right)^2 = \frac{(1-\,cos\theta)^2}{sin^2\theta} = \frac{(1-\,cos\theta)^2}{1-cos^2\theta} = \frac{1-\,cos\theta}{1+\,cos\theta}$ = RHS

67	If 5 sin θ + 3 cos θ = 4, find the value of 3 sin θ – 5 cos θ.
	ANS: 5 sin θ + 3 cos θ = 4 $\Rightarrow$ $(5 \sin\theta + 3 \cos\theta)^2 = (4)^2$ $\Rightarrow$ $25 \sin^2\theta + 9 \cos^2\theta + 30 \sin\theta \cos\theta = 16$ $\Rightarrow$ $25(1 - \cos^2\theta) + 9(1 - \sin^2\theta) + 30 \sin\theta \cos\theta = 16$ $\Rightarrow$ $(3 \sin\theta - 5 \cos\theta)^2 = 18$ $\Rightarrow$ $3 \sin\theta - 5 \cos\theta = \pm 3\sqrt{2}$
68	Prove that: $\frac{tanA}{secA-1} + \frac{tanA}{secA+1} = 2\ cosec\ A.$
	$$\frac{tanA}{secA-1} + \frac{tanA}{secA+1} = \frac{tanA(secA+1) + tanA(secA-1)}{(secA-1)(secA+1)}$$ $$= \frac{tanA \times secA + tanA + tanA \times secA - tanA}{(secA-1)(secA+1)}$$ $= \frac{2tanA \times secA}{(sec^2A-1)} = \frac{2tanA \times secA}{(tan^2A)} = 2 \times \frac{1}{cosA} \times \frac{cosA}{sina} = 2\ cosec\ A$
69	If 2 cos θ – sin θ = x and cos θ – 3 sin θ = y, prove that $2x^2 + y^2 - 2xy = 5$.
	ANS: 2 cos θ – sin θ = x ...(i) cos θ – 3 sin θ = y LHS = $2x^2 + y^2 - 2xy$ $= 2[2\cos\theta - \sin\theta]^2 + (\cos\theta - 3\sin\theta)^2 - 2(2\cos\theta - \sin\theta)(\cos\theta - 3\sin\theta)$ $= 2(4\cos^2\theta + \sin^2\theta - 4\cos\theta . \sin\theta) + \cos^2\theta + 9\sin^2\theta - 6\cos\theta\sin\theta - 2(2\cos^2\theta - 6\sin\theta\cos\theta - \sin\theta \cos\theta + 3\sin^2\theta)$ $= 8\cos^2\theta + 2\sin^2\theta - 8\cos\theta . \sin\theta + \cos^2\theta + 9\sin^2\theta - 6\cos\theta . \sin\theta - 4\cos^2\theta + 14\sin\theta . \cos\theta - 6\sin^2\theta$ $= 5\cos^2\theta + 5\sin^2\theta - 14\cos\theta\sin\theta + 14\cos\theta . \sin\theta$ $= 5(\cos^2\theta + \sin^2\theta) = 5 \times 1 = 5$ = RHS
70	Find the value of sin 38° – cos 52°
	ANS: sin 38° – cos 52° = sin 38° – sin (90° – 52°) [cos A = sin(90° – A] = sin 38° – sin 38° = 0
71	An electic pole is 10 m high. A steel wire fixed to the top of the pole is affixed at a point on the ground to keep the pole upright. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.
	ANS: $10\sqrt{2}$ m