SAMPLE PAPER - 2025

PERIODIC TEST – 2

Class: XII

Subject : Mathematics (041) M.M : 80
Date : 01– 09 –2025 Time : 3 Hours

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long answer (LA) – type questions carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study – based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in section B, 3 questions in section C, 2 questions in section D and one subpart each in 2 questions of section E.
9. Use of calculators is not allowed.

	SECTION- A (Multiple Choice Questions) Each question carries 1 mark	
1.	If a matrix $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ $and\ \|A\|^3 = 125$ then the value of $\alpha =$ ________ (A) ± 3 (B) ± 2 (C) ± 5 (D) ± 9	(1)
	ANS: (A) ± 3	
2.	Principal branch of $tan^{-1}x$ is ________. A) $\left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$ (B) $\left(0, \frac{\pi}{2}\right)$ (C) $(0, \pi)$ (D) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$	(1)
	ANS: (D) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$	
3.	If $E(\theta) = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ then $E(\alpha).E(\beta) =$ __________ (A) $(0°)$ (B) $E(\alpha\ \beta)$ (C) $E(\alpha + \beta)$ (D) $E(\alpha - \beta)$	(1)
	ANS: (B) $E(\alpha\ \beta)$	
4.	If $f(x) = x\tan^{-1}x$, then $f'(1) =$ ______ (A) $\frac{1}{2} - \frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{1}{2} + \frac{\pi}{4}$ (D) $\frac{1}{2}$	(1)
	ANS: (C) $\frac{1}{2} + \frac{\pi}{4}$	
5	Let $f(x) = \begin{cases} 3x - 4, & 0 \le x \le 2 \\ 2x + \lambda. & 2 \le x \le 3 \end{cases}$, if f is continuous at $x = 2$ then find the value of λ . (A) -2 (B) 2 (C) 4 (D) 0	(1)
	ANS: (A) $\lambda = -2$	
6.	Find the maximum and minimum values of the function $f(x) = -\|x+1\| + 3$ (A) no maximum , minimum 3 (B) no maximum , minimum 0 (C) maximum 3, minimum -1 (D) maximum 3, no minimum	(1)
	ANS: (D) maximum 3, no minimum	
7.	The side of an equilateral triangle is increasing at the rate of 0.5 cm/s. Find the rate of increase of its perimeter. (A) 1.5 cm/sec (B) 1 cm/sec (C) 0.5 cm/sec (D) 3 cm/sec	(1)

	ANS: (B) $a = \frac{1}{3},\ b = -1$	
8.	Find $\int \frac{(5+3\sqrt{x})^2}{\sqrt{x}} dx$ (A) $\frac{1}{9}(5+3\sqrt{x})^3 + C$ (B) $\frac{1}{3}(5+3\sqrt{x})^3 + C$ (C) $\frac{2}{9}(5+3\sqrt{x})^2 + C$ (D) $\frac{2}{9}(5+3\sqrt{x})^3 + C$	(1)
	ANS: $\frac{2}{9}(5+3\sqrt{x})^3 + C$	
9.	A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate. (A) $(4, 11)$ *and* $\left(-4, \frac{31}{3}\right)$ (B) $(4, -11)$ *and* $(-4, 4)$ (C) $(4, 11)$ *and* $\left(-4, -\frac{31}{3}\right)$ (D) $(4, -11)$ *and* $\left(-4, -\frac{31}{3}\right)$	(1)
	ANS : (C) $(4, 11)$ *and* $\left(-4, -\frac{31}{3}\right)$	
10.	Evaluate : $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{tanx}} dx$ (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 1 (D) -1	(1)
	ANS: (B) $\frac{\pi}{4}$	
11.	If $y = \sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$ find $\frac{dy}{dx}$. (A) $x + \frac{\pi}{4}$ (B) $x - \frac{\pi}{4}$ (C) 1 (D) -1	(1)
	ANS: (C) 1 $y = \sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$ $y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) y = \sin^{-1}\left(cos\frac{\pi}{4}\sin x + sin\frac{\pi}{4}\cos x\right)$ $y = \sin^{-1}\left(\sin(x + \frac{\pi}{4})\right) = x + \frac{\pi}{4}$ $\frac{dy}{dx} = 1$	
12.	The relation "less than" in the set of natural numbers is ______ (A) Only symmetric (B) Only transitive (C) Only reflexive (D) equivalence relation	(1)
	(B) Only transitive	
13.	Evaluate : $\int \frac{sinx+cosx}{\sqrt{1+sin2x}} dx$ (A) $x + C$ (B) $-x + C$ (C) $sinx + C$ (D) 1	(1)
	ANS: (A) $x + C$ $\int \frac{sinx+cosx}{\sqrt{sin^2x+cos^2x+2sinxcosx}} dx = \int \frac{sinx+cosx}{\sqrt{(sinx+cosx)^2}} dx = \int 1\, dx = x + C$	
14.	Let N be the set of natural numbers and relation R on N be defined by $R = \{(x, y): x, y \in N, x + 4y = 10\}$. R is _________. (A) reflexive (B) symmetric (C) not reflexive and not symmetric (D) reflexive but not symmetric	(1)
	ANS: (C) not reflexive and not symmetric R = {(2,2), (6,1)} , R is not reflexive because (1, 1) $\notin$ R. R is not symmetric, because (6,1) $\in$ R but (1, 6) $\notin$R.	

15.	Evaluate: $\int \tan^{-1}\sqrt{\frac{1-cos2x}{1+cos2x}}dx$ (A) $\frac{x^2}{2}+c$ (B) $x+c$ (C) $\tan^{-1}x+c$ (D) $\frac{x}{2}+c$	(1)
	ANS: (A) $\frac{x^2}{2}+c$	
16.	Evaluate: $\int_0^{\frac{\pi}{2}}\frac{dx}{1+sinx}$ (A) 0 (B) 1 (C) 2 (D) -1	(1)
	ANS: (B) 1 $\int_0^{\frac{\pi}{2}}\frac{dx}{1+sinx} = \int_0^{\frac{\pi}{2}}\frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)} = \int_0^{\frac{\pi}{2}}\frac{1}{2}\ sec^2(\frac{\pi}{4}-\frac{x}{2})$ dx. $-tan\left(\frac{\pi}{4}-\frac{\pi}{4}\right)+\ tan\left(\frac{\pi}{4}-0\right)=1$	
17	Evaluate : $\sin\left\{\frac{\pi}{3}-\ \sin^{-1}\left(-\frac{1}{2}\right)\right\}$ (A) 1 (B) -1 (C) -1 (D) $\frac{-1}{4}$	(1)
	ANS: (A) 1	
18	Given a skew – symmetric matrix $\begin{bmatrix}0 & a & 1\\ -1 & b & 1\\ -1 & c & 0\end{bmatrix}$, then the value of $(a+b-c)^2$ is ______. (A) 2 (B) 0 (C) 1 (D) 4	(1)
	ANS: D) 4 $A^T = -A \Rightarrow a=1, b=0,\ c=-1 \Rightarrow (a+b-c)^2=4$	
	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.	
19	Assertion (A): In set $A = \{1,2,3\}$ a relation R defined as R = {(1, 1), (2, 2)} is reflexive. Reason (R) : A relation R is reflexive in set A if (a, a) ∈ R for all a ∈ A	(1)
	ANS. (D) A is false but R is true.	
20	Assertion (A) : The value of determinant of a matrix and the value of determinant of its transpose are equal. Reason (R) : The value of determinant remains unchanged if its rows and columns are interchanged.	(1)
	ANS**:** (A) Both A and R are true and R is the correct explanation of A.	
	SECTION - B This section comprises of very short answer type-questions (VSA) of 2 marks each.	
21	Prove that: $tan^{-1}\left(\frac{cosx}{1-sinx}\right) = \frac{\pi}{4}+\frac{x}{2}$	(2)
	ANS: $\tan^{-1}\left(\frac{cosx}{1-sinx}\right) = \tan^{-1}\left(\frac{sin\left(\frac{\pi}{2}-x\right)}{1-cos\left(\frac{\pi}{2}-x\right)}\right)$ $= \tan^{-1}\left(\frac{2sin\left(\frac{\pi}{4}-\frac{x}{2}\right)cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right) = \tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right) = \tan^{-1}\tan\left\{\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}$ $=\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)=\frac{\pi}{4}+\frac{x}{2}$	
22	Evaluate : $\int\sqrt{1+sinx}\,dx$	(2)

	OR Evaluate : $\int sec^4x\,.\,tan\,x\,dx$	
	ANS: $= \int \sqrt{sin^2\frac{x}{2} + cos^2\frac{x}{2} + 2sin\frac{x}{2}cos\frac{x}{2}}\;dx =$ $= \int \sqrt{\left(sin\frac{x}{2} + cos\frac{x}{2}\right)^2}\;dx = \int [sin\frac{x}{2} + cos\frac{x}{2}\,]dx \;\; = -2\,cos\frac{x}{2} + 2sin\frac{x}{2} + C$ **OR** ANS: $\int sec^4x\,.\,tan\,x\,dx$ $\int sec^2x\,.\,tan\,x.sec^2x\,.dx$ $tanx = t\,, sec^2x\,dx = dt\; \int (1 + tan^2x)\,.\,tan\,x.sec^2x\,.dx$ $\int (1 + t^2)\,.\,t.dt = \int (t + t^3)\,.dt$ $\frac{t^2}{2} + \frac{t^4}{4}$ +C = $\frac{tan^2x}{2} + \frac{tan^4x}{4}$ + C	
23	If y = $x^{\cos^{-1}x}$ then find $\frac{dy}{dx}$	(2)
	ANS: y = $x^{\cos^{-1}x}$ $\log y = \cos^{-1}x.\,logx$ $\frac{1}{y}\frac{dy}{dx} = \cos^{-1}x\,.\,\frac{1}{x} + logx\frac{-1}{\sqrt{1-x^2}}.$ $\frac{dy}{dx} = y\left(\cos^{-1}x\,.\,\frac{1}{x} + logx\frac{-1}{\sqrt{1-x^2}}.\right)$ $= x^{\cos^{-1}x}\left(\frac{1}{x}\cos^{-1}x - \frac{logx}{\sqrt{1-x^2}}\right)$	
24	Show that the function $\tan^{-1}(cosx + sinx)$ is strictly increasing on $\left(0,\,\frac{\pi}{4}\right)$. **OR** Show that the function $f(x) = log\,sinx$ is strictly increasing on $\left(0,\,\frac{\pi}{2}\right)$ strictly decreasing on $\left(\frac{\pi}{2},\,\pi\right)$.	(2)
	ANS: y = $\tan^{-1}(cosx + sinx)$ $\frac{dy}{dx} = \frac{1}{1+(cosx+sinx)^2}(-sinx + cosx)$ $= \frac{-sinx+cosx}{1+(cosx+sinx)^2}$ $1 + (cosx + sinx)^2 > 0$ $\frac{dy}{dx} = \frac{-sinx+cosx}{1+(cosx+sinx)^2} > 0 \Rightarrow cosx - sinx > 0$ Since $x \in \left(0,\,\frac{\pi}{4}\right)$ $cosx > sinx$ $\Rightarrow \tan^{-1}(cosx + sinx)$ is strictly increasing on $\left(0,\,\frac{\pi}{4}\right)$. **OR** ANS : f($x$) = log $sinx$ f'(x) = cot x x $\in \left(0,\,\frac{\pi}{2}\right)$, cot x > 0, f'(x) > 0 f(x) is increasing on $\left(0,\,\frac{\pi}{2}\right)$ x $\in \left(\frac{\pi}{2},\,\pi\right)$, cot x < 0, f'(x) < 0 f(x) is decreasing on $\left(\frac{\pi}{2},\,\pi\right)$	
25	Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.	(2)
	ANS: Let A = {1, 2, 3, 4, 5, 6}.	

A relation R is defined on set A as:
$R = \{(a, b): b = a + 1\}$
$R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$
We observe, $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \notin R$
We can say $(a, a) \notin R$, where $a \in A$.
R is not reflexive.
It can be observed that $(2, 3) \in R$, but $(3, 2) \notin R$.
R is not symmetric.
Now, $(2, 3), (3, 4) \in R$ but, $(2, 4) \notin R$
As $(x,y) \in R, (y, z) \in R \nRightarrow (x, z) \in R$
R is not transitive
We observe, R is neither reflexive, nor symmetric, nor transitive.

SECTION - C

This section comprises of short answer type-questions (SA) of 3 marks each.

26 If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k so that $A^2 = 8A + kI$. (3)

OR

Find equation of line joining $(1, 2)$ and $(3, 6)$ using determinants.

ANS: $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$A^2 = 8A + kI$.

$\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56 \end{bmatrix}$

$1 = 8 + k \Rightarrow k = -7$

OR

Let point (x, y) lies on line joining the points $(1, 2)$ and $(3, 6)$.
points (x, y), $(1, 2)$ and $(3, 6)$ are collinear

$\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$

$x(2 - 6) - y(1 - 3) + 1(6 - 6) = 0$
$-4x + 2y = 0$ $2x - y = 0$ is the required equation.

27 Evaluate : $\int \frac{sinx+cosx}{9+16\,sin2x} dx$ (3)

Ans: $\int \frac{sinx+cosx}{9+16\,sin2x} dx$:

$I = \int \frac{sinx+cosx}{9+16\,sin2x} dx = \int \frac{sinx+cosx}{25-16+16\,sin2x} dx$

$\int \frac{sinx+cosx}{25-16\,(1-sin2x)} dx = \int \frac{sinx+cosx}{25-16\,(cosx-sinx)^2} dx$

$cosx - sinx = t$, $(sinx + cosx\,)dx = -dt$

$I == \int \frac{1}{25-16\,t^2} dt = \int \frac{1}{5^2-(4t)^2} dt$

$\frac{1}{40} \log\left|\frac{5+4(cosx-sinx\,)}{5-4(cosx-sinx\,)} + C\right|$

28 If $A = \begin{bmatrix} cos\alpha & -sin\alpha & 0 \\ sin\alpha & cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, verify that $A\,(adjA) = |A|I$. (3)

ANS: $A = \begin{bmatrix} cos\alpha & -sin\alpha & 0 \\ sin\alpha & cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{vmatrix} cos\alpha & -sin\alpha & 0 \\ sin\alpha & cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$

	$adjA = \begin{bmatrix} cos\alpha & sin\alpha & 0 \\ -sin\alpha & cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $A\,(adjA) = \begin{bmatrix} cos\alpha & -sin\alpha & 0 \\ sin\alpha & cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\alpha & sin\alpha & 0 \\ -sin\alpha & cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \|A\|I.$	
29	If $x = ae^{\theta}(sin\theta - cos\theta) and\ y = ae^{\theta}(sin\theta + cos\theta)$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ **OR** If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+logx)^2}{logy}$	(3)
	ANS: $x = ae^{\theta}(sin\theta - cos\theta) \implies \frac{dx}{d\theta} = 2ae^{\theta}(sin\theta)$ $y = ae^{\theta}(sin\theta + cos\theta) \implies \frac{dy}{d\theta} = 2ae^{\theta}(cos\theta)$ $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = cot\theta$ at $\theta = \frac{\pi}{4}$ $\frac{dy}{dx} = 1$ **OR** ANS: Consider, $y^x = e^{y-x}$ Taking log on both sides, we get $x \log y = (y - x)\ loge$ $x \log y = y - x$ $x(1 + logy) = y \Rightarrow x = \frac{y}{1+logy}$ *differentiate w:r:t* x $\frac{dx}{dy} = \frac{(1+logy).1-y.\frac{1}{y}}{(1+logy)^2} = \frac{logy}{(1+logy)^2}$ $\frac{dy}{dx} = \frac{(1+logx)^2}{logy}$	
30	Evaluate : $\int \frac{5x}{(x+1)(x^2+9)} dx$ **OR** Evaluate using properties of integration: $\int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1} + \sqrt[3]{11-x}} dx$	(3)
	ANS: $I = 5\int \frac{x}{(x+1)(x^2+9)} dx$ $\frac{x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$ $x = A(x^2 + 9) + (Bx + C)(x + 1)$ $Put\ x = -1 \Rightarrow -1 = 10A \Rightarrow A = -\frac{1}{10}$ $Put\ x = 0 \Rightarrow 0 = 9A + C \Rightarrow = \frac{9}{10}$ $Compare\ coefficient\ of\ x^2 \Rightarrow 0 = A + B \Rightarrow B = \frac{1}{10}$ $5\int \frac{x}{(x+1)(x^2+9)} dx = 5\int \left[\frac{-\frac{1}{10}}{x+1} + \frac{\frac{1}{10}x + \frac{9}{10}}{x^2+9} \right] dx$ $-\frac{1}{2}\log(x+1) + \frac{1}{4}\ log(x^2+1) + \frac{9}{2} \times \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$ $-\frac{1}{2}\log(x+1) + \frac{1}{4}\ log(x^2+1) + \frac{3}{2} \tan^{-1}\left(\frac{x}{3}\right) + C$ **OR** $I = \int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1} + \sqrt[3]{11-x}} dx$ $I = \int_2^8 \frac{\sqrt[3]{10-x+1}}{\sqrt[3]{10-x+1} + \sqrt[3]{11-10+x}} dx$ Using the property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$	

	$I = \int_2^8 \frac{\sqrt[3]{11-x}}{\sqrt[3]{11-x} + \sqrt[3]{1+x}}\, dx$ $2I = \int_2^8 1\, dx \quad = [x]_2^8 = \quad 6$ $I = 3$	
31	Let N be the set of all natural numbers and let R be a relation on $\mathrm{N} \times \mathrm{N}$ defined by $(\mathrm{a,b})\mathrm{R}(\mathrm{c,d}) \;\Rightarrow\; \mathrm{ad} = \mathrm{bc}$ $\;f$or all $(\mathrm{a,b}),(\mathrm{c,d}) \in \mathrm{N}\times\mathrm{N}$. Show that R is an equivalence relation on $\mathrm{N}\times\mathrm{N}$.	
	ANS: Relation R is defined by $(\mathrm{a,b})\mathrm{R}(\mathrm{c,d}) \;\Rightarrow\; \mathrm{ad} = \mathrm{bc}$ forall $(\mathrm{a,b}),(\mathrm{c,d}) \in \mathrm{N}\times\mathrm{N}$. For reflexive: $(\mathrm{a,b})\mathrm{R}(\mathrm{a,b})\mathrm{ab} = \mathrm{ba}$, which is true in N. Hence, reflexive. For symmetric: $(\mathrm{a,b})\mathrm{R}(\mathrm{c,d})$ $\mathrm{ad} = \mathrm{bc}$ $\mathrm{cb} = \mathrm{da}$ $(\mathrm{c,d})\mathrm{R}(\mathrm{a,b})$.Hence, symmetric. For transitive: $Consider(\mathrm{a,b})\mathrm{R}(\mathrm{c,d}) and (\mathrm{c,d})\mathrm{R}(\mathrm{e,f})$ $ad = bc$ and $cf = de$ $\mathrm{ad}\cdot\mathrm{cf} = \mathrm{bc}\cdot\mathrm{de}$ $\mathrm{af} = \mathrm{be}$ $(\mathrm{a,b})\mathrm{R}(\mathrm{e,f})$.Hence, transitive. Since relation R is reflexive , symmetric and transitive. Hence , relation R is an equivalence relation.	(3)
	SECTION- D This section comprises of Long Answer (LA) - type questions of 5 marks each	
32	If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations: $2x + y - 3z = 13\,;\; 3x + 2y + z = 4\,;\; x + 2y - z = 8$	(5)
	ANS: $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, $\|A\| = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{vmatrix} = -16$ $adjA = \begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}$, $A^{-1} = -\frac{1}{16}\begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}$ Matrix equation is $\begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ -3 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$ $A^T.X = B \Rightarrow \; X = (A^{-1})^T B$ $X = \left\{-\frac{1}{16}\begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}\right\}'.\begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$ $X = \left\{-\frac{1}{16}\begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}\right\}'.\begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$ $X = -\frac{1}{16}\begin{bmatrix} -52-20+56 \\ 52+4-88 \\ 52-12+8 \end{bmatrix} = -\frac{1}{16}\begin{bmatrix} -16 \\ -32 \\ 48 \end{bmatrix}$ $x = 1, y = 2 \; and \; z = -3$	
33.	If $x = sint,\; y = sinpt$ prove that $(1-x^2)\frac{d^2y}{dx^2} - x\,\frac{dy}{dx} + p^2 y = 0$. OR If $x^{16}y^9 = (x^2+y)^{17}$ Show that $\frac{dy}{dx} = \frac{2y}{x}$.	
	ANS: $x = sint$ $\frac{dx}{dt} = cost$, $\frac{dy}{dt} = p\, cospt$ $\frac{dy}{dx} = \frac{p\, cospt}{cost}$ $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx}$ $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{p\, cospt}{cost}\right) \times \frac{1}{cost}$	

	$= \frac{-p^2 sinpt\ cost + p\ cospt.sint}{cos^2 t} \times \frac{1}{cost}$ $cos^2 t \left(\frac{d^2y}{dx^2}\right) = sint\ \frac{p\ cospt}{cost} - p^2 sinpt$ $(1 - \ sin^2 t)\frac{d^2y}{dx^2} - x\ \frac{dy}{dx}\ p^2 y = 0$ **OR** $x^{16}y^9 = (x^2+y)^{17}$ Taking log on both sides $\log(x^{16}y^9) = log(x^2+y)^{17}$ $16\ logx + 9\log y = 17\log(x^2+y)$ Differentiate, $\Rightarrow\ \frac{16}{x} + \frac{9}{y} \times \frac{dy}{dx} = 17\ \times \frac{1}{x^2+y} \times \left(2x + \frac{dy}{dx}\right)$ $\Rightarrow\ \left(\frac{9}{y} - \frac{17}{x^2+y}\right)\frac{dy}{dx} = \frac{34x}{x^2+y} - \frac{16}{x}$ $\Rightarrow\ \left(\frac{9x^2+9y-17y}{y(x^2+y)}\right)\frac{dy}{dx} = \frac{34x^2-16x^2-16y}{x(x^2+y)}$ $\frac{dy}{dx} = \frac{18x^2-16y}{x(x^2+y)} \times \frac{y(x^2+y)}{9x^2-8y} = \frac{2(9x^2-8y).y}{x(9x^2-8y)}$ $\frac{dy}{dx} = \frac{2y}{x}$	
34	Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix	
	ANS: $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ $B^T = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$ $P = \frac{1}{2}\ (B + B^T) = \frac{1}{2}\left\{\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}\right\} = \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$ $Q = \frac{1}{2}\ (B - B^T) = \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$ $B = P + Q = \begin{bmatrix} 2 & -\frac{3}{2} & -\frac{3}{2} \\ -\frac{3}{2} & 3 & 1 \\ -\frac{3}{2} & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$	
35.	Evaluate : $\int_2^5 \{\ \|x-2\| + \|x-3\| + \|x-5\|\ \} dx.$ **OR** Evaluate : $\int \frac{\sqrt{x^2+1}\,[log(x^2+1) - 2\ logx]}{x^4}\ dx$	
	ANS: $\int_2^5 \{\|x-2\| + \|x-3\| + \|x-5\|\ \}\ dx$ $= \int_2^5 (x-2)\ dx + \int_2^3 (3-x)\ dx + \int_3^5 (x-3)\ dx + \int_2^5 (5-x)\ dx$ $\left[\frac{x^2}{2} - 2x\right]_2^5 + \left[3x - \frac{x^2}{2}\right]_2^3 + \left[\frac{x^2}{2} - 3x\right]_3^5 + \left[5x - \frac{x^2}{2}\right]_2^5$ Simplify $= \left(\frac{9}{2} - 0\right) - \left(0 - \frac{1}{2}\right) + (2-0) - \left(0 - \frac{9}{2}\right) = \frac{23}{2}$ **OR**	

ANS: $I = \int \frac{\sqrt{x^2+1}\,[log(x^2+1)-2\,logx]}{x^4}\,dx$

$I = \int \frac{\sqrt{x^2+1}\left(log\left(\frac{x^2+1}{x^2}\right)\right)}{x^4}\,dx$

$I = \int x\,\frac{\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)}{x^4}\,dx$

$Put\ 1+\frac{1}{x^2} = t \qquad -\frac{2}{x^3}\,dx = \ dt$

$$I = \int \frac{\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)}{x^3}\,dx \qquad = \int -\frac{1}{2}\sqrt{t}\log t\,dt$$

$$= -\frac{1}{2}\left[\frac{2}{3}logt\ t^{3/2} - \frac{2}{3}\int t^{3/2}\frac{1}{t}\,dt\right]$$

$$I = -\frac{1}{3}t^{\frac{3}{2}}\,logt - \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

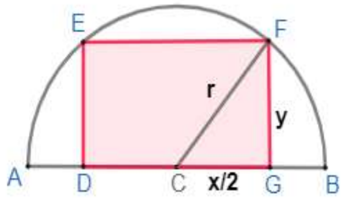
$$= -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + C$$

SECTION –E

36. A rectangle is inscribed in a semi- circle of radius r with one of its sides on the diameter of the semi- circle. Using the concept of maxima and minima, we need to find the dimensions of the rectangle, so that its area is maximum.
Use the figure to answer the following.
i) Find the area of rectangle A in terms of r and x. (1)
ii) The value of x in terms of r =_____. (1)
iii) Find the length and breadth of the rectangle (x and y) in terms of r. (2)

OR

iii) Maximum area = _______



ANS:

Let x and y be the sides of the rectangle

$\left(\frac{x}{2}\right)^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - \frac{x^2}{4}$

Area of rectangle A = xy $\qquad A^2 = x^2y^2$

Let Z = $A^2 = x^2y^2$

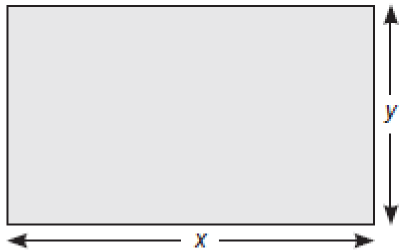
Z = $x^2(r^2 - \frac{x^2}{4})$ $\qquad \Rightarrow \qquad$ Z $= x^2r^2 - \frac{x^4}{4}$

$\frac{dZ}{dx} = 2r^2x - x^3. \qquad \frac{dZ}{dx} = 0 \Rightarrow \quad 2r^2x - x^3 = 0$

$\Rightarrow$ x$(2r^2 - x^2) = 0, \quad x \neq 0\,, x^2 = 2r^2 \Rightarrow x = \sqrt{2}\,r \ , y = \frac{r}{\sqrt{2}}$

$\frac{d^2Z}{dx^2}\ at\ x = \sqrt{2}\,r < 0$

Maximum area = A = $xy = \sqrt{2}\,r\sqrt{\left(r^2 - \frac{x^2}{4}\right)} = r^2$

37.	Arav wants to donate a rectangular plot of land for a school in her village. When she was asked to give dimensions of the plot, she told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by $5300\ m^2$. Based on the information given above, answer the following questions: 	
	i)) The equations in terms of $x\ and\ y$ are ______& _______. ii) Which of the following matrix equation is represented by the given information? (A) $\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$ (B) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-50\\-550\end{bmatrix}$ (C) $\begin{bmatrix}1 & 1\\2 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$ (D) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$ iii) The value of x (length of rectangular field) is ______. **OR** iii) How much is the area of rectangular field?	(1) (1) (2)
	ANS: i) x – y = 50, 2x + y = 550 ii) (A) $\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$ iii) 200 m **OR** 30000 Sq m	
38.	Mansi visited one Exhibition along with her family. The Exhibition had a huge swing, which attracted many children. Mansi found that the swing traced the path of a Parabola as given by $y = x^2$.	
	 Answer the following questions using the above information. i) Let $f : R \to R$ be defined by $f(x) = x^2$. Check whether f is bijective or not. ii) Let $f : N \to N$ be defined by $f(x) = x^2$. Show that f one – one.	(2) (2)
	ANS: i) f is not bijective . Neither Surjective nor Injective ii) f : {1, 2, 3, ...} → {1, 4, 9, ...} f is one- one	