CIRCLES

CLASS X (2025-26)

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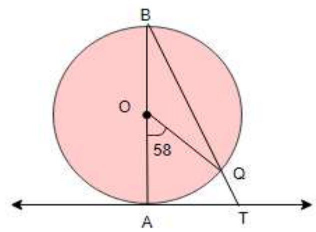
SECTION- A

1 In figure, AB is the diameter of a circle with centre O and AT is a tangent. If ∠ AOQ = 58°, find ∠ ATQ.

A) 51°   B) 58°   C) 71°   D) 61°



ANS: ∠ ABQ = $\frac{1}{2}$ × 58° = 29°

In Δ ABT,

∠ BAT + ∠ ABT + ∠ ATB = 180°

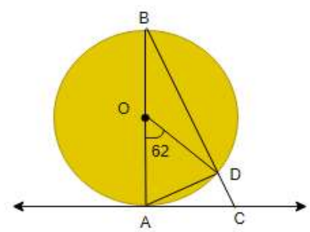
90° + 29° + ∠ ATB = 180°

∠ ATB = 61°   as ∠ ATB = ∠ ATQ ⇒ ∠ATQ = 61°

2 In the figure, AB is the diameter of a circle with centre O and AC is a tangent. If ∠ AOD = 62°, find ∠ ACD.

(A) 51°   (B) 60°

(C) 59°   (D) 61°

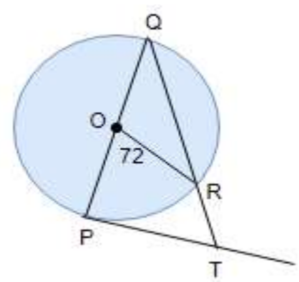


ANS: (C) 59°

3 In the given figure, PQ is a diameter of a circle with centre O and PT is a tangent at P, QT meets the circle at R. If ∠POR = 72° then ∠PTR= ____

A) 52°   (B) 60°

(C) 54°   (D) 64°

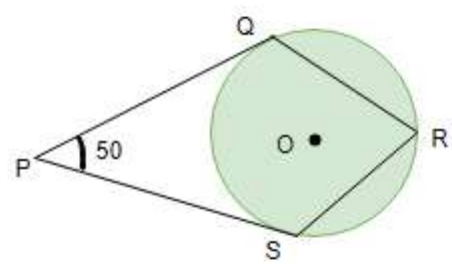


ANS: 54°

4 In the figure , O is the centre of the circle and PQ and PS are tangents to the circle at points Q and S respectively.

$\angle QRS =$ _____

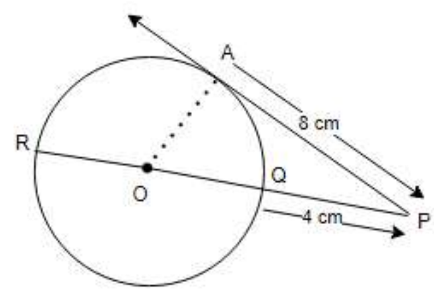
A) 65 °   B) 130 °   C) 55 °   D) 100 °



ANS: A) 65 °

5 In the figure, O is the centre of the circle and PA is tangent to the circle from the point P. PQR passes through the centre of the circle O. If PA = 8 cm, PQ = 4 cm, find the radius of the circle.

A) 3 cm B) 6 cm C) 12 cm D) 10 cm



ANS: Let OA = r cm

$OA^2 + AP^2 = OP^2$

$r^2 + 8^2 = (r+4)^2$

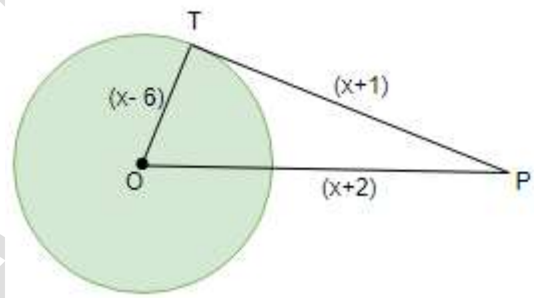
$r^2 + 8^2 = r^2 + 4^2 + 8r$

$64 - 16 = 8r \Rightarrow$ r = 6 . The radius of the circle 6 cm.

6 In the below figure, find the area of Δ OTP.

A) $30\ cm^2$ (B) $60\ cm^2$

(C) $15\ cm^2$ (D) $40\ cm^2$



ANS: $(x+1)^2 + (x-6)^2 = (x+2)^2$

$x^2 + 1 + 2x + x^2 + 36 - 12x = x^2 + 4 + 4x$

$x^2 - 14x + 33 = 0$

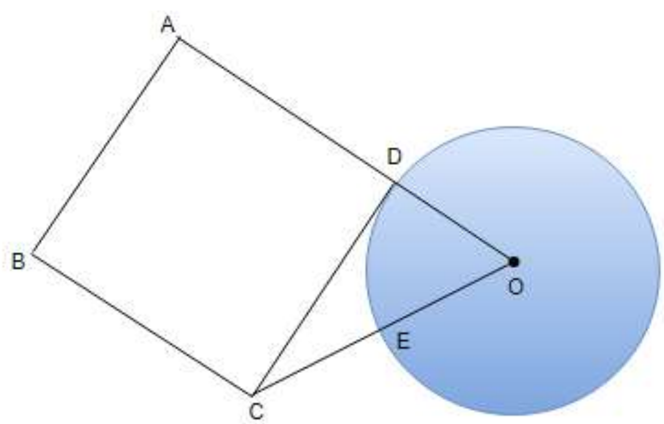
$(x-3)(x-11) = 0$

$x = 11, \quad x \neq 3$ sides are 5, 12,13

Area = $\frac{1}{2} \times 5 \times 12 = 30\ cm^2$

7 ABCD is a square , CD is a tangent to the circle with centre O. if OD = CE , the find the ratio of the area of circle to that of square.

A) $\frac{2\pi}{3}$ B) $\frac{3}{\pi}$ C) $\frac{\pi}{9}$ D) $\frac{\pi}{3}$



ANS: D) $\frac{\pi}{3}$

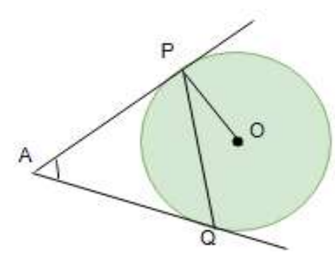
8 Tangents AP and AQ are drawn to a circle with centre O from external point A then _____

A) $\angle PAQ = 2\angle OPQ$

B) $\angle PAQ = \angle OPQ$

C) $\angle PQA = \angle OPA$

D) $\angle PQA = 2\angle OPA$



A) $\angle PAQ = 2\angle OPQ$

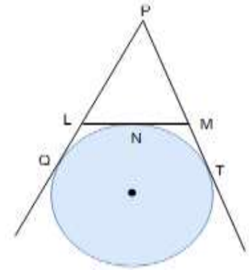
9 The distance between two parallel tangents of a circle of radius 10 cm is ____

A) 20 cm B) 10 cm C) 15 cm D) 5 cm

ANS: A) 20 cm

10 In the figure, If $PQ = 30\ cm$, then find the perimeter of $\Delta\ PLM$.

A) $30\ cm$ B) $60\ cm$ C) $40\ cm$ D) $35\ cm$



ANS: $PQ = PT$

$PL + LQ = PM + MT$

$\Rightarrow\ PL + LN = PM + MN$

perimeter of $\Delta\ PLM = PL + LN + MN + PM = PQ + PT = 2 \times PQ = 60cm.$

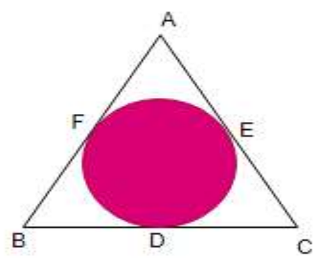
11 The two tangents from an external point P to a circle with centre O are PA and PB. If ∠ APB = x°, what is the value of ∠ AOB?

A) x ° B) $(180 - x\,)$° C) 90° D) $(90 - x\,)$°

ANS: B) $(180 - x\,)$°

12 A triangle ABC is drawn to circumscribe a circle. If AB = 13 cm, BC = 14 cm and AE = 7 cm, then AC is equal to ____

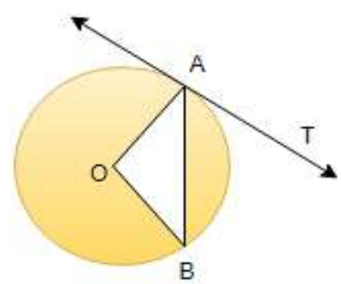
(A) 12 cm (B) 15 cm (C) 11 cm (D) 16 cm



ANS: (B) 15 cm

13 In given figure, O is the centre of the circle, AB is a chord and AT is the tangent at A. If ∠ AOB = 100° then find ∠ BAT .

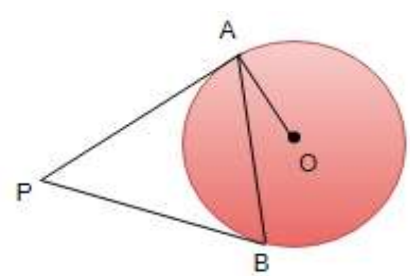
A) 100° B) 40° C) 50° D) 90°



ANS: C) 50°

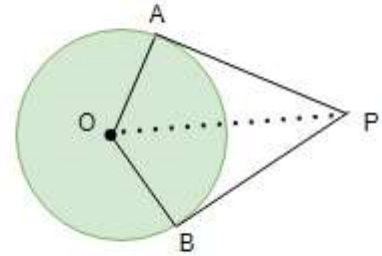
14 In the figure PA and PB are tangents to the circle with centre O. If ∠APB = 70°, then ∠OAB is ____

A) 35° B) 70° C) 30° D) 15°



ANS: A) 35°

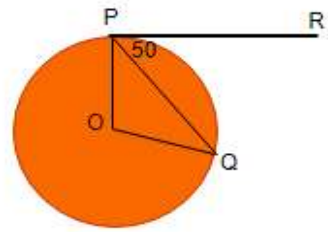
15 If angle between two tangents drawn from a point to a circle of radius a and centre O is 60° . then OP = ______



A) $\sqrt{3}a$ B) $\frac{a}{\sqrt{3}}$ C) $\frac{2a}{\sqrt{3}}$ D) $\frac{a}{2}$

ANS: $\frac{2a}{\sqrt{3}}$

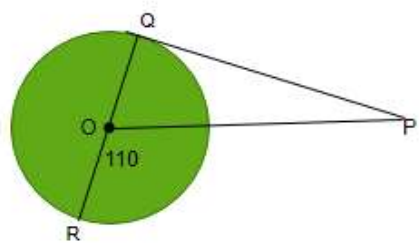
16 In figure if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then ∠POQ is equal ____



A) 100° B) 80° C) 90° D) 75°

ANS: A) 100°

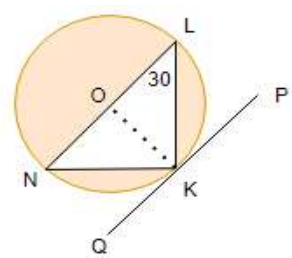
17 PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠POR = 110°. Find ∠OPQ.



A) 10° B) 20° C) 30° D) 25°

ANS: 20°

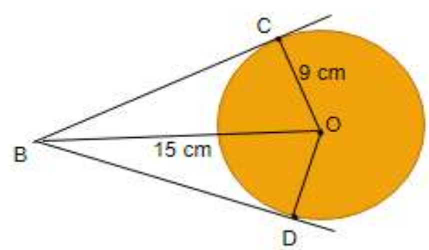
18 In Figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and ∠KLN = 30°, find ∠PKL.



A) 30° B) 20° C) 60° D) 25°

ANS: 60°

19 In the figure, BC and BD are tangents to the circle with centre O and radius 9 cm. If OP = 15 cm , then the length of (BC + BD) = _____ cm

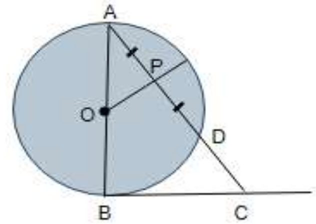


(A) 18 (B) 12
(C) 24 (D) 21

ANS: (C) 24

20 In the figure, O is the centre of the circle. BC is a tangent to the circle at B. If OP bisects the chord AD and $\angle AOP = 60°$, Then find $\angle C$.

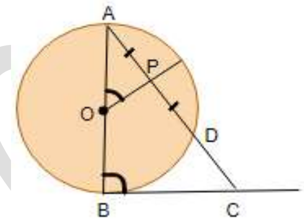
A) 60° B) 30° C) 45° D) 25°



ANS: OP bisects the chord AD ⇒ $OP \perp AD$

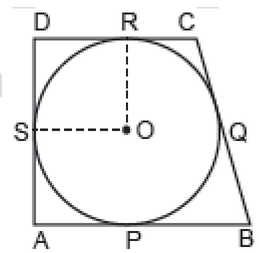
$\angle OPA = 90°$ and $\angle AOP = 60°$ ⇒ $\angle A = 30°$

In $\Delta ABC, \angle C = 180° - 120° = 60°$



21 A quadrilateral ABCD is drawn so that ∠ D = 90°, BC = 38 cm and CD = 25 cm. A circle is inscribed in the quadrilateral and it touches the side AB, BC, CD and DA at P, Q, R and S respectively. If BP = 27 cm, find the radius of the inscribed circle.

A) 100° B) 105° C) 130° D)1 25°



ANS: To find: $\angle 1 + \angle 2$

Solution: OQ is perpendicular to the tangent PQ. (radius is perpendicular to the tangent at the point of contact).

In Δ OPQ, ∠ POR = ∠ 1 + ∠ RQP (external angle is equal to the sum of interior opposite angles)

⇒ 130° = ∠ 1 + 90° ⇒ ∠ 1 = 40° ....(i)

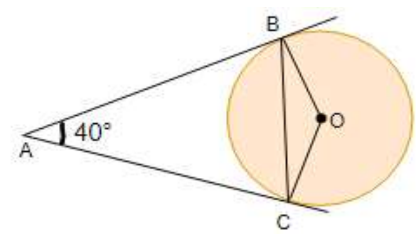
∠ ROT is the angle subtended by arc RT at centre and ∠ RST is the angle subtended by same arc at point S on circumference.

$\angle 2 = \frac{1}{2} \times \angle ROT = \frac{1}{2} \times 130° = 65°$ ....(ii)

Adding (i) and (ii), we get ∠ 1 + ∠ 2 = 40° + 65° = 105°

22 In the given figure, AB and AC are tangents to the circle with centre O such that ∠ BAC = 40°, then ∠ BOC is equal to ____.

A) 130° B) 120° C) 140° D) 150°



ANS: In quadrilateral ABOC

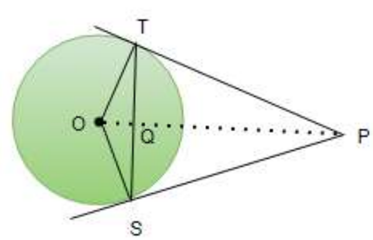
ABO + ∠ BOC + ∠ OCA + ∠ BAC = 360º

⇒ 90º + ∠ BOC + 90º + 40º = 360º

⇒ ∠ BOC = 360º – 220º = 140º

23 In figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r . If OP = $2r$ ∠ OTS = ∠ OST = ______

A) 20° B) 40° C) 30° D) 25°



ANS: In Δ OTS , OT = OS [radii]

$\Rightarrow \angle OTS = \angle OST$ ....(i)

In right Δ OTP , $\frac{OT}{OP} = \sin \angle TPO$

$\Rightarrow \frac{r}{2r} = \sin \angle TPO$

$\Rightarrow \sin \angle TPO = \frac{1}{2} \quad \Rightarrow \angle TPO = 30°$

Similarly $\angle$ OPS = 30°

$\Rightarrow \angle TPS = 30° + 30° = 60°$

Also $\angle TPS + \angle SOT = 180°$

$\angle SOT = 120°$

In Δ SOT,

$\angle SOT + \angle OTS + \angle OST = 180°$

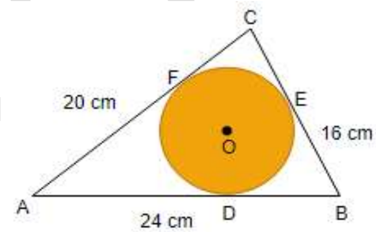
$120° + 2\angle OTS = 180°$

$\Rightarrow \angle OTS = 30°$ ...(ii)

From (i) and (ii)

$\angle OTS = \angle OST = 30°$

24 A circle is inscribed in a ΔABC having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF



ANS: Let $AD = AF = x$ [Tangents from external point are equal]

$BD = BE = y$ and $CE = CF = z$

According to the question,

AB = x + y = 24 cm ... (i)

BC = y + z = 16 cm ... (ii)

AC = x + z = 20 cm ...(iii)

Subtracting (iii) from (i), we get

y – z = 4 ... (iv)

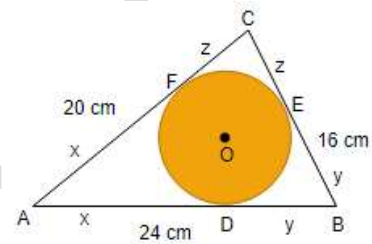
Adding (ii) and (iv), we get

$2y = 20 \Rightarrow y = 10\ cm.$

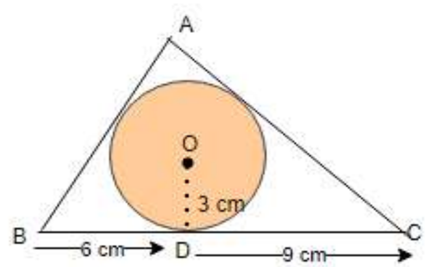
Substituting the value of y in (ii) and (i) we get

z = 6 cm; x = 14 cm

AD = 14 cm, BE = 10 cm and CF = 6 cm.



25 In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of Δ ABC is 54 cm^2, then find the lengths of sides AB and AC.



Let AF = x cm

AF = AE = x [tangents from A]

Also BD = BF = 6 cm and CD = CE = 9 cm

AB = (6 + x) cm and AC = (9 + x) cm

Area Δ ABC = Area ∆BOC + Area ∆COA + Area ∆AOB

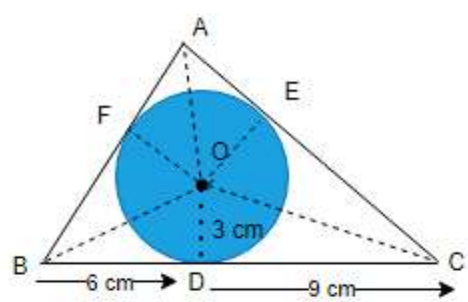
$\Rightarrow 54 = \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE + \frac{1}{2} AB \times OF$

$\Rightarrow 54 \times 2 = 15 \times 3 + (6 + x) \times 3 + (9 + x) \times 3$

$108 = 45 + 18 + 3x + 27 + 3x$

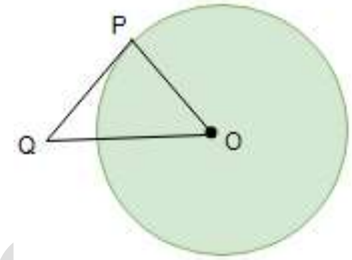
$6x = 18 \Rightarrow x = 3$

$\Rightarrow$ AB = 6 + x = 6 + 3 = 9 cm and AC = 9 + x = 9 + 3 =



12 cm

26 PQ is a tangent to a circle with centre O at point P. If $\Delta\, OPQ$ is isosceles triangle, then find $\angle\, OQP$.

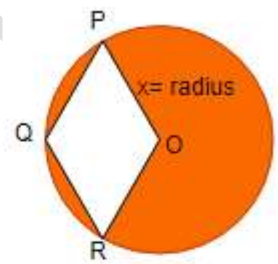


ANS: $\Delta\, OPQ$ is isosceles triangle , $\angle Q = \angle O$ and QP is tangent $\Rightarrow\ \angle P = 90°$

$\angle Q + \angle O + \angle P = 180°$

$\Rightarrow\ 2\angle Q = 90°\ \Rightarrow\ \angle\, OQP = 45°$

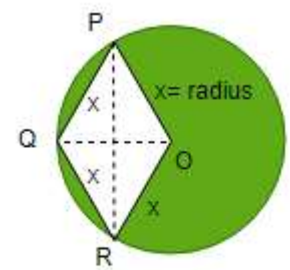
27 In the given figure, OPQR is a rhombus, three of whose vertices lie on a circle with centre O. If the area of the rhombus is $32\sqrt{3}$ cm^2, find the radius of the circle.



Side of the rhombus = radius of the circle

OP = OQ = PQ = x = OR = QR

ΔORQ and ΔOPQ are equilateral triangles



Area of the equilateral triangle OPQ = $\frac{\sqrt{3}}{4}\, a^2$

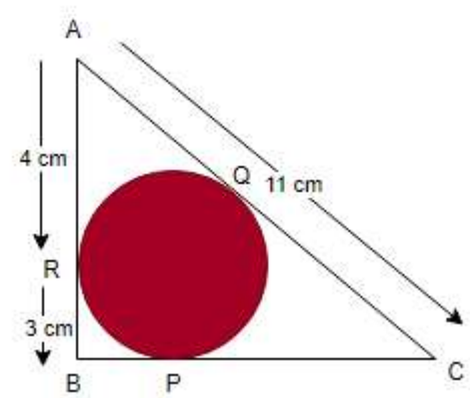
Area of the equilateral ΔOPQ = $\frac{\sqrt{3}}{4}\, x^2$

Area of the rhombus = 2 ar (ΔOPQ)

$2 \times \frac{\sqrt{3}}{4}\, x^2 = \frac{\sqrt{3}}{2}\, x^2 = 32\sqrt{3}$

$x^2 = 64$ cm^2 $\quad x = 8$ cm

28 In figure, Δ ABC is circumscribing a circle. Find the length of BC.



ANS: AR = 4 cm

Also, AR = AQ ⇒ AQ = 4 cm

Now, QC = AC – AQ = 11 cm – 4 cm = 7 cm ...(i)

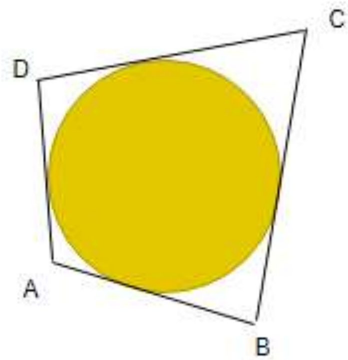
Also, BP = BR

BP = 3 cm and PC = QC

PC = 7 cm [From (i)]

BC = BP + PC = 3 cm + 7 cm = 10 cm

29 In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.



ANS: If a circle touches all the four sides of quadrilateral ABCD, then

AB + CD = AD + BC

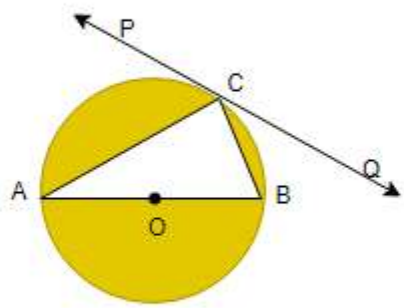
6 + 8 = AD + 9

⇒14 = AD + 9

⇒14 – 9 = AD

⇒AD = 5 cm

30 In figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and ∠ CAB = 30°, find ∠ PCA.



In Δ AOC,

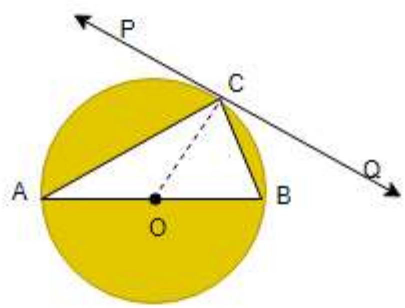
AO = CO [Radii]

∠ OCA = ∠ CAB = 30°

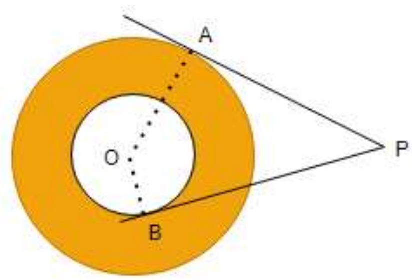
OC ⊥ PQ, ⇒ ∠ OCP = 90°

⇒ ∠ PCA + ∠ OCA = 90°

⇒ ∠ PCA + 30° = 90° ⇒ ∠ PCA = 60°



31 Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radius 8 cm and 5 cm respectively, as shown in the figure. If AP = 15 cm, then BP = _____

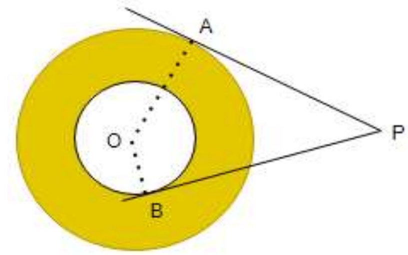


A) $2\sqrt{66}$ B) $4\sqrt{66}$ C) $2\sqrt{33}$ D) $\sqrt{66}$

ANS: $OP = \sqrt{8^2 + 15^2} = \sqrt{289} = 17$

$PB = \sqrt{17^2 - 5^2} = \sqrt{264} = 2\sqrt{66}$

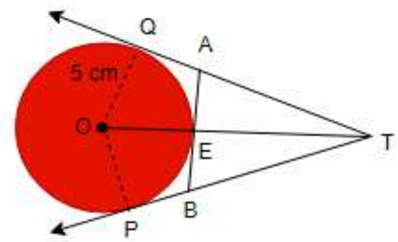
32 In figure, there are two concentric circles with centre of radii 5 cm and 3 cm. Tangents PA and PB are drawn from an external point P to these circles with centre O as shown in the figure. If AP = 12 cm , then BP = _____



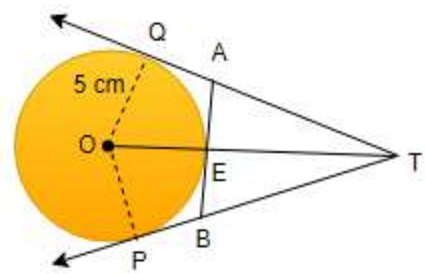
A) $\sqrt{160}$ B) $\sqrt{150}$ C) 10 D) $2\sqrt{10}$

ANS: A) $\sqrt{160}$

33 In figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. if AB is a tangent to the circle at E, find the length of AB, where TP and TQ are tangents to the circle.



ANS: $TQ = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$



$TQ = PT = 12\ cm$

Let AE = AQ $= x$

AT = $12 - x,$ ET = 13 -5 = 8

$AT^2 = AE^2 + ET^2$

$(12 - x)^2 = (x)^2 + 8^2$

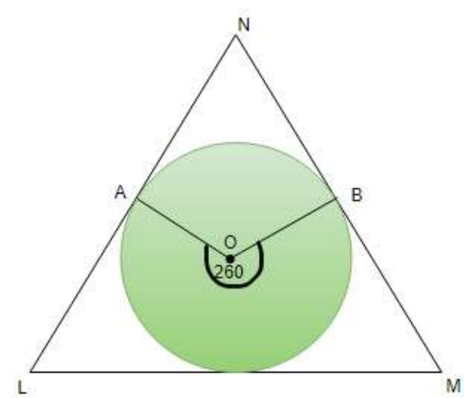
$$144 + x^2 - 24x = x^2 + 64$$

$$x = \frac{80}{24} = \frac{10}{3}$$

AB = $2 \times \frac{10}{3} = \frac{20}{3}\ cm$

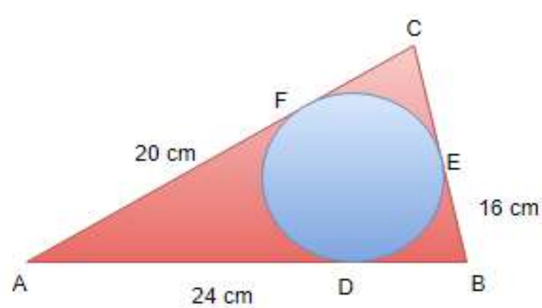
34 A circle with centre O is inscribed in a triangle LMN . A and B are points of tangency. Reflex $\angle AOB = 260°$, find ANB.

A) 50° B) 100° C) 80° D) 130°

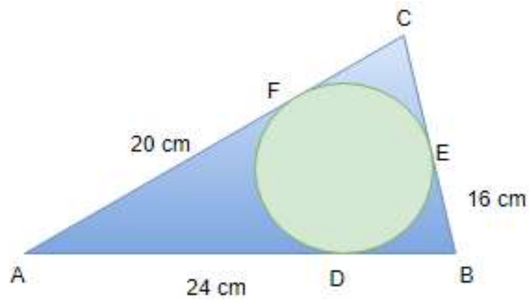


ANS: C) 80°

35 A circle is inscribed in a ΔABC having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF.



ANS: Let AD = AF = x [Tangents from external point are equal]



BD = BE = y and CE = CF = z

According to the question,

AB = $x + y$ = 24 cm ... (i)

BC = $y + z$ = 16 cm ... (ii)

AC = $x + z$ = 20 cm ...(iii)

Subtracting (iii) from (i), we get

$y - z = 4$ ... (iv)

Adding (ii) and (iv), we get

$2y = 20 \Rightarrow y = 10$ cm.

Substituting the value of y in (ii) and (i) we get $z = 6$ cm; $x = 14$ cm

AD = 14 cm, BE = 10 cm and CF = 6 cm.

SECTION- B

1 Two parallel lines touch the circle at points A and B respectively. If area of the circle is 25 π cm^2, then AB is equal to ____________

(A) 5 cm (B) 8 cm (C) 10 cm (D) 25 cm

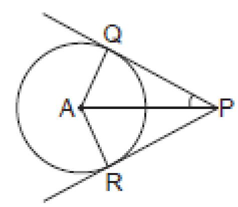
ANS: (c) Let radius of circle = R

$\therefore \pi R^2 = 25\pi$

$\Rightarrow$ R = 5 cm

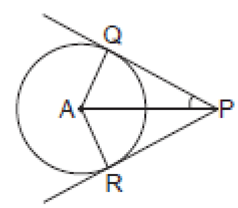
$\therefore$ Distance between two parallel tangents = diameter = 2 × 5 = 10 cm.

2 In figure, PQ and PR are tangents to a circle with centre A. If ∠QPA = 27°, then ∠QAR equals to__



(A) 63° (B) 153° (C) 126° (D) 117°

ANS: (C) 126° ∠QPA = ∠RPA



[∵ Δ AQP ≅ ΔARP (RHS congruence rule)]

⇒ ∠RPA = 27°

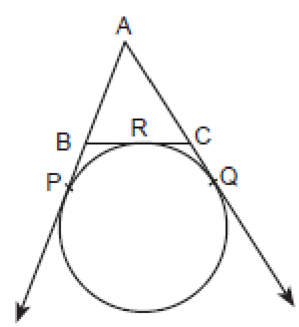
∴ ∠QPR = ∠QPA + ∠RPA = 27° + 27° = 54° Now,

∠QAR + ∠AQP + ∠ARP + ∠QPR = 360°

⇒ ∠QAR = 90° + 90° + 54° = 360°

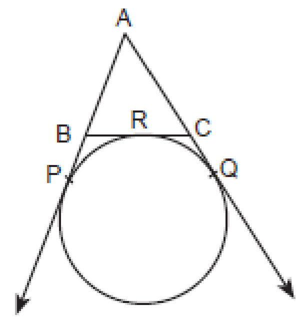
⇒ ∠QAR = 360° – 234° = 126°

3 In figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then the length of AP (in cm) is _____



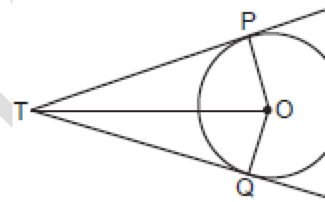
(A) 7.5 (B) 15 (C) 10 (D) 9

ANS: (A) AP = AQ



⇒ AB + BP = AC + CQ
⇒ 5 + BP = 6 + CQ
BP = 1 + CQ
BP = 1 + CR
(∵ CQ = CR)
BP = 1 + (BC – BR)
BP = 1 + (4 – BP) (∵ BR = BP)
2 BP = 5 ⇒ BP = $\frac{5}{2}$ = 2.5 cm
Now, AP = AB + BP = 5 + 2.5 = 7.5 cm

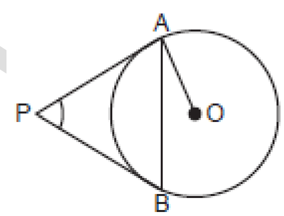
4 In the given figure, TP and TQ are two tangents to a circle with centre O, such that ∠ POQ = 110°. Then ∠ PTQ is equal to _____



(a) 55° (b) 70° (c) 110° (d) 90°

ANS: (A) In quadrilateral POQT,
∠ PTQ + ∠ TPO + ∠ TQO + ∠ POQ = 360°
⇒ ∠ PTQ + 90° + 90° + 110° = 360°
⇒ ∠ PTQ + 290° = 360°
⇒ ∠ PTQ = 360° – 290° = 70°

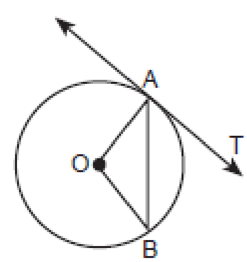
5 In the figure PA and PB are tangents to the circle with centre O. If ∠ APB = 60°, then ∠ OAB is____



(A) 30° (B) 60° (C) 90° (D) 15°

ANS: (A) Given ∠ APB = 60°
∵ ∠ APB + ∠ PAB + ∠ PBA = 180°
⇒ ∠ APB + x + x = 180°
[∵ PA = PB ∴ ∠ PAB = ∠ PBA = x (say)]
⇒ 60° + 2x = 180°
⇒ 2x = 180° – 60°
⇒ 2x = 120°
⇒ x = 60°
Also, ∠ OAP = 90°
⇒ ∠ OAB + ∠ PAB = 90°
⇒ ∠ OAB + 60° = 90°
⇒ ∠ OAB = 30°

6 In figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If ∠ AOB = 100°, then ∠ BAT is equal to ______

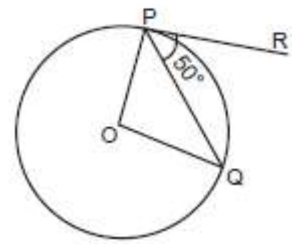


(A) 100° (B) 40° (C) 50° (D) 90°

ANS: (C) AOB = 100°
∠ OAB = ∠ OBA (∵ OA and OB are radii)
Now, in Δ AOB,

∠ AOB + ∠ OAB + ∠ OBA = 180° (Angle sum property of Δ)
⇒ 100° + x + x = 180° [Let ∠ OAB = ∠ OBA = x]
⇒ 2x = 180° – 100° ⇒ 2x = 80° ⇒ x = 40°
Also, ∠ OAB + ∠ BAT = 90°
[∵ OA is radius and TA is tangent at A]
⇒ 40° + ∠ BAT = 90° ⇒ ∠ BAT = 50°

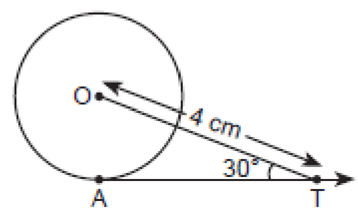
7 In figure if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then ∠ POQ is equal to_____



(A) 100° (B) 80° (C) 90° (D) 75°

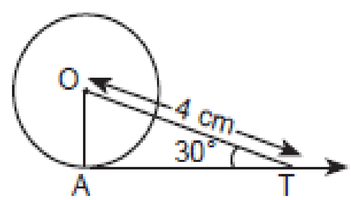
ANS: (A) OP ⊥ PR [∵ Tangent and radius are ⊥ to each other at the point of contact]
∠ OPQ = 90º – 50º = 40º
OP = OQ [Radii]
∴ ∠ OPQ = ∠ OQP = 40º
In ΔOPQ,
⇒ ∠ POQ + ∠ OPQ + ∠ OQP = 180º
⇒ ∠ POQ + 40º + 40º = 180º
∠ POQ = 180° – 80° = 100º

8 In figure AT is a tangent to the circle with centre O such that OT = 4 cm and ∠ OTA = 30°. Then AT is equal to _______



(A) 4 cm (B) 2 cm (C) $2\sqrt{3}$ cm (D) $4\sqrt{3}$ cm

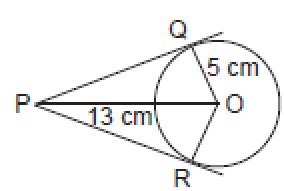
ANS: $\angle OAT = 90°$



In $\Delta\, OAT,\ \frac{AT}{OT} = cos30°$

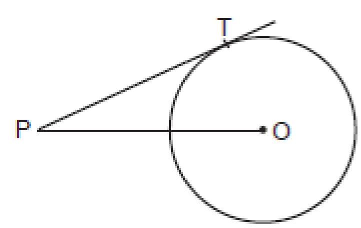
$\frac{AT}{4} = \frac{\sqrt{3}}{2} \Rightarrow AT = 2\sqrt{3}$ cm

9 From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is _____
(A) 60 cm^2 (B) 65 cm^2 (C) 30 cm^2 (D) 32.5 cm^2
ANS: (A) 60 cm^2



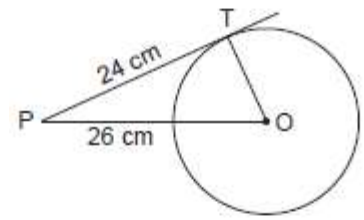
$PQ = \sqrt{OP^2 - OQ^2}$
$PQ = 12$
area of the quadrilateral PQOR = $ar\ \Delta\, POQ + ar\ of\ \Delta\, POR$
$\frac{1}{2} \times 12 \times 5 + \frac{1}{2} \times 12 \times 5 = 60\ cm^2$

10 In the given figure, point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 24 cm. Then the radius of the circle is ____



(A) 25 cm (B) 26 cm (C) 24 cm (D) 10 cm

ANS: (D) ∵ OT is radius and PT is tangent ∴ OT ⊥ PT



Now, in ΔOTP,

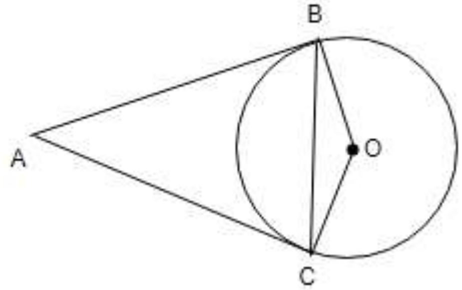
$OP^2 = PT^2 + OT^2$

$\Rightarrow 26^2 = 24^2 + OT^2$

$\Rightarrow 676 - 576 = OT^2$

$\Rightarrow 100 = OT^2 \Rightarrow OT = 10$ cm

11 In the given figure, AB and AC are tangents to the circle with centre O such that ∠ BAC = 40°, then ∠ BOC is equal to____



(A) 40° (B) 50° (C) 140° (D) 150°

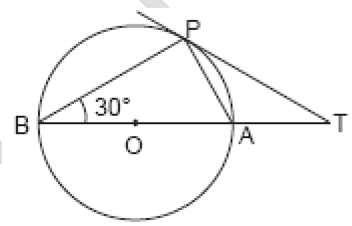
ANS: (C) In quadrilateral ABOC

∠ABO + ∠BOC + ∠ OCA + ∠ BAC = 360º

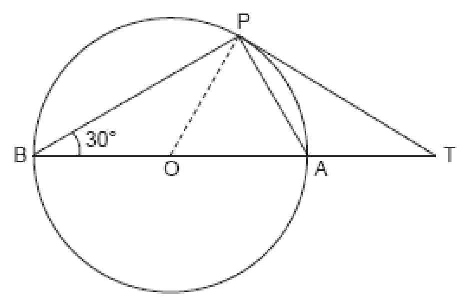
⇒ 90º + ∠ BOC + 90º + 40º = 360º

⇒ ∠ BOC = 360º – 220º = 140º

12 In figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If ∠ PBT = 30°, prove that BA : AT = 2 : 1.



ANS: ∠ BPA = 90° (Angle in semicircle)



In Δ BPA,

∠ABP + ∠BPA + ∠PAB = 180°

30° + 90° + ∠ PAB = 180°

∠ PAB = 60°

Also ∠ POA = 2∠ PBA

∠ POA = 2 × 30° = 60°

OP = AP (sides opposite to equal angles) ...(i)

In Δ OPT, ∠ OPT = 90°

∠ POT = 60° and ∠ PTO = 30° [angle sum property]

Also ∠ APT + ∠ ATP = ∠ PAO (exterior angle property)

∠ APT + 30° = 60°

APT = 30°

AP = AT (sides opposite to equal angles) ... (ii)

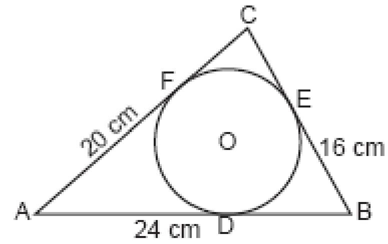
From (i) and (ii) AT = OP = radius of the circle;

and AB = $2r$

AB = 2AT $\frac{AB}{AT} = 2$

AB : AT = 2 : 1

13 A circle is inscribed in a Δ ABC having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF



ANS: Let AD = AF = x [Tangents from external point are equal]

BD = BE = y and CE = CF = z

According to the question,

AB = $x + y$ = 24 cm ... (i)

BC = $y + z$ = 16 cm ... (ii)

AC = $x + z$ = 20 cm ...(iii)

Subtracting (iii) from (i), we get

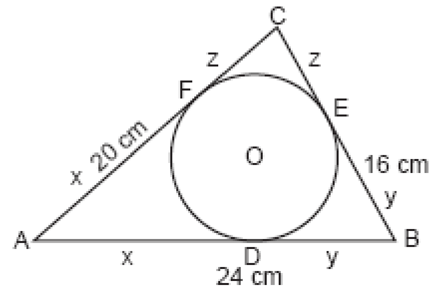
$y - z = 4$ ... (iv)

Adding (ii) and (iv), we get

$2y = 20 \Rightarrow y = 10$ cm.

Substituting the value of y in (ii) and (i) we get $z = 6$ cm; $x = 14$ cm

AD = 14 cm, BE = 10 cm and CF = 6 cm.



14 A circle touches x-axis at A and y-axis at B. If O is origin and OA = 5 units, then diameter of the circle is __

(A) 8 units (B) 10 units (C) $10\sqrt{2}$ units (D) $8\sqrt{2}$ units

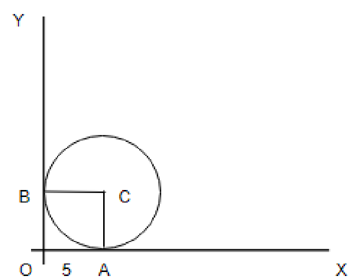
ANS: (B) OA = OB ⇒ OB = 5

AC = BC [Radii]

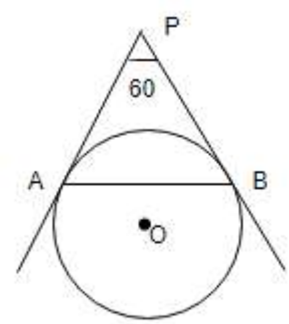
⇒ OACB is a square.

⇒ AC = OA = 5

⇒ Diameter = 10 units.



15 In figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and ∠ APB = 60°. Find the length of chord AB.



ANS: AP = BP [tangents from external point P]

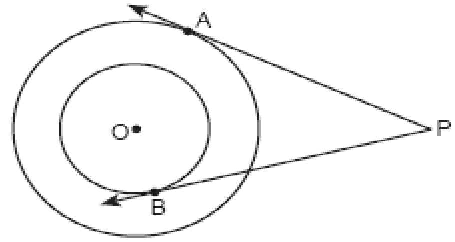
∠ PAB = ∠ PBA [Angles opposite to equal sides]

Now ∠ APB + ∠ PAB + ∠ PBA = 180°

60° + 2 ∠ PAB = 180°

∠ PAB = 60° ⇒ Δ APB is an equilateral ⇒ AB = AP = 5 cm

16 In figure, there are two concentric circles, with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.



PA = 12 cm, OA = 5 cm, OB = 3 cm

$OP^2 = OA^2 + AP^2 = OB^2 + BP^2$

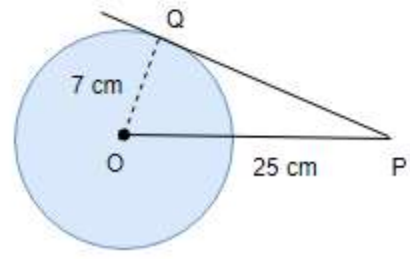
$25 + 144 = 9 + BP^2$

$169 – 9 = BP^2$

$\Rightarrow BP = \sqrt{160}$ cm = 12.65 cm. (Approx.)

17 Find the length of the tangent drawn from a point whose distance from the centre of a circle is 25 cm. Given that radius of the circle is 7 cm.

Let O is the centre of the circle and P is a point such that OP = 25 cm and PQ is the tangent to the circle.



OQ = radius = 7 cm

In Δ OQP, we have Q = 90°

$OP^2 = OQ^2 + PQ^2$

$\Rightarrow (25)^2 = 7^2 + PQ^2 \Rightarrow PQ^2 = 625 – 49 = 576$

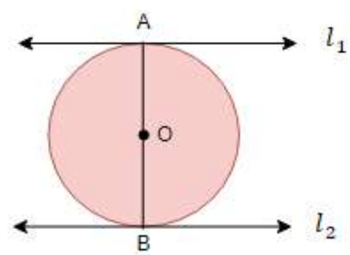
$\Rightarrow$ PQ = 24 cm Hence, the length of the tangent = 24 cm

18 What is the angle between a tangent to a circle and the radius through the point of contact? Justify your answer.

ANS: 90°. Because radius through point of contact of tangent to a circle is perpendicular to the tangent.

19 What is the distance between two parallel tangents of a circle of radius 7 cm?

Two parallel tangents of a circle can be drawn only at the end points of the diameter

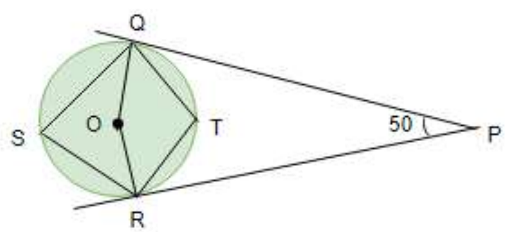


$\Rightarrow l_1 \parallel l_2$

$\Rightarrow$ Distance between l_1 and l_2 = AB = Diameter of the circle

$= 2 \times r = 2 \times 7$ cm = 14 cm

20 In the given figure, O is the centre . find ∠ QSR



ANS: Given: PQ and PR are tangents to a circle with centre O and ∠ QPR = 50°.

To find: ∠ QSR

Sol. ∠ QOR + ∠ QPR = 180°

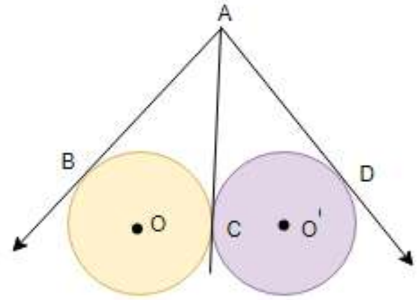
∠ QOR + 50° = 180°

$\Rightarrow$ ∠ QOR = 130°

⇒ ∠ QSR = $\frac{1}{2}$ ∠ QOR [Degree measure theorem]

⇒ ∠ QSR = $\frac{1}{2} \times 130° = 65°$

21 In the given figure, AB, AC and AD are tangents. If AB = 5 cm, find AD



ANS: Given: AB, AC and AD are tangents. AB = 5 cm.

To find: AD

Sol. AB and AC are tangents from the same point to the circle with centre O.

⇒ AB = AC ...(*i*)

(Length of the tangents from the same external point are equal).

AC and AD are tangents from the same point to the circle with centre O.

⇒ AC = AD ...(*ii*)

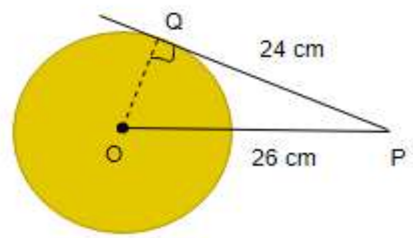
(Length of the tangents from the same external point are equal)

From (*i*) and (*ii*)

AB = AC = AD = 5 cm

22 A point P is 26 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

Let O is the centre of the circle and PQ is the tangent from P. A.T.Q., OP = 26 cm and PQ = 24 cm



In Δ OQP, we have ∠ Q = 90°

$OP^2 = OQ^2 + PQ^2$

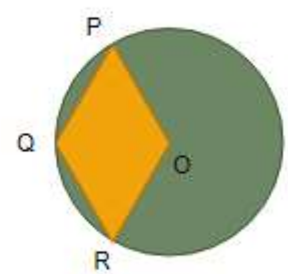
⇒ $(26)^2 = OQ^2 + (24)^2$

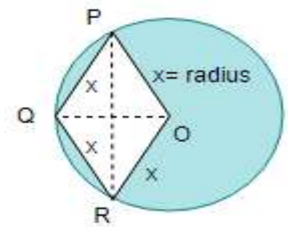
⇒ $OQ^2 = 676 - 576 = 100$

⇒ OQ = 10 cm

Radius of the circle = 10 cm

23 In the given figure, OPQR is a rhombus, three of whose vertices lie on a circle with centre O. If the area of the rhombus is $32\sqrt{3}$ cm^2, find the radius of the circle.





Side of the rhombus = radius of the circle

OP = OQ = PQ = x = OR = QR

ΔORQ and ΔOPQ are equilateral triangles

Area of the equilateral triangle OPQ = $\frac{\sqrt{3}}{4}a^2$

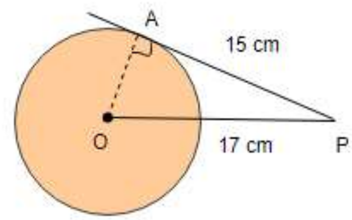
Area of the equilateral ΔOPQ = $\frac{\sqrt{3}}{4}x^2$

Area of the rhombus = 2 ar (ΔOPQ)

$2 \times \frac{\sqrt{3}}{4}x^2 = \frac{\sqrt{3}}{2}x^2 = 32\sqrt{3}$

$x^2 = 64\ cm^2$ $\quad x = 8$ cm

24 From a point P, the length of the tangent to a circle is 15 cm and distance of P from the centre of the circle is 17 cm. Then what is the radius of the circle?



∠ OAP = 90°

⇒ $172 = r^2 + 15^2$ [By Pythagoras theorem]

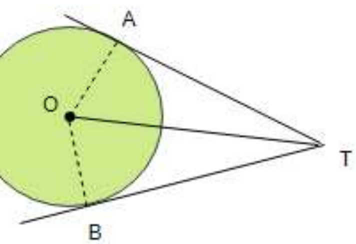
⇒ $r^2 = 17^2 - 15^2 = (17 - 15)(17 + 15) = 2 \times 32$

⇒ $r^2 = 64$ ⇒ $r = \pm 8$ cm

Ignoring negative value as length cannot be negative.

⇒ $r = 8$ cm

25 In figure , O is the centre of the circle , if ∠ ATO = 40°, find ∠ AOB.



ANS: In ΔOAT, ∠ ATO = 40°, ∠ OAT = 90°

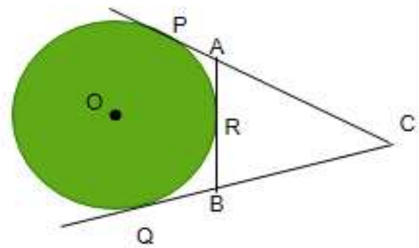
∠ AOT = 50° [Angle sum property]

Now ∠ BTO = 40° as OT bisects ATB

Similarly, ∠ BOT = 50°

∠ AOB = ∠ AOT + ∠ BOT = 50° + 50° = 100°

26 In figure, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, and BC = 7 cm, then find the length of BR.



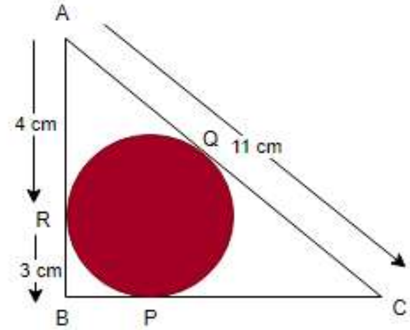
ANS: Since CP = CQ = 11cm [Length of the two tangents from same external point]

CQ = CB + BQ

But BQ = BR

Therefore, 11 = 7 + BR ⇒ BR = 4 cm

27 In figure, Δ ABC is circumscribing a circle. Find the length of BC.



ANS: AR = 4 cm

Also, AR = AQ ⇒ AQ = 4 cm

Now, QC = AC – AQ = 11 cm – 4 cm = 7 cm ...(i)

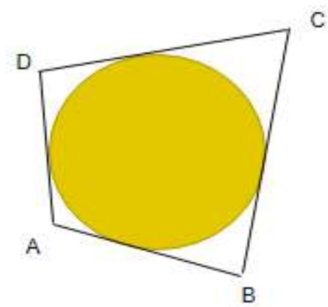
Also, BP = BR

BP = 3 cm and PC = QC

PC = 7 cm [From (i)]

BC = BP + PC = 3 cm + 7 cm = 10 cm

28 In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.



ANS: If a circle touches all the four sides of quadrilateral ABCD, then

AB + CD = AD + BC

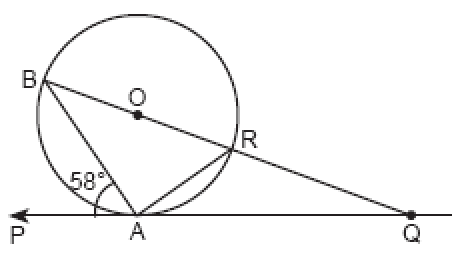
6 + 8 = AD + 9

⇒14 = AD + 9

⇒14 – 9 = AD

⇒AD = 5 cm

29 In figure, O is the centre of the circle, PQ is a tangent to the circle at A. If ∠ PAB = 58°, find ∠ ABQ and ∠ AQB.



Join OA.

OA ⊥ PAQ.

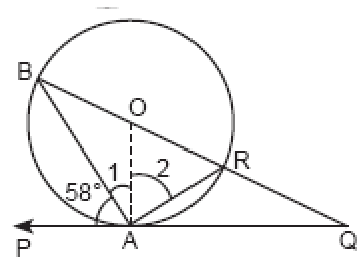
∠ OAP = 90°

⇒∠ 1 + 58° = 90°

⇒∠ 1 = 90° – 58° = 32°

In ΔBOA, OA = OB.

Now ∠ 1 = ∠ ABQ



⇒∠ ABQ = 32°

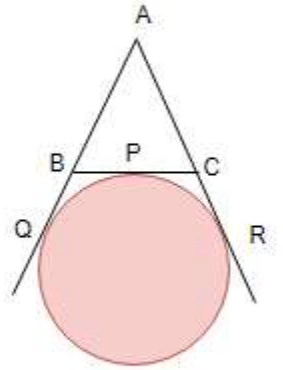
∠ PAB + ∠ BAQ = 180° ⇒∠ BAQ = 180° – 58° = 122°

In ΔABQ,

∠ ABQ + ∠ BAQ + ∠ AQB = 180°

∠ AQB = 180° – 122° – 32° = 26°

30 In figure, a circle touches the side BC of ΔABC at P and touches AB and AC produced at Q and R respectively. If AQ = 5 cm, find the perimeter of Δ ABC.



ANS: Given: A circle touches the side BC of ΔABC at P and touches AB and AC produced at Q and R respectively and AQ = 5 cm.

To find: Perimeter of ΔABC.

Sol.: AQ and AR are tangents from the same point

AQ = AR = 5 cm ...(i) [Tangents from the same external points are equal]

BQ and BP are tangents from same point

⇒BQ = BP ...(ii)

CP and CR are also tangents from the same point

⇒ CP = CR ...(iii)

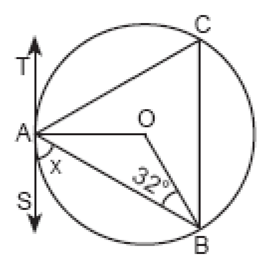
In Δ ABC, Perimeter of Δ ABC = AB + BC + AC = AB + BP + CP + AC

AB + BQ + CR + AC = AQ + AR [From (ii) and (iii)]

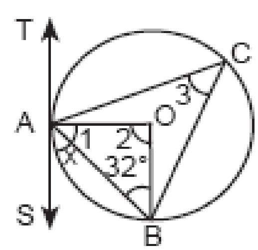
= 5 cm + 5 cm = 10 cm [From (i)]

Perimeter of Δ ABC = 10 cm

31 In the given figure, TAS is a tangent to the circle, with centre O, at the point A. If ∠ OBA = 32°, find the value of *x*.



Given: TAS is tangent to the circle with centre O at point A.



∠ OBA = 32°

To find: *x*

Sol. In Δ OAB , OA = OB [Radii of the same circle]

⇒ ∠ 1 = 32° [Angle opposite to equal sides of a triangle are equal]

In ΔOAB,

∠ 1 + ∠ 2 + 32° = 180° [Angle sum property of a triangle]

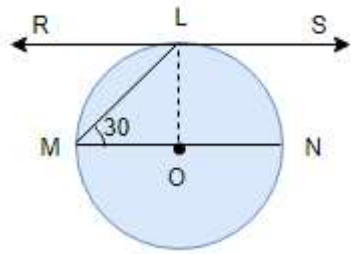
⇒ ∠ 2 + 32° + 32° = 180° ⇒ ∠ 2 = 180° – 64° = 116°

∠ 3 = $\frac{1}{2}$ ∠ 2 = $\frac{1}{2}$ × 116° = 58° [Degree measure theorem]

TAS is tangent to the circle (Given)

x = ∠ ACB = 58° [Angles in the alternate segments are equal]

32 In the given figure, RS is the tangent to the circle at L and MN is a diameter. If ∠ NML = 30°, determine ∠ RLM.



Join OL

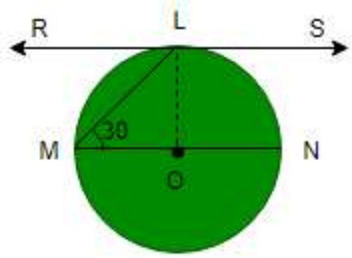
OL ⊥ RS.

Also OL = OM [Radii of the same circle]

∠ OML = ∠ OLM

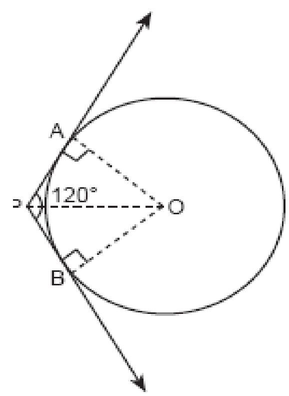
⇒ ∠ OLM = 30°

⇒ ∠ RLM = 90° – 30° = 60°



33 Two tangents PA and PB are drawn to the circle with centre O, such that APB = 120°. Prove that OP = 2AP.

Given. A circle C(O, r). PA and PB are tangents to the circle from point P, outside the circle such that APB = 120°. OP is joined.



To Prove. OP = 2AP.

Construction. Join OA and OB.

Proof. Consider Δs PAO and PBO

PA = PB [Tangents to a circle, from a point outside it, are equal.]

OP = OP [Common]

∠ OAP = ∠ OBP = 90°

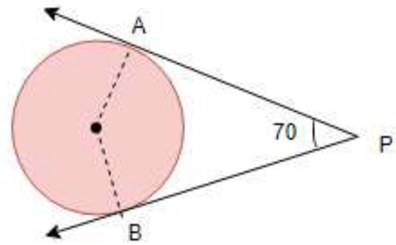
ΔOAP ≅ ΔOBP [RHS]

∠ OPA = ∠ OPB = $\frac{1}{2}$ ∠ APB = $\frac{1}{2}$ × 120° = 60°.

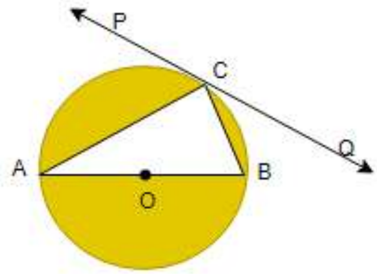
In right angled ΔOAP, $\frac{AP}{OP}$ = cos 60° = $\frac{1}{2}$ ⇒ OP = 2AP.

34 The two tangents from an external point P to a circle with centre O are PA and PB. If ∠ APB = 70°, what is the value of ∠ AOB?

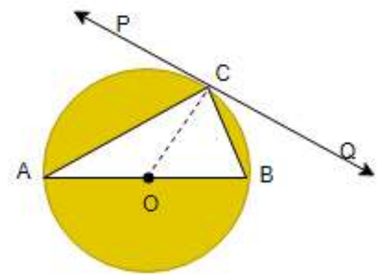
PA and PB are tangents to the circle.
∠ A = ∠ B = 90° [Tangent makes 90° angle with the radius at the point of contact]
In quadrilateral OAPB
∠ AOB + ∠ A + ∠ P + ∠ B = 360° [Angle sum property of a quadrilateral].
⇒ ∠ AOB + 90° + 70° + 90° = 360°
⇒ ∠ AOB + 250° = 360°
⇒ ∠ AOB = 360° – 250° = 110°



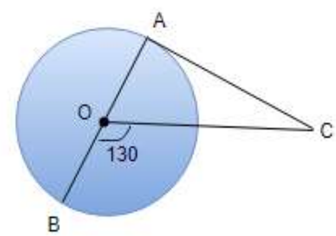
35 In figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and ∠ CAB = 30°, find ∠ PCA.



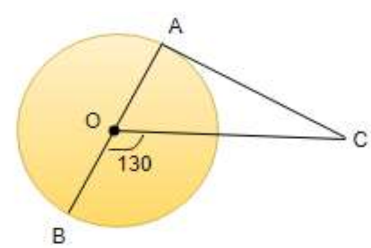
In Δ AOC,
AO = CO [Radii]
∠ OCA = ∠ CAB = 30°
OC ⊥ PQ, ⇒ ∠ OCP = 90°
⇒ ∠ PCA + ∠ OCA = 90°
⇒ ∠ PCA + 30° = 90° ⇒ ∠ PCA = 60°



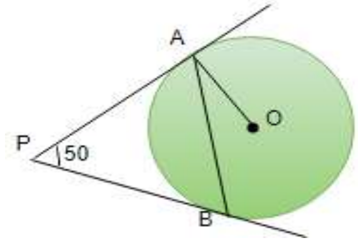
36 In figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If ∠ BOC = 130°, then find ∠ ACO



∠ AOC = 180° – 130° = 50°
∠ OAC = 90° [OA ⊥ AC]
In Δ OAC,
∠ OAC + ∠ AOC + ∠ ACO = 180°
90° + 50° + ∠ ACO = 180°
⇒ ∠ ACO = 40°



37 In figure, PA and PB are tangents to the circle with centre O such that ∠ APB = 50°. Write the measure of ∠ OAB



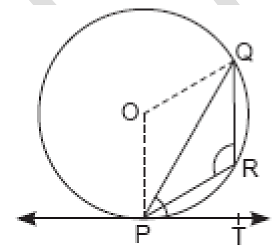
ANS: In Δ APB, ∠ BAP = ∠ ABP (angle opp. to equal side)

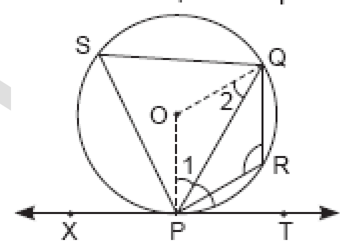
∠ BAP = $\frac{1}{2}$ (180° – APB) = $\frac{1}{2}$ (130°) = 65°

∠ OAP = 90°

∠ OAB = 90° – ∠ BAP = 90° – 65° = 25°

38 In figure, PQ is a chord of a circle with centre O and PT is a tangent. If ∠ QPT = 60°, find ∠ PRQ





∠ OPQ + ∠ QPT = 90°

∠ OPQ = 90° – ∠ QPT = 90° – 60° = 30°

In Δ OPQ,

∠POQ + ∠1 + ∠2 = 180°

∠POQ + 2 ∠1 = 180° (∠1 = ∠2)

∠POQ + 2(30°) = 180°

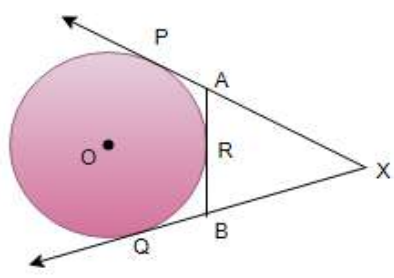
∠POQ = 120°

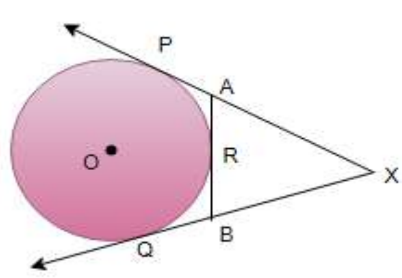
∠PSQ = $\frac{1}{2}$ POQ = $\frac{1}{2}$ × 120° = 60°

PSQ + PRQ = 180° (sum of opp. angles of cyclic quadrilateral)

∠ PRQ = 180° – ∠PRQ = 180° – 60° = 120°

39 In figure, XP and XQ are two tangents to a circle with centre O from a point X outside the circle. ARB is tangent to circle at R. Prove that XA + AR = XB + BR.





Given: XP, XQ and ARB are three tangents.

To prove: XA + AR = XB + BR

Proof: AP = AR ... (i)

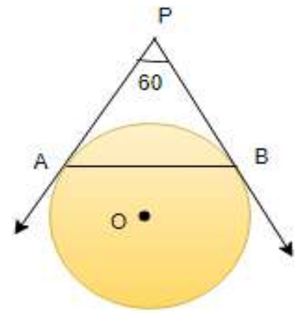
and BQ = BR ... (ii)

Also XQ = XP [Tangents drawn from an external point]

XA + AP = XB + BQ

XA + AR = XB + BR [From (i) and (ii)]

40 In figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and ∠ APB = 60°. Find the length of chord AB.



ANS: AP = BP [tangents from external point P]

∠ PAB = ∠PBA [Angles opposite to equal sides]

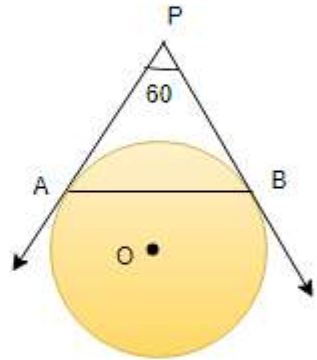
Now ∠ APB + ∠ PAB + ∠ PBA = 180°

60° + 2 ∠ PAB = 180°

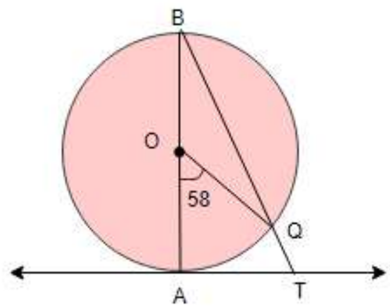
∠ PAB = 60°

Δ APB is an equilateral Δ

AB = AP = 5 cm



41 In figure, AB is the diameter of a circle with centre O and AT is a tangent. If ∠ AOQ = 58°, find ∠ ATQ.



∠ ABQ = $\frac{1}{2}$ × 58° = 29°

In Δ ABT,

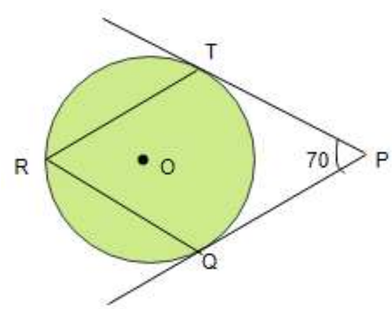
∠ BAT + ∠ ABT + ∠ ATB = 180°

90° + 29° + ∠ ATB = 180°

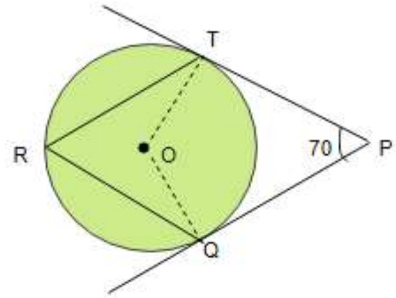
∠ ATB = 61°

as ∠ ATB = ∠ ATQ ⇒ ∠ATQ = 61°

42 In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠ TPQ = 70°, find ∠ TRQ

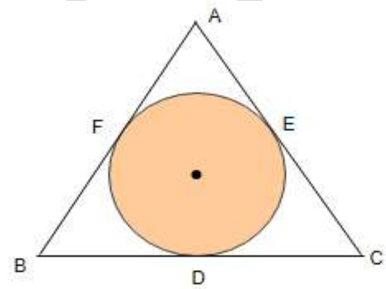


$\angle TOQ + \angle TPQ = 180°$
$\Rightarrow \angle TOQ = 110°$
Also $\angle TOQ = 2 \angle TRQ$
[angle subtended by an arc at centre of the circle is twice the angle subtended by it in alternate segment]
$\Rightarrow 110° = 2\angle TRQ \Rightarrow \angle TRQ = 55°$



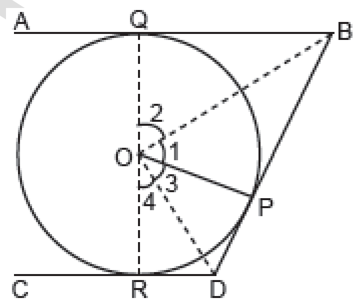
43 ABC is an isosceles triangle, in which AB = AC, circumscribed about a circle. Show that BC is bisected at the point of contact

Here, AB = AC (Given) ...(*i*)
AF = AE (Tangents from A) ...(*ii*)
AB – AF = AC – AE
$\Rightarrow$ BF = CE ...(*iii*)
Now, BF = BD (Tangents from B)
Also, CE = CD (Tangents from C)
$\Rightarrow$ BD = CD

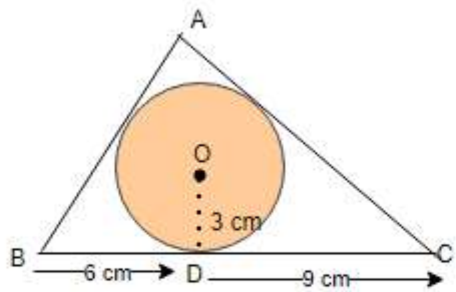


44 Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre

Given. AB and CD are two tangents to a circle and AB || CD.
Tangent BD intercepts an angle BOD at the centre.
To Prove. BOD = 90°.
Construction. Join OQ, OB, OD and OR.
Proof. OP ⊥ BD.
[A tangent at any point of a circle is perpendicular to the radius through the point of contact.]
In right angled ∆s OQB and OPB,
$\angle 1 = \angle 2$,
Similarly in right angled ∆s OPD and ORD
$\angle 3 = \angle 4$
$\angle BOD = \angle 1 + \angle 3 = \frac{1}{2}$ $[2\angle 1 + 2\angle 3)] = \frac{1}{2}$ $(\angle 1 + \angle 1 + \angle 3 + \angle 3)$
$= \frac{1}{2} (\angle 1 + \angle 2 + \angle 3 + \angle 4) = \frac{1}{2}$ $(180°) = 90°$.



45 In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ∆ ABC is 54 cm^2, then find the lengths of sides AB and AC.



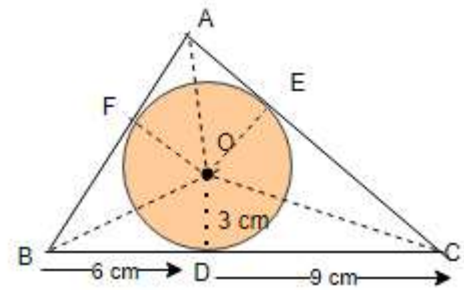
Let AF = x cm

AF = AE = x [tangents from A]

Also BD = BF = 6 cm and CD = CE = 9 cm

AB = $(6 + x)$ cm and AC = $(9 + x)$ cm

Area ∆ ABC = Area ∆BOC + Area ∆COA + Area ∆AOB



$\Rightarrow\ 54 = \frac{1}{2}$ BC × OD + $\frac{1}{2}$ AC × OE + $\frac{1}{2}$ AB × OF

$\Rightarrow\ 54 \times 2 = 15 \times 3 + (6 + x) \times 3 + (9 + x) \times 3$

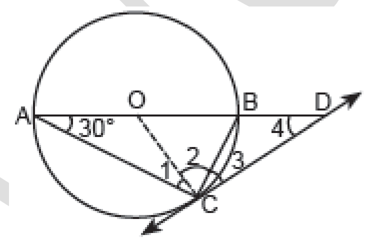
$108 = 45 + 18 + 3x + 27 + 3x$

$6x = 18 \Rightarrow x = 3$

$\Rightarrow$ AB = $6 + x = 6 + 3 = 9$ cm and AC = $9 + x = 9 + 3$ = 12 cm

46 AB is diameter and AC is a chord of a circle such that BAC = 30°. If tangent at C intersects AB produced in D, prove that BC = BD.

Join OC



OC ⊥ CD ∴ ∠2 + ∠3 = 90°

OC = OA ∴ ∠1 = 30°

Now ∠1 + ∠2 = 90° [Angles in a semicircle]

∴ ∠2 = 90° – 30° = 60°

⇒ ∠3 = 30°

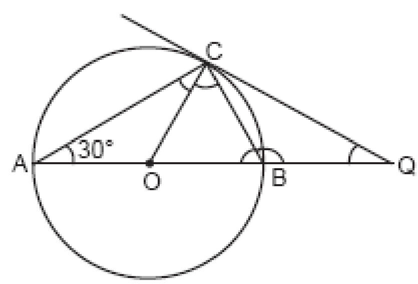
In ∆ACD, ∠ ACD + ∠CAD + ∠ 4 = 180°

⇒ 120° + 30° + ∠ 4 = 180° ⇒ ∠ 4 = 30°

In ∆BCD, ∠ 3 = ∠ 4

∴ BC = BD.

47 In the figure, AB is diameter of a circle with centre O and QC is a tangent to the circle at C. If ∠CAB = 30°, find ∠CQA and ∠ CBA.



ANS: In ∆ AOC, OA = OC [Radii of the same circle]

∠ACO = ∠CAO = 30° [Opp. angles of equal sides are equal.]

Also ∠ ACB = 90° [Angle in semicircle]

∠OCB = 90° – 30° = 60°

In ∆ COB, OC = OB [Radii of the semicircle]

∠ OCB = ∠ OBC = 60° [Opposite angles of equal sides]

Now OC ⊥ CQ

∠ OCQ = 90° ⇒ ∠ BCQ = 90° – 60° = 30°

Also ∠ OBC + ∠ CBQ = 180°

⇒60° + ∠ CBQ = 180° ⇒ ∠ CBQ = 120°

In ∆ CBQ,

$\angle BCQ + \angle CBQ + \angle CQB = 180°$

$\Rightarrow 30° + 120° + \angle CQB = 180° \Rightarrow \angle CQB = 30°$

$\angle CQA = 30°$,

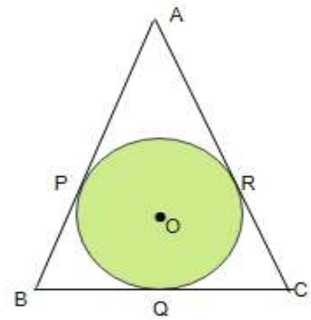
Now $\angle CBA = 180° - \angle CBQ = 180° - 120° = 60°$.

48 In figure, the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P, Q and R respectively.

Prove that

(i) AB + CQ = AC + BQ

(ii) Area (Δ ABC) = $\frac{1}{2}$ (perimeter of Δ ABC) × r



i) AP = AR [Tangents from A] ...(i)

Similarly, BP = BQ ...(ii)

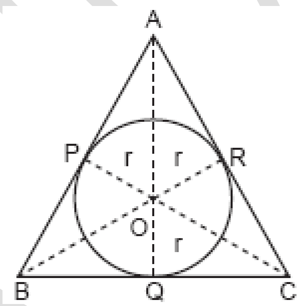
CR = CQ ...(iii)

Now, AP = AR

⇒ (AB – BP) = (AC – CR)

⇒AB + CR = AC + BP

⇒AB + CQ = AC + BQ [Using eq. (ii) and (iii)]



(ii) Let AB = x, BC = y, AC = z

Perimeter of Δ ABC = $x + y + z$ ...(iv)

Area of Δ ABC = $\frac{1}{2}$ [area of ΔAOB + area of ΔBOC + area of ΔAOC]

⇒Area of Δ ABC = $\frac{1}{2}$ AB × OP + $\frac{1}{2}$ × BC × OQ + $\frac{1}{2}$ × AC × OR

Area of Δ ABC = $\frac{1}{2}\, x \times r + \frac{1}{2}\, y \times r + \frac{1}{2}\, z \times r$

⇒Area of Δ ABC = $\frac{1}{2}\,(x + y + z) \times r$

⇒ Area of Δ ABC = $\frac{1}{2}$ (Perimeter of Δ ABC) × r [Using (iv)]

49 PQR is a right angled triangle right angled at Q. PQ = 5 cm, QR = 12 cm. A circle with centre O is inscribed in ΔPQR, touching its all sides. Find the radius of the circle.

Let QS = x; SR = $12 - x$

PT = $5 - x$; PM = PT

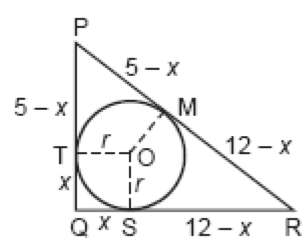
PM = $5 - x$

Also SR = MR ⇒ MR = $12 - x$

Also $PQ^2 + QR^2 = PR^2$

⇒ PR = 13 ⇒PM + MR = 13

⇒ $5 - x + 12 - x = 13 \Rightarrow 2x = 4 \Rightarrow x = 2$

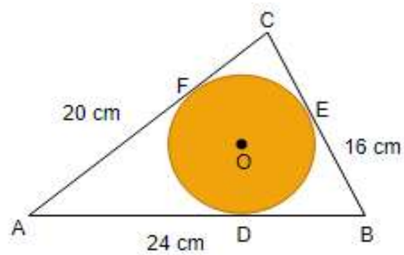


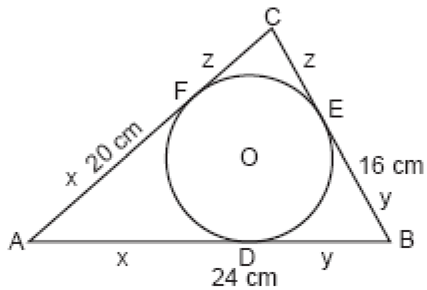
Also OSQT is a square

∴ OS = QS ⇒ OS = 2 cm

∴ Radius of incircle = 2 cm.

50 A circle is inscribed in a ΔABC having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF.





Let AD = AF = x [Tangents from external point are equal]

BD = BE = y and CE = CF = z

According to the question,

AB = $x + y$ = 24 cm ... (i)

BC = $y + z$ = 16 cm ... (ii)

AC = $x + z$ = 20 cm ...(iii)

Subtracting (iii) from (i), we get $y - z = 4$ ... (iv)

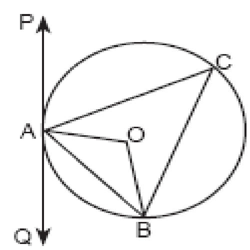
Adding (ii) and (iv), we get

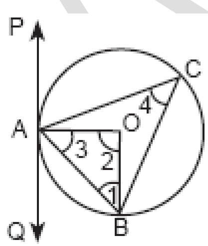
$2y = 20 \Rightarrow y = 10$ cm.

Substituting the value of y in (ii) and (i) we get $z = 6$ cm; $x = 14$ cm

AD = 14 cm, BE = 10 cm and CF = 6 cm.

51 PAQ is a tangent to the circle with centre O at a point A as shown in figure. If ∠ OBA = 35°, find the value of ∠ BAQ and ∠ ACB.





Given: PAQ is a tangent to the circle with centre O at a point A as shown in figure OBA = 35°.

To find: BAQ and ACB

Proof: OA = OB [Radii of the same circle]

⇒∠ 3 = 35° [Angles opposite to equal sides of a triangle are equal]

But, ∠ 1 + ∠ 2 + ∠ 3 = 180° [Angle sum property]

⇒ 35° + 35° + ∠ 2 = 180° ⇒ ∠ 2 = 180° – 70° = 110°

$4 = \frac{1}{2} \angle 2 = \frac{1}{2} \times 110° = 55°$

$\Rightarrow \angle ACB = 55°$ [Degree measure theorem]

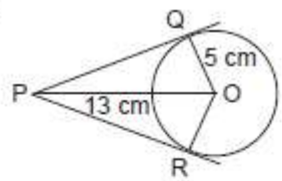
$\angle BAQ = \angle ACB = 55°$ [Angles in the same segment]

52 From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is

(a) 60 cm^2 (b) 65 cm^2 (c) 30 cm^2 (d) 32.5 cm^2

(*a*) Here, $PQ = \sqrt{OP^2 - OQ^2}$

$= \sqrt{13^2 - 5^2} = 12$ cm



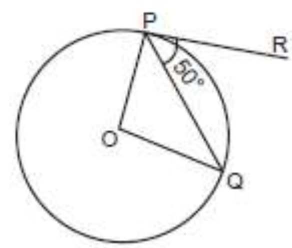
Area of quadrilateral PQOR

= ar. of ΔPOQ + ar. of ΔPOR

$= \frac{1}{2} \times 12 \times 5 + \frac{1}{2} \times 12 \times 5 = 30 + 30 = 60\ cm^2$.

53 In figure if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then $\angle POQ$ is equal to

a) 100° (b) 80° (c) 90° (d) 75°



ANS: (a) OP ⊥ PR [∵ Tangent and radius are ⊥ to each other at the point of contact]

$\angle OPQ = 90^o - 50^o = 40^o$

OP = OQ [Radii]

$\therefore \angle OPQ = \angle OQP = 40^o$

In Δ OPQ, $\Rightarrow \angle POQ + \angle OPQ + \angle OQP = 180^o$

$\Rightarrow \angle POQ + 40^o + 40^o = 180^o$ $\angle POQ = 180° - 80° = 100^o$.

54 Two concentric circles are of radii 13 cm and 5 cm. The length of the chord of larger circle which touches the smaller circle is _____________.

∵ AB touches the smaller circle

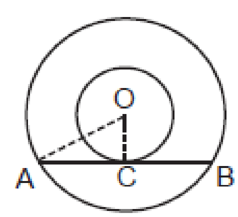
∴ OC ⊥ AB and hence AC = BC

In right ΔOCA, $OA^2 = OC^2 + AC^2$

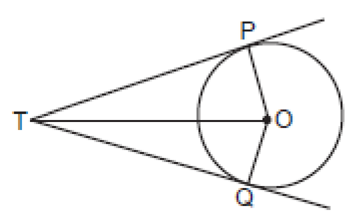
$\Rightarrow AC^2 = 13^2 - 5^2$

$\Rightarrow AC = 12$

$\therefore AB = 2 \times 12 = 24$ cm.



55 In the given figure, TP and TQ are two tangents to a circle with centre O, such that POQ = 110°. Then PTQ is equal to (a) 55° (b) 70° (c) 110° (d) 90°



ANS: (b) In quadrilateral POQT,

PTQ + TPO + TQO + POQ = 360°

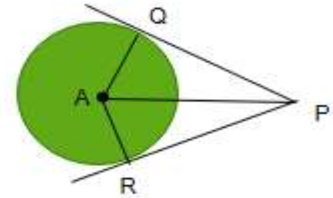
⇒ PTQ + 90° + 90° + 110° = 360°

⇒ PTQ + 290° = 360°

⇒ PTQ = 360° – 290° = 70°

56 In figure, PQ and PR are tangents to a circle with centre A. If ∠ QPA = 27°, then ∠ QAR equals to

_____



(a) 63° (b) 153° (c) 126° (d) 117°

ANS: (c) ∠ QPA = ∠ RPA

[∵ ΔAQP ≅ ΔARP (RHS congruence rule)]

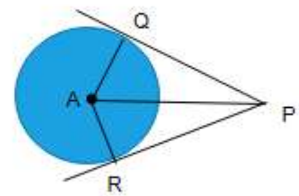
⇒ ∠ RPA = 27°

∴ ∠ QPR = ∠ QPA + ∠ RPA = 27° + 27° = 54° Now,

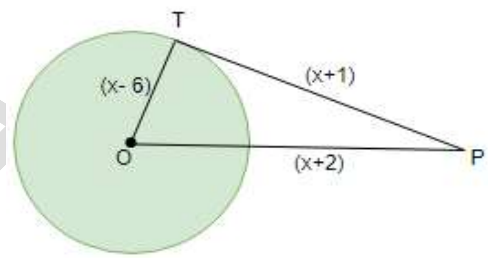
∠ QAR + ∠ AQP + ∠ ARP + ∠ QPR = 360°

⇒ ∠ QAR = 90° + 90° + 54° = 360°

⇒ ∠ QAR = 360° – 234° = 126°



57 In the below figure, find the actual length of sides of Δ OTP.



In the figure, find the value of x.